EUNOIA JUNIOR COLLEGE

JC2 Preliminary Examination 2024

General Certificate of Education Advanced Level

Higher 2

CANDIDATE NAME

CIVICS GROUP

INDEX NO.

MATHEMATICS

Paper 1

9758/01

09 September 2024

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name, civics group and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** questions.
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You are expected to use an approved graphing calculator.
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You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **22** printed pages and **2** blank page(s).

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total

1 The sum of the digits of a certain three-digit number is 15. The sum of the digit in the tens placing and twice the digit in the hundreds placing is equal to the digit in the units placing. If the digits are reversed, the new number is 21 more than 5 times the old number. What is the number? [4]

2 **Do not use a calculator in answering this question.**

Find the complex numbers z and w which satisfy the following simultaneous equations

$$2z - w = 5\mathrm{i},$$

$$3\mathrm{i}z - w^* = -5.$$

Give your answers in the form $a + b\mathrm{i}$, where a and b are real numbers. [5]

3 The curve with equation $y = p - \frac{1}{x+q}$, where p and q are constants, is transformed by a reflection in the x-axis, followed by a translation of 3 units in the negative y-direction, followed by a stretch of scale factor 4 parallel to the x-axis. A point on the curve $y = p - \frac{1}{x+q}$ has coordinates $\left(-\frac{3}{4}, -3\right)$.

(a) Find the resulting coordinates of this point after it undergoes the same sequence of transformations. [1]

The resulting curve has asymptotes $x = -1$ and $y = 2$.

(b) Find the values of p and q. [5]

4 A curve C has equation $y = \frac{x^2 + 25x + 114}{x+1}$.

(a) Sketch C, indicating on the diagram the equations of any asymptotes, and the coordinates of any turning points and any axial intercepts. [4]

A curve D has parametric equations

$$x = 23\sin 2\theta - 1,\ y = 23 + 23\cos 2\theta \text{ for } -\frac{\pi}{2} < \theta < \frac{\pi}{2}.$$

(b) Find the cartesian equation of curve D. [2]

(c) Hence, on the same diagram as in part **(a)**, sketch the curve D and determine the number of intersection points between curve C and curve D. [3]

5 **(a)** **(i)** It is given that $2xy - x^2\frac{dy}{dx} = 2x^4 + y^2$. Using the substitution $wy = x^2$, show that the differential equation can be transformed to $\frac{dw}{dx} = 1 + 2w^2$. [3]

(ii) Hence, solve the differential equation $2xy - x^2\frac{dy}{dx} = 2x^4 + y^2$. [3]

(b) It is given that u satisfies $\frac{d^2u}{dx^2} = e^{2x}$ and has a stationary value of $\frac{e^2}{4}$ when $x = 1$. Find u in terms of x. [3]

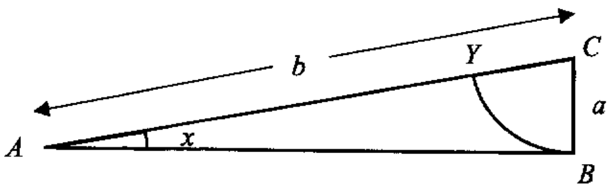
6 **(a)** Show that $\frac{r^2+3r+1}{(r+2)!}$ can be expressed as $\frac{1}{r!} - \frac{1}{(r+2)!}$. [2]

(b) Find an expression in terms of n for $\sum_{r=1}^{n}\frac{r^2+3r+1}{(r+2)!}$. You should not simplify your answer. [3]

(c) Explain why $\sum_{r=1}^{\infty}\frac{r^2+3r+1}{(r+2)!}$ is convergent, and state the limit. [2]

(d) Using your answer to part **(b)**, find the exact value of $\sum_{r=6}^{\infty}\frac{r^2-r-1}{r!}$. [3]

7 For a right-angled triangle ABC with $BC = a$ and $AC = b$, $\measuredangle CAB = x$ and $\measuredangle ABC = \frac{\pi}{2}$ as shown below, a sector with radius a is extended from the vertex C. This sector meets the side AC at the point Y.



(a) Show that $\left(\frac{AC}{AY}\right)^2 = \frac{1}{(1-\sin x)^2}$. [2]

(b) Using appropriate expansions from the List of Formulae (MF26), find the Maclaurin series for $f(x) = (1-\sin x)^{-2}$, up to and including the term in x^3. [3]

(c) Deduce the series expansion for $\frac{2\cos x}{(1-\sin x)^3}$, up to and including the term in x^2. [2]

(d) Using your answer in part **(b)** or otherwise, show that $\frac{AC}{AY} \approx 1 + x + x^2$, when x is sufficiently small. [3]

8 **Do not use a calculator in answering this question.**

(a) The equation $z^2(2+a\mathrm{i})+2z+1+b\mathrm{i}=0$, where a and b are real, has a root $\frac{1}{2}-\frac{1}{2}\mathrm{i}$.

(i) Find the values of a and b. [2]

(ii) Find the second root of this equation. [3]

(b) The complex numbers w_1 and w_2 are given by $-1+\sqrt{3}\mathrm{i}$ and $2-2\mathrm{i}$ respectively.

(i) Find the modulus and argument of $\frac{w_1}{w_2}$ in exact form. [3]

(ii) Hence show that $\sin\frac{11}{12}\pi=\frac{\sqrt{3}-1}{2\sqrt{2}}$. [3]

9 The function f is given by $\mathrm{f}(x)=\frac{1}{x^2-4}, x\in\mathbb{R}, x\neq -2, x\neq 2$.

(a) Sketch the graph of $y=\mathrm{f}(x)$. Give the equations of any asymptotes and the coordinates of any turning points. [2]

(b) If the domain of f is further restricted to $x\geq k$, state the least value of k for which the function f^{-1} exists. For this value of k, find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

For the rest of the question, the domain of f is $x\in\mathbb{R}, x\neq -2, x\neq 2$ as originally defined. The function g is given by $\mathrm{g}(x)=\frac{1}{x-1}, x\in\mathbb{R}, x\neq\frac{1}{2}, x\neq 1, x\neq\frac{3}{2}$.

(c) Find $\mathrm{fg}(x)$. [2]

(d) Find the range of fg. [3]

10 A mining company has identified a mineral layer below ground. Points (x, y, z) are defined relative to the mining office located at $(0, 0, 0)$, where units are metres. The ground is modelled as a horizontal plane with equation $z=0$. The top surface of the mineral layer is modelled as part of the plane containing the points $A(8, 4, -50)$, $B(12, 14, -42)$ and $C(-6, 20, -60)$.

(a) Show that a cartesian equation of the top surface of the mineral layer is $19x+6y-17z=1026$. [2]

(b) Find the acute angle between the ground and the top surface of the mineral layer. [2]

The mineral layer is found to be of thickness $14\sqrt{14}$ metres, with the bottom surface modelled as part of a plane parallel to the top surface.

(c) Find a cartesian equation of the bottom surface of the mineral layer. [3]

The mining company plans to drill vertically downwards from a point on the ground to reach the mineral layer. After the drill touches the top surface, it continues to penetrate through the mineral layer until it touches the bottom surface.

(d) Find the length of the drill that is found inside the mineral layer. [2]

It is found that in fact, cost effectiveness and safety could be increased by performing directional drilling. As such, the mining company proposes a new plan to drill at a certain angle from the point $D(-14,\ 3,\ 0)$ towards the mineral layer.

(e) If the shortest path is taken, find the position vector of the point at which the drill touches the top surface of the mineral layer. [3]

11 On his 45th birthday on 1 July 2024, Mr Eu purchases an annuity plan called EuRetire for a principal sum of $\$x$. The plan works like this:

Phase 1: The plan pays compound interest at a rate of 3% per annum on the last day of June each year, until Mr Eu turns 65. The total sum at the end of Phase 1 is called the "accumulated sum".

Phase 2: Upon turning 65, Mr Eu receives a monthly payout, $\$y$, on the 1st of each month, from the accumulated sum. The amount remaining in the plan continues to draw interest at 0.2% per month on the last day of each month.

(a) Write down an expression for the accumulated sum at the end of Phase 1, giving your answer to 5 significant figures. [1]

(b) Show that the amount remaining in the plan at the end of the n^{th} month of Phase 2, after interest has been credited, is

$$\$1.8061\left(1.002^n\right)x-501\left(1.002^n-1\right)y. \quad [4]$$

(c) Mr Eu wishes to receive \$1000 a month in Phase 2 for at least 10 years. What is the minimum principal sum, correct to the nearest dollar, he would need? [2]

Mr Eu decides to put in a principal sum of \$200 000.

(d) If he chooses to receive \$2000 monthly in Phase 2, in which month will he receive the last payout of \$2000? [2]

(e) An alternative plan is such that Phase 1 remains the same but Phase 2 provides a payout of \$1500 in the first month, and the subsequent monthly payouts increase by \$100 each month. However, the remaining amount in the accumulated sum no longer earns interest. If Mr Eu takes up the alternative plan, calculate the highest payout he receives and determine the month that he receives it in. [4]

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Paper 2 Click here to enter text.

17 September 2024

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Section A: Pure Mathematics [40 marks]

1 It is given that a is a real number such that $a < -1$.

(a) Factorise $x^3 - ax^2 + a^2x - a^3$. [1]

(b) Hence solve the inequality $\dfrac{x^3 - ax^2 + a^2x - a^3}{(x+a)(x+1)} \geq 0$. [4]

(c) Using your answer to part **(b)**, solve the inequality $\dfrac{|x|^3 + 2x^2 + 4|x| + 8}{(|x|-2)(|x|+1)} \geq 0$. [3]

2 A curve C has equation $y^{y+1} = x^x$, for $x > 0$ and $y > 0$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [4]

(b) Hence find the equation of the tangent to the curve $y^{y+1} = x^x$ which is parallel to the x-axis. [3]

(c) Using the result $t > \ln t$ for all $t > 0$, determine with reason whether there exists any tangent parallel to the y-axis. [2]

3 Referred to the origin O, the points A, B and X are such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OX} = \mathbf{a} + 3\mathbf{b}$. It is given that $|\mathbf{a}| = 2$, $|\mathbf{b}| = \sqrt{5}$ and $|\overrightarrow{OX}| = \sqrt{73}$.

(a) By considering the scalar product of $\overrightarrow{OX}$ with itself, or otherwise, find the value of $\mathbf{a}.\mathbf{b}$. [3]

(b) Find the area of triangle OAX. [5]

(c) The variable point V, not necessarily coplanar with A, B and X, has position vector $\mathbf{v}$. Given that $|\mathbf{v} - \mathbf{a}| = |\mathbf{v} - \mathbf{b}|$, describe geometrically the set of all possible positions of the point V. [2]

4 A curve C has parametric equations

$$x = \ln(4t), \qquad y = \frac{1}{\sqrt{4t^2 - 1}}, \qquad \text{for } \frac{1}{2} < t \leq 2.$$

The region R is bounded by C, the lines $x = \ln(2\sqrt{2})$, $x = \ln 4$, and the x-axis.

(a) Sketch the curve C, stating the equations of any asymptotes, coordinates of any endpoints, and denoting the region R clearly. [3]

(b) Show that the area of region R is given by $\displaystyle\int_a^b \frac{1}{t\sqrt{4t^2 - 1}}\,\mathrm{d}t$ where a and b are constants, and state the exact values of a and b. [3]

(c) Hence, by using the substitution $u = \dfrac{1}{t}$, find the exact area of the region R. [5]

(d) It is given that the volume of the solid formed by revolving an area under a parametric curve, with equations $x = \mathrm{f}(t)$, $y = \mathrm{g}(t)$, from $x = \mathrm{f}(\alpha)$ to $x = \mathrm{f}(\beta)$ through 2π about the x-axis is given by

$$\pi \int_{\alpha}^{\beta} [\mathrm{g}(t)]^2 \frac{\mathrm{d}x}{\mathrm{d}t} \mathrm{d}t .$$

Hence, find the volume of the solid obtained when the region R is rotated through 2π radians about the x-axis. [2]

Section B: Probability and Statistics [60 marks]

5 A company sells consumer hobby drones, and they buy the rotor motor parts from two suppliers, AE Rotors and Brushless Inc. 25% of the motors are from AE Rotors and the rest are from Brushless Inc. The probability that a randomly chosen motor supplied by AE Rotors is faulty is $\frac{1}{20}$. The probability that a randomly chosen motor supplied by Brushless Inc is faulty is q.

(a) Complete the tree diagram below to illustrate the possibilities for a randomly chosen motor. You should use the following notations for the events:

A for the motor is supplied by AE Rotors. B for the motor is supplied by Brushless Inc.

F for the motor is faulty, and F' for its complement, i.e. the motor is not faulty. [1]

It is known that the probability that a randomly picked motor that is faulty is supplied by Brushless Inc is $\frac{3}{8}$.

(b) Use the above information to set up an equation involving q, and hence find the value of q. [2]

(c) Three motors are randomly picked. Find the probability that all of them are supplied by AE Rotors and exactly one of them is faulty. [2]

6 The random variable X has distribution $\mathrm{B}(n, p)$, where $n \geq 2$. It is known that the mean of X and the standard deviation of X are equal.

(a) Find an expression for p in terms of n. [1]

(b) Show that $\frac{\mathrm{P}(X=1)}{\mathrm{P}(X=2)} = \frac{2n}{n-1}$. Hence find the the mode(s) of X. [4]

7 A random variable X has the probability distribution given in the following table.

x	1	2	4	8
$\mathrm{P}(X=x)$	0.4	0.3	0.2	0.1

(a) Find $\mathrm{Var}(X)$ and show that $\mathrm{Var}(3X-2)=39.96$. [3]

(b) The mean of 40 independent observations of X is denoted by M. Find an approximate value for $\mathrm{P}(M>3)$. [3]

8 In a game show, a contestant must draw 4 balls without replacement from a box containing 20 balls: 1 black, 3 red, 6 green and 10 yellow.

(a) Find the probability that no yellow ball is drawn. [1]

(b) Find the probability that a contestant draws at most 1 yellow ball and at least 2 green balls. [4]

In the game show, the amount of cash won by a contestant is determined by the colours of the balls drawn. Each ball has an associated cash value based on its colour as follows:

Colour	Amount ($)
Red	100
Green	200
Yellow	400

- If no black ball is drawn, then the contestant wins cash equal to the sum of the cash values of the four balls drawn.
- If the black ball is drawn, then the contestant wins cash equal to half the sum of the cash values of the remaining three balls drawn.

(c) Given that a contestant wins $600, find the probability that he has drawn a black ball. [3]

9 Workers from a certain company have a choice of two routes for travelling to work each day. The travel time in minutes using route A and route B have independent normal distributions. The means and standard deviations of these distributions are shown in the following table.

	Mean	Standard Deviation
Route A	45	7
Route B	53	5

(a) Given that the probability that the travel time using route A takes longer than k minutes is less than 0.2, find the least value of k, giving your answer to the nearest minute. [2]

James and John both work at the company and leave for work at the same time every day. James takes route A and John takes route B.

(b) **(i)** Find the probability that on a randomly chosen day, James and John arrive at the company more than 5 minutes apart. [3]

(ii) Find the probability that out of 5 randomly chosen days, James arrives at the company later than John on at most 1 day. [2]

When there is a traffic jam, it will increase the travel time using route A by 5 minutes and increase the travel time using route B by 8%.

(c) Peter uses route A for 3 occasions and route B for 1 occasion. Given that a traffic jam occurs on all occasions, find the probability that average travel time for the 4 occasions is between 50 minutes and 1 hour. You may assume that the travel time for all the occasions are independent of each other. [4]

10 A researcher wants to study the correlation between Body Mass Index (BMI) and the total cholesterol in the body. She selected 9 young adults of age 25 at random and recorded their BMI, x kg/m^2 and total cholesterol, y mg/dL. The results are given in the following table:

x	17.5	19.5	21.3	24.4	25.6	27.1	28.3	30.7	32.8
y	187	197	210	237	241	250	254	257	260

(a) The researcher calculates the product moment correlation coefficient between x and y to be 0.964 and thus concludes that an increase in BMI causes an increase in total cholesterol. State, with a reason, whether you agree with her conclusion. [1]

(b) Draw a scatter diagram for these values, labelling the axes. Using your diagram, explain why the relationship between x and y should not be modelled by an equation of the form $y = a + bx$, where a and b are constants. [3]

(c) By calculating the values of the relevant product moment correlation coefficients, explain why the relationship between x and y is modelled better by $y = a + b\ln x$ as compared to $y = a + b\sqrt{x}$. [3]

(d) Find the equation of the least squares regression line of y on $\ln x$. Use this equation to find estimates for the

(i) total cholesterol when the BMI is 20 kg/m^2,

(ii) BMI when the total cholesterol is 230 mg/dL. [3]

(e) Comment on the reliability of each of your estimates for part **(d)**. [2]

11 The heel stack of a running shoe refers to the thickness of the sole of the shoe at the point where the heel of the foot sits in the shoe.

A running shoe manufacturer makes a model called "SuperFly" and claims that the average heel stack is 42.7 millimetres. After reading some online reviews, the production manager wishes to test if the heel stack of the Superfly has been overstated.

(a) Explain whether the manager should carry out a 1-tail or 2-tail test. State appropriate hypotheses for the test, defining any symbols you use. [3]

The production manager goes to the factory to measure a random sample of 16 shoes. The heel stacks, x millimetres, are summarised as follows.

$$n = 16 \qquad \sum(x - 42.7) = -6.4 \qquad \sum(x - 42.7)^2 = 13.2$$

(b) Explain in this context what is meant by a "random sample". [1]

(c) What assumption(s), if any, does the production manager need to make about the population distribution of the heel stacks? [1]

(d) Calculate unbiased estimates for the population mean and population variance of the heel stacks, and carry out the hypothesis test in part **(a)** at 2.5% level of significance. [5]

The manufacturer upgrades the machine used to produce the sole of the Superfly. It is later established that the heel stack of the Superfly has a variance of 0.987 millimetres^2. The production manager tests another random sample of 20 shoes to see whether the mean heel stack has changed, at 2.5% level of significance.

(e) With the same assumption(s) as in part **(c)**, find the range of values of the sample mean, in millimetres to 1 decimal place, such that he can conclude that the mean heel stack has changed. (Answers obtained by trial and improvement from a calculator will obtain no marks.) [3]

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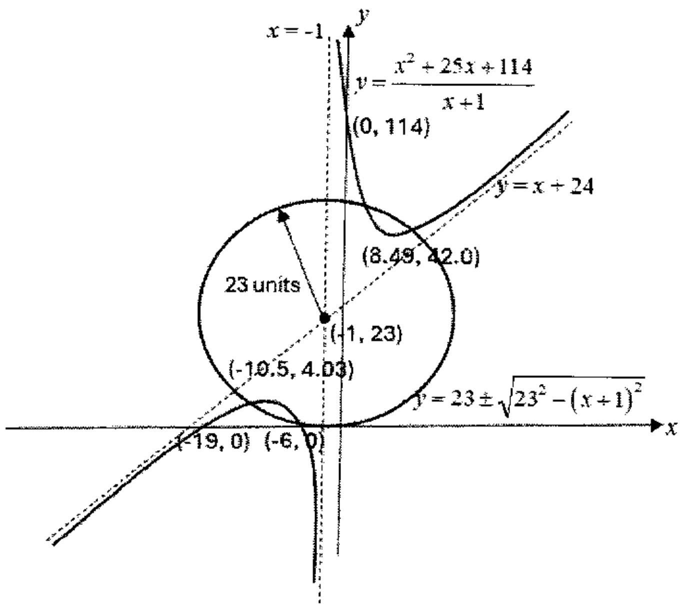
1	**Solution**
	Let the number be $100x+10y+z$. $x+y+z=15$ ----------(1) $2x+y=z$ $\Rightarrow 2x+y-z=0$ ----------(2) $100z+10y+x-5(100x+10y+z)=21$ $\Rightarrow -499x-40y+95z=21$ ----------(3) Solving using GC, $x=1,\ y=6,\ z=8$. The number is 168.
2	**Solution**
	$2z-w=5\text{i}$ $z=\dfrac{5\text{i}+w}{2}$ ....(1) Sub (1) into $3\text{i}z-w^*=-5$ $3\text{i}\left(\dfrac{5\text{i}+w}{2}\right)-w^*=-5$ $-15+3\text{i}w-2w^*=-10$ Let $w=a+b\text{i}$ $-15+3\text{i}(a+b\text{i})-2(a-b\text{i})=-10$ $-15+3a\text{i}-3b-2a+2b\text{i}=-10$ Comparing imaginary parts, $3a+2b=0$ Comparing real parts, $-15-3b-2a=-10$ $-2a-3b=5$ Solving, $a=2,b=-3$ $w=2-3\text{i}$ $z=\dfrac{5\text{i}+2-3\text{i}}{2}=1+\text{i}$
3	**Solution**

(a)	$\left(-\frac{3}{4},-3\right)\to\left(-\frac{3}{4},3\right)\to\left(-\frac{3}{4},0\right)\to(-3,0)$
(b)	$y=p-\frac{1}{x+q}$ Reflection in the x-axis $-y=p-\frac{1}{x+q}$ Translation of 3 units in the negative y-direction $-(y+3)=p-\frac{1}{x+q}\Rightarrow y=-3-p+\frac{1}{x+q}$ Scaling of factor 4 parallel to the x-axis $y=-3-p+\frac{1}{\frac{x}{4}+q}=-3-p+\frac{4}{x+4q}$ $x+4q=0\Rightarrow -4q=-1\Rightarrow q=\frac{1}{4}$ $-3-p=2\Rightarrow p=-5$ Alternative $y=2\to y-3=2\to y=5$ $-y=5\to y=-5\therefore p=-5$ $x=-1\to 4x=-1\to x=-\frac{1}{4}$ $x=-q\Leftrightarrow x=-\frac{1}{4}$ $\therefore q=\frac{1}{4}$
4	**Solution**

(a)	 $\frac{x^2+25x+114}{x+1}=\frac{x(x+1)+24(x+1)+90}{(x+1)}=x+24+\frac{90}{x+1}$
(b)	We have $\sin 2\theta=\frac{x+1}{23}$ and $\cos 2\theta=\frac{y-23}{23}$ Using $\sin^2 2\theta+\cos^2 2\theta=1$, $\left(\frac{x+1}{23}\right)^2+\left(\frac{y-23}{23}\right)^2=1$ $(x+1)^2+(y-23)^2=23^2$
(c)	$(x+1)^2+(y-23)^2=23^2$ Circle, centre (-1, 23), radius 23. $(y-23)^2=23^2-(x+1)^2$ $y=23\pm\sqrt{23^2-(x+1)^2}$ Number of intersection points = 4
5	**Solution**
(a)(i)	<u>Implicit differentiation</u> $wy=x^2 \Rightarrow y\frac{dw}{dx}+w\frac{dy}{dx}=2x$ Sub into DE:

$$2(x)\left(\frac{x^2}{w}\right)-x^2\,\frac{2x-\frac{x^2}{w}\left(\frac{dw}{dx}\right)}{w}=2x^4+\left(\frac{x^2}{w}\right)^2$$

$$2\left(\frac{x^3}{w}\right)-\frac{2x^3}{w}+\left(\frac{dw}{dx}\right)\left(\frac{x^4}{w^2}\right)=2x^4+\frac{x^4}{w^2}$$

$$\frac{dw}{dx}=\frac{w^2}{x^4}\left(2x^4+\frac{x^4}{w^2}\right)$$

$$\frac{dw}{dx}=1+2w^2$$

Alternative: (make y the subject first)

$$wy=x^2 \Rightarrow y=x^2w^{-1} \Rightarrow \frac{dy}{dx}=2xw^{-1}-x^2w^{-2}\frac{dw}{dx}$$

Sub into DE:

$$2(x)\left(x^2w^{-1}\right)-x^2\left(2xw^{-1}-x^2w^{-2}\frac{dw}{dx}\right)=2x^4+\left(x^2w^{-1}\right)^2$$

$$x^4w^{-2}\frac{dw}{dx}=x^4\left(2+w^{-2}\right)$$

$$\frac{dw}{dx}=\frac{2+w^{-2}}{w^{-2}}$$

$$=1+2w^2 \text{ (shown)}$$

Alternative 2: implicit but avoid denominators

$$wy=x^2 \Rightarrow y\frac{dw}{dx}+w\frac{dy}{dx}=2x$$

$$\times x^2w:\quad x^2wy\frac{dw}{dx}+w^2x^2\frac{dy}{dx}=2x^3w$$

$$x^4\frac{dw}{dx}+w^2x^2\frac{dy}{dx}=2x^3w$$

$$\Rightarrow w^2x^2\frac{dy}{dx}=2x^3w-x^4\frac{dw}{dx}---(*)$$

Multiply given DE by w^2:

$$2xyw^2-x^2w^2\frac{dy}{dx}=2x^4w^2+w^2y^2$$

$$2x^3w-x^2w^2\frac{dy}{dx}=2x^4w^2+x^4$$

$$x^2w^2\frac{dy}{dx}=2x^3w-2x^4w^2-x^4---(**)$$

Equate (*) and (**),

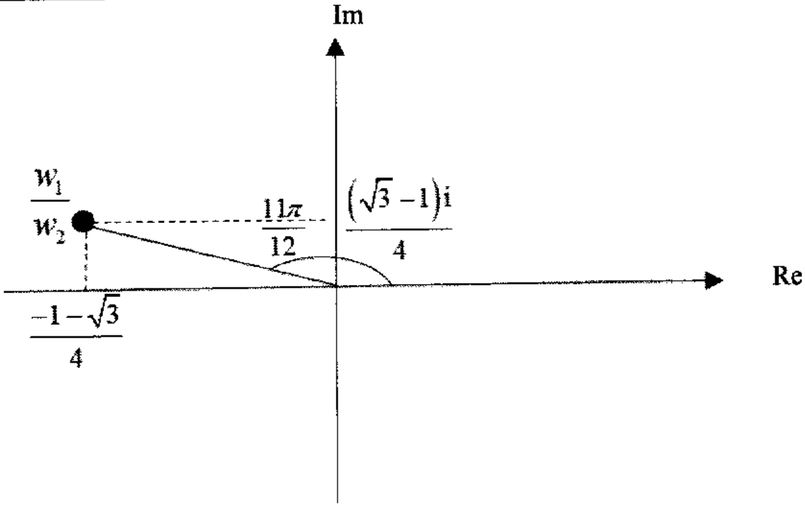
	$2x^3w - x^4\dfrac{\mathrm{d}w}{\mathrm{d}x} = 2x^3w - 2x^4w^2 - x^4$ $\dfrac{\mathrm{d}w}{\mathrm{d}x} = 1 + 2w^2$ (shown)
(a) **(ii)**	$\int \dfrac{1}{1+2w^2}\mathrm{d}w = \int 1\,\mathrm{d}x$ $\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}w\right) = x + c$ $\sqrt{2}w = \tan\sqrt{2}(x+c)$ $\dfrac{\sqrt{2}x^2}{y} = \tan\sqrt{2}(x+c)$ $y = \dfrac{\sqrt{2}x^2}{\tan\sqrt{2}(x+c)}$
(b)	$\dfrac{\mathrm{d}^2u}{\mathrm{d}x^2} = \mathrm{e}^{2x} \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{\mathrm{e}^{2x}}{2} + c \Rightarrow u = \dfrac{\mathrm{e}^{2x}}{4} + cx + d$ When $x = 1, \dfrac{\mathrm{d}u}{\mathrm{d}x} = 0$: $0 = \dfrac{\mathrm{e}^2}{2} + c \Rightarrow c = -\dfrac{\mathrm{e}^2}{2}$ When $x = 1, u = \dfrac{\mathrm{e}^2}{4}$: $\dfrac{\mathrm{e}^2}{4} = \dfrac{\mathrm{e}^2}{4} - \dfrac{\mathrm{e}^2}{2} + d \Rightarrow d = \dfrac{\mathrm{e}^2}{2}$ $u = \dfrac{\mathrm{e}^{2x}}{4} - \dfrac{\mathrm{e}^2}{2}x + \dfrac{\mathrm{e}^2}{2}$
6	**Solution**
(a)	$\dfrac{r^2+3r+1}{(r+2)!} = \dfrac{r^2+3r+2-1}{(r+2)!}$ $= \dfrac{(r+2)(r+1)}{(r+2)!} - \dfrac{1}{(r+2)!}$ $= \dfrac{1}{r!} - \dfrac{1}{(r+2)!}$ (shown) <u>Alternative 1</u> $\dfrac{1}{r!} - \dfrac{1}{(r+2)!} = \dfrac{(r+2)(r+1)}{(r+2)(r+1)r!} - \dfrac{1}{(r+2)!}$ $= \dfrac{r^2+3r+1}{(r+2)!}$ (shown) <u>Alternative 2</u> $\dfrac{r^2+3r+1}{(r+2)!} = \dfrac{A}{r!} + \dfrac{B}{(r+2)!} \Rightarrow r^2+3r+1 = A(r+1)(r+2) + B$

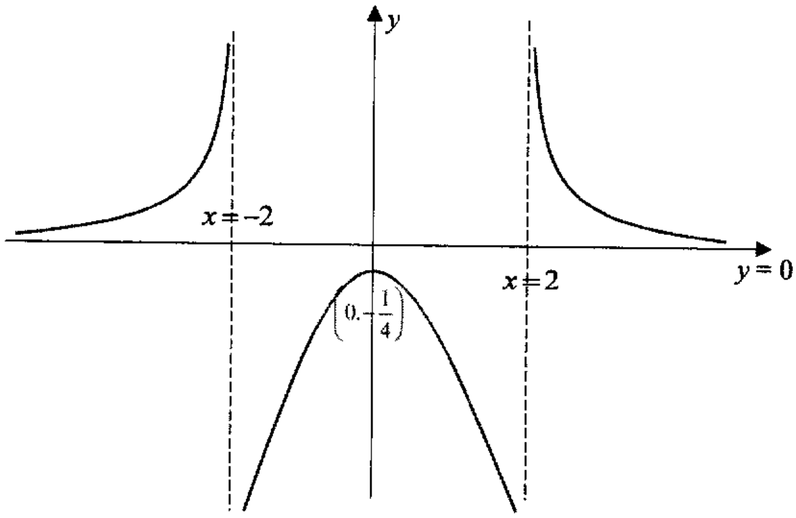
	Comparing $r^2 : A=1$ Comparing constant: $2A+B=1 \Rightarrow B=-1$ $\therefore \frac{r^2+3r+1}{(r+2)!}=\frac{1}{r!}-\frac{1}{(r+2)!}$ (shown)
(b)	$\sum_{r=1}^{n}\frac{r^2+3r+1}{(r+2)!}=\sum_{r=1}^{n}\left(\frac{1}{r!}-\frac{1}{(r+2)!}\right)$ $=\frac{1}{1!}-\frac{1}{3!}$ $+\frac{1}{2!}-\frac{1}{4!}$ $+\frac{1}{3!}-\frac{1}{5!}$ $+\ldots$ $+\frac{1}{(n-2)!}-\frac{1}{(n)!}$ $+\frac{1}{(n-1)!}-\frac{1}{(n+1)!}$ $+\frac{1}{(n)!}-\frac{1}{(n+2)!}$ $=1+\frac{1}{2}-\frac{1}{(n+1)!}-\frac{1}{(n+2)!}$ $=\frac{3}{2}-\frac{1}{(n+1)!}-\frac{1}{(n+2)!}$
(c)	As $n\to\infty$, $\frac{1}{(n+1)!}\to 0$ and $\frac{1}{(n+2)!}\to 0$. So $\sum_{r=1}^{\infty}\frac{r^2+3r+1}{(r+2)!}=\frac{3}{2}-0-0=\frac{3}{2}$. Since $\sum_{r=1}^{\infty}\frac{r^2+3r+1}{(r+2)!}$ is a finite number, series is convergent.
(d)	$\sum_{r=6}^{\infty}\frac{r^2-r-1}{r!}=\sum_{r+2=6}^{r+2=\infty}\frac{(r+2)^2-(r+2)-1}{(r+2)!}$ $=\sum_{r=4}^{\infty}\frac{r^2+3r+1}{(r+2)!}$ $=\sum_{r=1}^{\infty}\frac{r^2+3r+1}{(r+2)!}-\sum_{r=1}^{3}\frac{r^2+3r+1}{(r+2)!}$ $=\frac{3}{2}-\left(\frac{3}{2}-\frac{1}{4!}-\frac{1}{5!}\right)$ $=\frac{1}{20}$ <u>Alternative:by modifying (b)</u> $\ldots=\sum_{r=4}^{\infty}\frac{r^2+3r+1}{(r+2)!}$ $=\frac{1}{4!}+\frac{1}{5!}$ $=\frac{1}{20}$

7	**Solution**
(a)	Observe that $\sin x = \frac{a}{b} \Rightarrow a = b\sin x$. $\left(\frac{AC}{AY}\right)^2 = \left(\frac{b}{b-a}\right)^2 = \left(\frac{b}{b-b\sin x}\right)^2 = \frac{1}{(1-\sin x)^2}$ (shown) Alternative $\frac{1}{(1-\sin x)^2} = \left(\frac{1}{1-\frac{a}{b}}\right)^2 = \left(\frac{b}{b-a}\right)^2 = \left(\frac{AC}{AY}\right)^2$ (shown)
(b)	$\mathrm{f}(x) = (1-\sin x)^{-2}$ $= 1 + (-2)(-\sin x) + \frac{(-2)(-3)}{2!}(-\sin x)^2 + \frac{(-2)(-3)(-4)}{3!}(-\sin x)^3 + \ldots$ $= 1 + 2(\sin x) + 3(\sin x)^2 + 4(\sin x)^3 + \ldots$ $= 1 + 2\left(x - \frac{x^3}{3!} + \ldots\right) + 3\left(x - \frac{x^3}{3!} + \ldots\right)^2 + 4\left(x - \frac{x^3}{3!} + \ldots\right)^3 + \ldots$ $= 1 + 2x - \frac{x^3}{3} + 3x^2 + 4x^3 + \ldots = 1 + 2x + 3x^2 + \frac{11}{3}x^3 + \ldots$ <u>Alternatively:</u> $\mathrm{f}(x) = (1-\sin x)^{-2}$ $= \left[1 - \left(x - \frac{x^3}{3!} + \ldots\right)\right]^{-2} = \left[1 + \left(-x + \frac{x^3}{3!} + \ldots\right)\right]^{-2}$ $= 1 + (-2)\left(-x + \frac{x^3}{3!} + \ldots\right) + \frac{(-2)(-3)}{2!}\left(-x + \frac{x^3}{3!} + \ldots\right)^2$ $+ \frac{(-2)(-3)(-4)}{3!}\left(-x + \frac{x^3}{3!} + \ldots\right)^3 + \ldots$ $= 1 + \left(2x - \frac{x^3}{3} + \ldots\right) + 3\left(x^2 + \ldots\right) + 4\left(x^3 + \ldots\right) + \ldots$ $= 1 + 2x + 3x^2 + \frac{11}{3}x^3 + \ldots$
(c)	Since $(1-\sin x)^{-2} = 1 + 2x + 3x^2 + \frac{11}{3}x^3 + \ldots$, differentiating both sides, $-2(1-\sin x)^{-3}(-\cos x) = 2 + 6x + 11x^2 + \ldots$ $\therefore \frac{2\cos x}{(1-\sin x)^3} = 2 + 6x + 11x^2 + \ldots$
(d)	When x is sufficiently small, the terms in x^3 and above can be ignored,

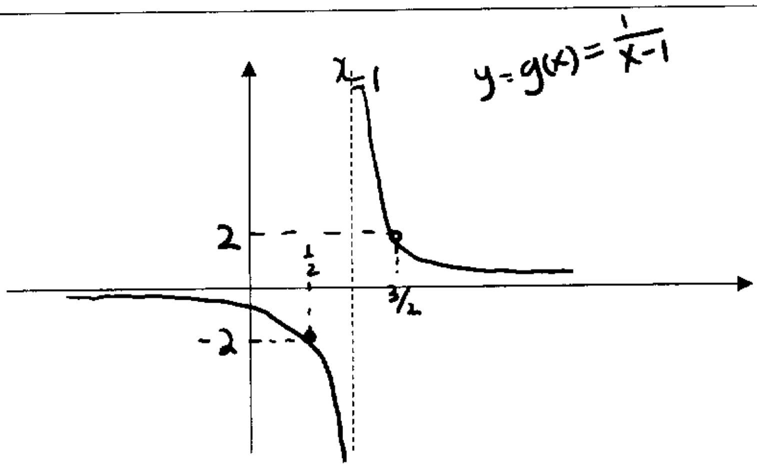
	Hence $\left(\dfrac{AC}{AY}\right)^2 = \mathrm{f}(x) = \dfrac{1}{(1-\sin x)^2} \approx 1+2x+3x^2$ $\dfrac{AC}{AY} \approx \left(1+2x+3x^2\right)^{1/2}$ $= 1+\dfrac{1}{2}\left(2x+3x^2\right)+\dfrac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(2x+3x^2\right)^2+\ldots$ $\approx 1+x+x^2$ <u>Otherwise:</u> From (a), $\dfrac{AC}{AY} = \dfrac{1}{(1-\sin x)} = (1-\sin x)^{-1}$ $\dfrac{AC}{AY} = 1+(-1)(-\sin x)+\dfrac{(-1)(-2)}{2!}(-\sin x)^2+\ldots$ $\approx 1+(-1)(-x)+\dfrac{(-1)(-2)}{2!}(-x)^2$ $= 1+x+x^2$
8	**Solution**
(a)(i)	Since $\frac{1}{2}-\frac{1}{2}\mathrm{i}$ is a root, $z^2(2+a\mathrm{i})+2z+1+b\mathrm{i}=0$ $\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)^2(2+a\mathrm{i})+2\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+1+b\mathrm{i}=0$ $\left(\frac{1}{4}-\frac{1}{2}\mathrm{i}+\left(-\frac{1}{2}\mathrm{i}\right)^2\right)(2+a\mathrm{i})+2\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+1+b\mathrm{i}=0$ $-\frac{\mathrm{i}}{2}(2+a\mathrm{i})+2-\mathrm{i}+b\mathrm{i}=0$ Comparing real and imaginary parts, $\frac{a}{2}+2=0$ $a=-4$ $-2+b=0$ $b=2$

(a)(ii)	$z^2(2-4i)+2z+1+2i=0$ $z=\frac{-2\pm\sqrt{2^2-4(2-4i)(1+2i)}}{2(2-4i)}$ $=\frac{-2\pm\sqrt{4-40}}{2(2-4i)}$ $=\frac{-1\pm 3i}{2-4i}\times\frac{2+4i}{2+4i}$ $=\frac{10-10i}{20}$ or $\frac{-14+2i}{20}$ $=\frac{1}{2}-\frac{1}{2}i$ (reject) or $-\frac{7}{10}+\frac{1}{10}i$ Alternative $z^2(2-4i)+2z+1+2i=\left(z-\frac{1}{2}+\frac{1}{2}i\right)\left[(2-4i)z-z_2\right]$ $=(2-4i)z^2-(1-2i)z+(1-2i)iz-zz_2+\frac{1}{2}z_2-\frac{1}{2}iz_2$ Comparing coefficient of z $3i+1-z_2=2$ $z_2=-1+3i$ The second root $=\frac{-1+3i}{2-4i}=\frac{2i-14}{20}=\frac{1}{10}i-\frac{7}{10}$ Alternative Sum of roots, $\alpha+\beta=-\frac{b}{a}$ Let 2nd root be z. $\frac{1}{2}-\frac{1}{2}i+z=-\frac{2}{2-4i}$ $z=\frac{-2}{2-4i}\times\frac{2+4i}{2+4i}-\frac{1}{2}+\frac{1}{2}i$ $z=-\frac{1}{5}-\frac{2}{5}i-\frac{1}{2}+\frac{1}{2}i$ $z=-\frac{7}{10}+\frac{1}{10}i$
(b)(i)	$w_1=-1+\sqrt{3}i=2e^{\frac{2\pi}{3}i}$ $w_2=2-2i=2\sqrt{2}e^{-\frac{\pi}{4}i}$

	$\frac{w_1}{w_2}=\frac{2e^{\frac{2\pi}{3}i}}{2\sqrt{2}e^{-\frac{\pi}{4}i}}$ $=\frac{e^{\frac{2\pi}{3}i+\frac{\pi}{4}i}}{\sqrt{2}}$ $=\frac{\sqrt{2}e^{\frac{11\pi}{12}i}}{2}$ $\left\lvert\frac{w_1}{w_2}\right\rvert=\frac{\sqrt{2}}{2}$ and $\arg\left(\frac{w_1}{w_2}\right)=\frac{11\pi}{12}$
(b)(ii)	$\frac{w_1}{w_2}=\frac{-1+\sqrt{3}i}{2-2i}$ $=\frac{\left(-1+\sqrt{3}i\right)\left(2+2i\right)}{4+4}$ $=\frac{-2+2\sqrt{3}i-2i-2\sqrt{3}}{8}$ $=\frac{-1-\sqrt{3}+\sqrt{3}i-i}{4}$ $\frac{w_1}{w_2}=\frac{\sqrt{2}}{2}\left(\cos\left(\frac{11\pi}{12}\right)+i\sin\left(\frac{11\pi}{12}\right)\right)$ Comparing imaginary parts, $\frac{\sqrt{2}}{2}\sin\left(\frac{11\pi}{12}\right)=\frac{\sqrt{3}-1}{4}$ $\sin\left(\frac{11\pi}{12}\right)=\frac{\sqrt{3}-1}{2\sqrt{2}}$ (Shown) <u>Alternative</u> 

	$\sin\left(\frac{11\pi}{12}\right)=\frac{\sqrt{3}-1}{4}\div\frac{\sqrt{2}}{2}=\frac{\sqrt{3}-1}{2\sqrt{2}}$ (Shown)
9	**Solution**
(a)	
(b)	Least value of $k=0$. $y=\frac{1}{x^2-4}$ $x^2-4=\frac{1}{y}$ $x=\pm\sqrt{\frac{1}{y}+4}$ Since $R_{f^{-1}}=D_f=[0,2)\cup(2,\infty)$, $f^{-1}(x)=\sqrt{\frac{1}{x}+4}$, $x\in\mathbb{R}, x\le-\frac{1}{4}$ or $x>0$ or $\left(-\infty,-\frac{1}{4}\right]\cup(0,\infty)$
(c)	$fg(x)=f\left(\frac{1}{x-1}\right)$ $=\frac{1}{\left(\frac{1}{x-1}\right)^2-4}$ $=\frac{1}{\frac{1-4(x-1)^2}{(x-1)^2}}$ $=\frac{(x-1)^2}{1-4(x^2-2x+1)}=\frac{x^2-2x+1}{-4x^2+8x-3}$

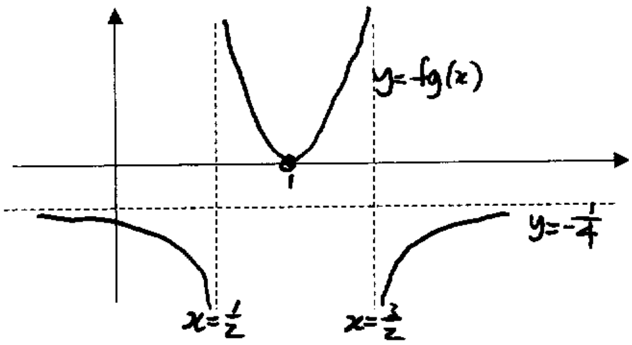
(d)



$R_g = \mathbb{R} \setminus \{-2,0,2\} \xrightarrow{\text{f}} R_{\text{fg}} = \left(-\infty, -\frac{1}{4}\right) \cup (0, \infty)$.

<u>Alternative</u>

$\text{fg}(x) = \dfrac{x^2 - 2x + 1}{-4x^2 + 8x - 3} = -\dfrac{1}{4} + \dfrac{1}{4(-2x+1)(2x-3)}$, where $x \neq \frac{1}{2}, x \neq 1, x \neq \frac{3}{2}$.



So $R_{\text{fg}} = \left(-\infty, -\frac{1}{4}\right) \cup (0, \infty)$.

10 **Solution**

(a)

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 4 \\ 10 \\ 8 \end{pmatrix} \times \begin{pmatrix} -14 \\ 16 \\ -10 \end{pmatrix} = \begin{pmatrix} -228 \\ -72 \\ 204 \end{pmatrix} = -12 \begin{pmatrix} 19 \\ 6 \\ -17 \end{pmatrix}$$

A normal to the top surface is $\mathbf{n} = \begin{pmatrix} 19 \\ 6 \\ -17 \end{pmatrix}$.

$$\overrightarrow{OA} \cdot \mathbf{n} = \begin{pmatrix} 8 \\ 4 \\ -50 \end{pmatrix} \cdot \begin{pmatrix} 19 \\ 6 \\ -17 \end{pmatrix} = 1026$$

A cartesian equation of the top surface is $19x + 6y - 17z = 1026$.

(b)

Acute angle $\theta = \cos^{-1}\dfrac{\left|\begin{pmatrix}0\\0\\1\end{pmatrix}\cdot\begin{pmatrix}19\\6\\-17\end{pmatrix}\right|}{\left|\begin{pmatrix}0\\0\\1\end{pmatrix}\right|\left|\begin{pmatrix}19\\6\\-17\end{pmatrix}\right|} = \cos^{-1}\dfrac{17}{\sqrt{686}} = 49.5°$

(c)

Let cartesian equation of the bottom surface be $19x+6y-17z=d$.

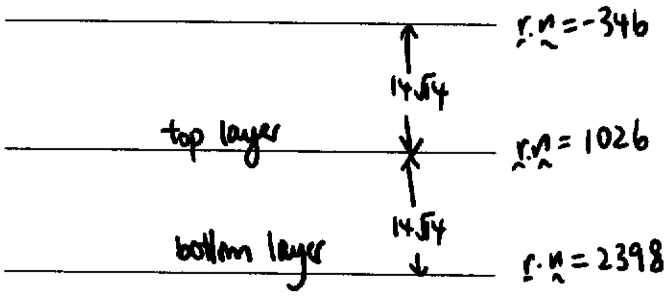
Method 1

> Recall:
> For a plane of equation $\mathbf{r.n}=d$, the distance between the plane and the origin is given by $\dfrac{|d|}{|\mathbf{n}|}$. [Can you remember what the implication is if d is positive, versus negative?]
>
> Based on that, we can further deduce that the distance between two parallel planes say $p_1:\mathbf{r.n}=d_1$ and $p_2:\mathbf{r.n}=d_2$ is given by $\dfrac{|d_1-d_2|}{|\mathbf{n}|}$.

Distance between the top and bottom surfaces $=\dfrac{|d-1026|}{|\mathbf{n}|}=14\sqrt{14}$.

$\Rightarrow |d-1026| = (14\sqrt{14})\sqrt{686} = 1372$

$d = 2398$ or -346

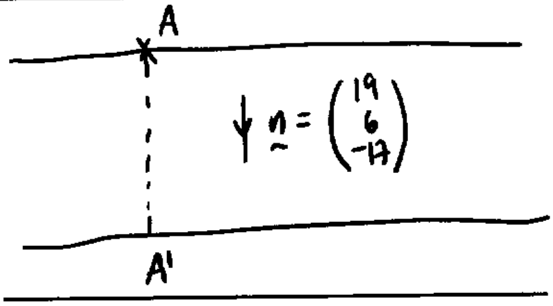


$\Rightarrow d = 1026+1372 = 2398$ (distance between top surface and origin < distance between bottom surface and origin)

Cartesian equation of the bottom surface is $19x+6y-17z=2398$.

Method 2

> General approach: Since we know the planes are parallel, we already know the normals. All we need is a point on the bottom layer. We can get that point by taking for example $\overrightarrow{OA}+14\sqrt{14}\mathbf{n}$.



$$\overrightarrow{OA'} = \overrightarrow{OA} + 14\sqrt{14}\,\frac{\begin{pmatrix}19\\6\\-17\end{pmatrix}}{\sqrt{686}}$$

$$= \begin{pmatrix}8\\4\\-50\end{pmatrix} + 2\begin{pmatrix}19\\6\\-17\end{pmatrix} = \begin{pmatrix}46\\16\\-84\end{pmatrix}$$

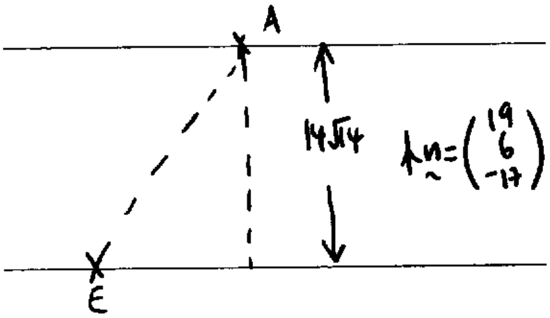
Equation of bottom layer:

$$\mathbf{r}.\begin{pmatrix}19\\6\\-17\end{pmatrix} = \begin{pmatrix}46\\16\\-84\end{pmatrix}.\begin{pmatrix}19\\6\\-17\end{pmatrix} = 2398$$

Cartesian equation: $19x + 6y - 17z = 2398$.

Method 3

Let E be a point on the bottom surface.

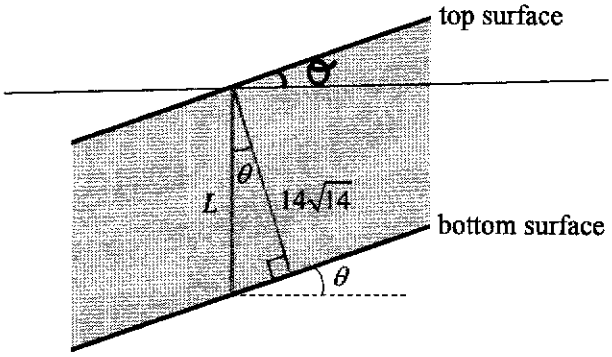


Then $\left|\overrightarrow{AE}\cdot\mathbf{n}\right| = 14\sqrt{14}$.

$$\Rightarrow \frac{\left|\overrightarrow{OE}\cdot\mathbf{n} - \overrightarrow{OA}\cdot\mathbf{n}\right|}{|\mathbf{n}|} = 14\sqrt{14}$$

$$\Rightarrow \frac{\left|\overrightarrow{OE}\cdot\mathbf{n} - 1026\right|}{\sqrt{686}} = 14\sqrt{14}$$

$$\Rightarrow \overrightarrow{OE}\cdot\mathbf{n} = 1026 \pm 14\sqrt{14}\sqrt{686}$$

	∴ The bottom surface has equation $\mathbf{r}\cdot\mathbf{n} = 1026 + 1372 = 2398$. Cartesian equation of the bottom surface is $19x + 6y - 17z = 2398$.
(d)	Let L be the length of the drill that is found inside the mineral layer.  $\cos\theta = \dfrac{14\sqrt{14}}{L}$ Using **(b)**, $L = \dfrac{14\sqrt{14}}{\frac{17}{\sqrt{686}}} = \dfrac{1372}{17}$ metres.
(e)	Let F be the foot of the perpendicular from D to the top surface. Line DF: $\mathbf{r} = \begin{pmatrix} -14 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 19 \\ 6 \\ -17 \end{pmatrix}$, $\lambda \in \mathbb{R}$ ----------(1) Top surface: $\mathbf{r}\cdot\begin{pmatrix} 19 \\ 6 \\ -17 \end{pmatrix} = 1026$ ----------(2) Sub (1) into (2): $\left[\begin{pmatrix} -14 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 19 \\ 6 \\ -17 \end{pmatrix}\right]\cdot\begin{pmatrix} 19 \\ 6 \\ -17 \end{pmatrix} = 1026$ $\Rightarrow -248 + 686\lambda = 1026$ $\Rightarrow \lambda = \dfrac{13}{7}$ $\therefore \overrightarrow{OF} = \begin{pmatrix} -14 \\ 3 \\ 0 \end{pmatrix} + \dfrac{13}{7}\begin{pmatrix} 19 \\ 6 \\ -17 \end{pmatrix} = \dfrac{1}{7}\begin{pmatrix} 149 \\ 99 \\ -221 \end{pmatrix}$
11	**Solution**
(a)	$(1.8601x)

(b)

From (a), accumulated sum at start of Phase 2 is $1.8601x$.

Month	\$ remaining at the end of the month
1	$(1.8601x-y)\times 1.002$
2	$((1.8601x-y)\times 1.002-y)\times 1.002$ $=1.8601(1.002^2)x-y(1.002^2+1.002)$
3	$(1.8601(1.002^2)x-y(1.002^2+1.002)-y)\times 1.002$ $=1.8601(1.002^3)x-y(1.002^3+1.002^2+1.002)$
n	$1.8601(1.002^n)x-y(1.002^n+\ldots+1.002)$ $=1.8601(1.002^n)x-y\left(\dfrac{1.002(1.002^n-1)}{1.002-1}\right)$ $=1.8601(1.002^n)x-501(1.002^n-1)y$ (shown)

(c)

For $n=120$ and $y=1000$, we want

$$1.8061(1.002^{120})x-501(1.002^{120}-1)(1000)\geq 0$$

Solving, $x\geq 59135.69$ so least $x=59136$.

(d)

For $x=200000$ and $y=2000$, we want

$$1.8061(1.002^n)(200000)-501(1.002^n-1)(2000)\geq 0$$

$n=223$, sum $=1514.38$
$n=224$, sum $=-486.59$

So the number of payouts of \$2000 is 223. He will receive the final payout of \$2000 in the 223rd month (or Jan 2063).

(e)

For $x=200000$

Accumulated sum = 361222.25 [or 361220 if 1.8061 is used]

We want $\frac{n}{2}(2(1500)+(n-1)(100))\leq 361222.25$

$n=71$, sum $=355000$
$n=72$, sum $=363600$

So month of final payout is the 71st month.

Final (highest) amount $=1500+70\times 100=8500$.

EUNOIA JUNIOR COLLEGE

JC2 Preliminary Examination 2024

General Certificate of Education Advanced Level

Higher 2

CANDIDATE NAME	

CIVICS GROUP		INDEX NO.	

MATHEMATICS

9758/01

Paper 2 Click here to enter text.

17 September 2024

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

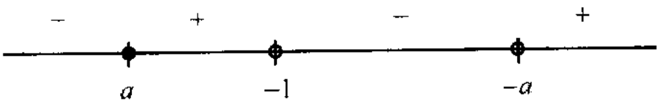
Write your name, civics group and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **21** printed pages and **1** blank page(s).

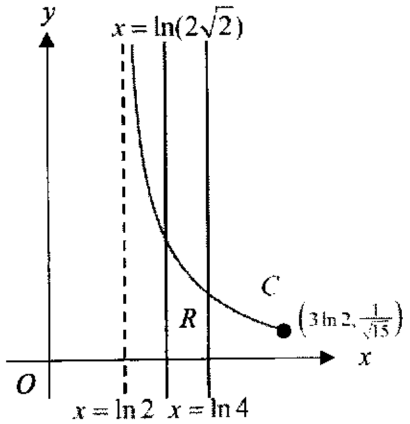
Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total

1	**Solution**
(a)	Let $x^3 - ax^2 + a^2x - a^3 = (x-a)(x^2 + Ax + a^2)$ Comparing coefficients of x^2: $-a = A - a \Rightarrow A = 0$ $\therefore\ x^3 - ax^2 + a^2x - a^3 = (x-a)(x^2 + a^2)$ <u>Alternative</u> $x^3 - ax^2 + a^2x - a^3 = x^2(x-a) + a^2(x-a)$ $= (x^2 + a^2)(x-a)$
(b)	$\dfrac{x^3 - ax^2 + a^2x - a^3}{(x+a)(x+1)} \geq 0 \qquad (*)$ $\dfrac{(x-a)(x^2+a^2)}{(x+a)(x+1)} \geq 0$ $\dfrac{x-a}{(x+a)(x+1)} \geq 0$ ($\because x^2 + a^2$ is always positive)  $\therefore\ a \leq x < -1$ or $x > -a$
(c)	Replacing x with $\lvert x\rvert$ and substituting $a = -2$ in (*), we get $\dfrac{\lvert x\rvert^3 + 2x^2 + 4\lvert x\rvert + 8}{(\lvert x\rvert - 2)(\lvert x\rvert + 1)} \geq 0$. $\therefore\ -2 \leq \lvert x\rvert < -1$ (no solution) or $\lvert x\rvert > 2$ $\Rightarrow x > 2$ or $x < -2$

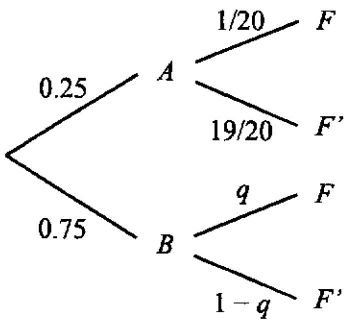
2	Solution
(a)	$y^{y+1}=x^x,\ x,y>0$ $x\ln x=(y+1)\ln y$ $x\left(\frac{1}{x}\right)+\ln x=(y+1)\frac{1}{y}\frac{dy}{dx}+\frac{dy}{dx}\ln y$ $\left(\frac{y+1+y\ln y}{y}\right)\frac{dy}{dx}=1+\ln x$ $\frac{dy}{dx}=\frac{y(1+\ln x)}{(y+1+y\ln y)}$
(b)	Tangent parallel to the x-axis $\frac{dy}{dx}=0$ $y(1+\ln x)=0$ $y=0$ or $\ln x+1=0$ (rej $\because y>0$) $\ln x=-1$ $x=\frac{1}{e}$ Sub $x=\frac{1}{e}$ into equation of curve. $y^{y+1}=\left(\frac{1}{e}\right)^{\frac{1}{e}}$ From GC, $y=0.817$ (3.s.f)
(c)	For a tangent parallel to the y-axis, $\frac{dy}{dx}$ is undefined i.e. $y+1+y\ln y=0$ Consider $y+1+y\ln y=y\left(1+\frac{1}{y}+\ln y\right)=y\left(1+\frac{1}{y}-\ln\frac{1}{y}\right)$ Using the given result: $t>\ln t$ for all $t>0$, $\Rightarrow\frac{1}{y}-\ln\frac{1}{y}>0$ for $y>0$ $\Rightarrow y\left(1+\frac{1}{y}-\ln\frac{1}{y}\right)>0$ for all $y>0$ Therefore there is no tangent parallel to the y- axis.

3	Solution
(a)	$\overrightarrow{OX}.\overrightarrow{OX}=\left\|\overrightarrow{OX}\right\|^2$ $(\mathbf{a}+3\mathbf{b}).(\mathbf{a}+3\mathbf{b})=73$ $\|\mathbf{a}\|^2+3\mathbf{a}.\mathbf{b}+3\mathbf{b}.\mathbf{a}+9\|\mathbf{b}\|^2=73$ $4+6\mathbf{a}.\mathbf{b}+9(5)=73$ $\mathbf{a}.\mathbf{b}=4$ <u>Alternative (by geometry)</u> Using cosine rule on $\triangle OAX$, where $\overrightarrow{AX}=3\mathbf{b}$ and $\measuredangle OAX=\pi-\measuredangle AOB$ $\left\|\overrightarrow{OX}\right\|^2=\left\|\overrightarrow{OA}\right\|^2+\left\|\overrightarrow{AX}\right\|^2-2\left\|\overrightarrow{OA}\right\|\left\|\overrightarrow{AX}\right\|\cos\measuredangle OAX$ $\Rightarrow\ \left(\sqrt{73}\right)^2=(2)^2+\left(3\sqrt{5}\right)^2-2\|\mathbf{a}\|\|3\mathbf{b}\|\cos(\pi-\measuredangle AOB)$ $\Rightarrow\ 73=4+45+6\|\mathbf{a}\|\|\mathbf{b}\|\cos(\measuredangle AOB)$ $\Rightarrow\ 4=\|\mathbf{a}\|\|\mathbf{b}\|\cos(\measuredangle AOB)=\mathbf{a}.\mathbf{b}$
(b)	Area of triangle $OAX=\frac{1}{2}\left\|\overrightarrow{OA}\times\overrightarrow{OX}\right\|$ $=\frac{1}{2}\|\mathbf{a}\times(\mathbf{a}+3\mathbf{b})\|$ $=\frac{1}{2}\|\mathbf{a}\times\mathbf{a}+3\mathbf{a}\times\mathbf{b}\|=\frac{3}{2}\|\mathbf{a}\times\mathbf{b}\|$ $=\frac{3}{2}\|\mathbf{a}\|\|\mathbf{b}\|\sin\theta$ (where θ is the angle between **a** and **b**) To find $\sin\theta$: $\mathbf{a}.\mathbf{b}=4\Rightarrow\|\mathbf{a}\|\|\mathbf{b}\|\cos\theta=4$ $(2)\left(\sqrt{5}\right)\cos\theta=4$ $\cos\theta=\frac{2}{\sqrt{5}}\Rightarrow\sin\theta=\frac{1}{\sqrt{5}}$ Thus, area of triangle $OAX=\frac{3}{2}\|\mathbf{a}\|\|\mathbf{b}\|\sin\theta=\frac{3}{2}(2)\left(\sqrt{5}\right)\frac{1}{\sqrt{5}}=3$ <u>Alternatively</u>

	From $\triangle OAX$, $\cos\measuredangle OAX = \dfrac{\lvert\overrightarrow{OA}\rvert^2 + \lvert\overrightarrow{AX}\rvert^2 - \lvert\overrightarrow{OX}\rvert^2}{2\lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{AX}\rvert}$ $\cos\measuredangle OAX = \dfrac{(2)^2 + (3\sqrt{5})^2 - (\sqrt{73})^2}{2(2)(3\sqrt{5})} = \dfrac{-2}{\sqrt{5}}$ $\sin\measuredangle OAX = \sqrt{1-\left(\dfrac{-2}{\sqrt{5}}\right)^2} = \dfrac{1}{\sqrt{5}}$ Area of $\triangle OAX = \dfrac{1}{2}\lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{AX}\rvert\sin\measuredangle OAX$ $= \dfrac{1}{2}(2)(3\sqrt{5})\left(\dfrac{1}{\sqrt{5}}\right)$ $= 3$
(c)	Since $\lvert\mathbf{v}-\mathbf{a}\rvert = \lvert\mathbf{v}-\mathbf{b}\rvert$, point V with position vector v is equidistant from points A and B. Thus the set of points on the plane that is equidistant from points A and B, i.e. passes through mid-point of AB and is perpendicular to vector $\overrightarrow{AB}$. <u>Alternatively (Algebraically)</u> $\lvert\mathbf{v}-\mathbf{a}\rvert = \lvert\mathbf{v}-\mathbf{b}\rvert$ $\Rightarrow \lvert\mathbf{v}-\mathbf{a}\rvert^2 = \lvert\mathbf{v}-\mathbf{b}\rvert^2$ $(\mathbf{v}-\mathbf{a}).(\mathbf{v}-\mathbf{a}) = (\mathbf{v}-\mathbf{b}).(\mathbf{v}-\mathbf{b})$ $\lvert\mathbf{v}\rvert^2 - 2\mathbf{a}.\mathbf{v} + \lvert\mathbf{a}\rvert^2 = \lvert\mathbf{v}\rvert^2 - 2\mathbf{b}.\mathbf{v} + \lvert\mathbf{b}\rvert^2$ $2\mathbf{a}.\mathbf{v} - 2\mathbf{b}.\mathbf{v} = \lvert\mathbf{a}\rvert^2 - \lvert\mathbf{b}\rvert^2$ $\mathbf{v}.(\mathbf{a}-\mathbf{b}) = \dfrac{\lvert\mathbf{a}\rvert^2 - \lvert\mathbf{b}\rvert^2}{2} = -\dfrac{1}{2}$ Thus the set of points on the plane with normal $(\mathbf{a}-\mathbf{b})$ and point $\mathbf{v}$ with equation $\mathbf{v}.(\mathbf{a}-\mathbf{b}) = -\dfrac{1}{2}$.

4	**Solution**
(a)	 Explanation: Consider $x=\ln(4t)$, x is undefined for $t\le 0$, but that's not in the given range of t. So x is defined for $\frac{1}{2}<t\le 2$. Consider $y=\frac{1}{\sqrt{4t^2-1}}$, y is undefined for $t=\frac{1}{2}$. As $t\to\frac{1}{2}$, $y\to+\infty$. Thus there is an asymptote at $x=\ln\left(4\left(\frac{1}{2}\right)\right)=\ln 2$.
(b)	When $x=\ln(2\sqrt{2})$, $x=\ln(4t)=\ln(2\sqrt{2})$ $t=\frac{1}{\sqrt{2}}$ When $x=\ln 4$, $x=\ln(4t)=\ln 4$ $t=1$ Area of Region $R=\int_{\ln(2\sqrt{2})}^{\ln 4} y\,dx=\int_{\frac{1}{\sqrt{2}}}^{1} y\frac{dx}{dt}\,dt$ $=\int_{\frac{1}{\sqrt{2}}}^{1}\frac{1}{\sqrt{4t^2-1}}\left(\frac{1}{t}\right)dt=\int_{\frac{1}{\sqrt{2}}}^{1}\frac{1}{t\sqrt{4t^2-1}}\,dt$ (shown) $a=\frac{1}{\sqrt{2}}, b=1$
(c)	$u=\frac{1}{t}\Rightarrow t=\frac{1}{u}$; $\frac{dt}{du}=-\frac{1}{u^2}$ When $t=1$, $u=1$. When $t=\frac{1}{\sqrt{2}}$, $u=\sqrt{2}$.

	Area of Region $R=\int_{\frac{1}{\sqrt{2}}}^{1}\frac{1}{t\sqrt{4t^2-1}}\,\mathrm{d}t=\int_{\sqrt{2}}^{1}\frac{1}{\frac{1}{u}\sqrt{\frac{4}{u^2}-1}}\left(-\frac{1}{u^2}\right)\mathrm{d}u$ $=\int_{1}^{\sqrt{2}}\frac{1}{\sqrt{4-u^2}}\,\mathrm{d}u=\left[\sin^{-1}\left(\frac{u}{2}\right)\right]_{1}^{\sqrt{2}}$ $=\frac{\pi}{4}-\frac{\pi}{6}=\frac{\pi}{12}$
(d)	Volume of Revolution $=\pi\int_{\frac{1}{\sqrt{2}}}^{1}\left(\frac{1}{4t^2-1}\right)\left(\frac{1}{t}\right)\mathrm{d}t$ $=\pi\int_{\frac{1}{\sqrt{2}}}^{1}\frac{1}{4t^3-t}\,\mathrm{d}t$ $=0.63690$ $=0.637$ (3sf)

5	**Solution**
(a)	
(b)	$\mathrm{P}(B\mid F)=\frac{0.75q}{0.75q+0.25(0.05)}=\frac{3}{8}$ $\Rightarrow 6q=2.25q+0.0375$ $\Rightarrow q=0.01$
(c)	Required probability $=\left[(0.25)\left(\frac{1}{20}\right)\right](0.25)^2\left(\frac{19}{20}\right)^2\frac{3!}{2!}=0.00212$

6	Solution
(a)	$\mathrm{E}(X)=\sqrt{\mathrm{Var}(X)}$ $np=\sqrt{np(1-p)}$ $n^2p^2=np(1-p)$ $np=1-p$ (as $n>0, p>0$) $p=\dfrac{1}{n+1}$
(b)	$\dfrac{\mathrm{P}(X=1)}{\mathrm{P}(X=2)}=\dfrac{\dfrac{n!}{1!(n-1)!}\left(\dfrac{1}{n+1}\right)\left(\dfrac{n}{n+1}\right)^{n-1}}{\dfrac{n!}{2!(n-2)!}\left(\dfrac{1}{n+1}\right)^2\left(\dfrac{n}{n+1}\right)^{n-2}}$ $=\dfrac{n\left(\dfrac{n}{n+1}\right)}{\dfrac{n(n-1)}{2}\left(\dfrac{1}{n+1}\right)}$ $=\dfrac{2n}{n-1}$ Since $\dfrac{2n}{n-1}>1$ for positive integers n, $\mathrm{P}(X=1)>\mathrm{P}(X=2)$. Also note that $\mathrm{P}(X=0)=\left(\dfrac{n}{n+1}\right)^n=\mathrm{P}(X=1)$ Hence the modes of X are 0 and 1.

7	Solution
(a)	$\mathrm{E}(X)=1\times0.4+2\times0.3+4\times0.2+8\times0.1=2.6$ $\mathrm{E}(X^2)=1\times0.4+4\times0.3+16\times0.2+64\times0.1=11.2$ $\mathrm{Var}(X)=E(X^2)-[E(X)]^2=11.2-2.6^2=4.44$ $\mathrm{Var}(3X-2)=9\mathrm{Var}(X)+\mathrm{Var}(2)=9(4.44)+0=39.96$
(b)	Since $n=40$ is large, $M=\dfrac{X_1+...+X_{40}}{40}\sim N\left(2.6, \dfrac{4.44}{40}\right)$ approximately by Central Limit Theorem $\mathrm{P}(M>3)=0.11495=0.115$ (3 s.f.)

8 **Solution**

(a) Method 1 (P&C – treat all balls as distinct)

$$P(0\ Y)=\frac{{}^{10}C_4}{{}^{20}C_4}=\frac{14}{323} \quad \text{or } 0.0433 \text{ (3 sf)}$$

Method 2 (Probability)

$$P(0\ Y)=\left(\frac{10}{20}\right)\left(\frac{9}{19}\right)\left(\frac{8}{18}\right)\left(\frac{7}{17}\right)=\frac{14}{323}=0.0433 \text{ (3 sf)}$$

(b) Method 1 (P&C)

Let X represent black or red ball.

G	Y	Case	Probability
2	1	GGYX	$\frac{{}^{6}C_2\,{}^{10}C_1\,{}^{4}C_1}{{}^{20}C_4}=\frac{600}{4845}=\frac{40}{323}$
2	0	GGXX	$\frac{{}^{6}C_2\,{}^{4}C_2}{{}^{20}C_4}=\frac{90}{4845}=\frac{6}{323}$
3	1	GGGY	$\frac{{}^{6}C_3\,{}^{14}C_1}{{}^{20}C_4}=\frac{280}{4845}=\frac{56}{969}$
3	0	GGGX	
4	0	GGGG	$\frac{{}^{6}C_4}{{}^{20}C_4}=\frac{15}{4845}=\frac{1}{323}$

$$\text{Required probability}=\frac{40}{323}+\frac{6}{323}+\frac{56}{969}+\frac{1}{323}=\frac{197}{969} \quad \text{or } 0.203 \text{ (3s.f.)}$$

Method 2 (probability)

Let X represent black or red ball.

G	Y	Case	Probability
2	1	GGYX	$\left(\frac{6}{20}\right)\left(\frac{5}{19}\right)\left(\frac{10}{18}\right)\left(\frac{4}{17}\right)\times\frac{4!}{2!}=\frac{40}{323}$
2	0	GGXX	$\left(\frac{6}{20}\right)\left(\frac{5}{19}\right)\left(\frac{4}{18}\right)\left(\frac{3}{17}\right)\times\frac{4!}{2!2!}=\frac{6}{323}$
3	1	GGGY	non-green $\left(\frac{6}{20}\right)\left(\frac{5}{19}\right)\left(\frac{4}{18}\right)\left(\frac{14}{17}\right)\times\frac{4!}{3!}=\frac{56}{969}$
3	0	GGGX	
4	0	GGGG	$\left(\frac{6}{20}\right)\left(\frac{5}{19}\right)\left(\frac{4}{18}\right)\left(\frac{3}{17}\right)=\frac{1}{323}$

$$\text{Required probability}=\frac{40}{323}+\frac{6}{323}+\frac{56}{969}+\frac{1}{323}=\frac{197}{969} \quad \text{or } 0.203 \text{ (3s.f.)}$$

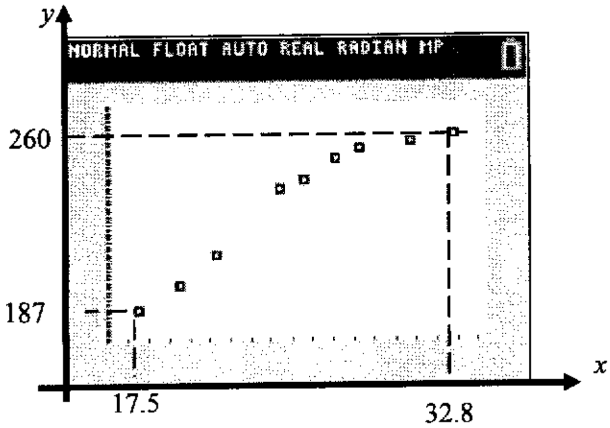
(c) Method 1: P&C

$$P(\text{black ball} | \$600 \text{ won}) = \frac{P(\text{black ball and } \$600)}{P(\$600)}$$
$$= \frac{P(BYYY)}{P(BYYY) + P(RRGG)}$$
$$= \frac{\text{no. of ways BYYY}}{\text{no. of ways BYYY} + \text{no. of ways RRGG}}$$
$$= \frac{{}^1C_1 {}^{10}C_3}{{}^1C_1 {}^{10}C_3 + {}^3C_2 {}^6C_2}$$
$$= \frac{8}{11}$$

Method 2: Prob

$$P(\text{black ball} | \$600 \text{ won}) = \frac{P(\text{black ball and } \$600)}{P(\$600)}$$
$$= \frac{P(BYYY)}{P(BYYY) + P(RRGG)}$$
$$= \frac{\left(\frac{1}{20}\right)\left(\frac{10}{19}\right)\left(\frac{9}{18}\right)\left(\frac{8}{17}\right) \times \frac{4!}{3!}}{\left(\frac{1}{20}\right)\left(\frac{10}{19}\right)\left(\frac{9}{18}\right)\left(\frac{8}{17}\right) \times \frac{4!}{3!} + \left(\frac{3}{20}\right)\left(\frac{2}{19}\right)\left(\frac{6}{18}\right)\left(\frac{5}{17}\right) \times \frac{4!}{2!2!}}$$
$$= \frac{8}{11}$$

9	Solution
(a)	Let A be the random variable denoting the travel time in mins using route A. $A \sim N(45,7^2)$ $P(A>k)<0.2$ $k>50.9$ Least k is 51 minutes
(b)(i)	Let B be the random variable denoting the travel time in mins using route B. $B \sim N(53,5^2)$ $E(A-B)=-8$ $\text{Var}(A-B)=74$ $A-B \sim N(-8,74)$ Required Probability = $P(\lvert A-B\rvert>5)=1-P(-5\le A-B\le 5)=0.702$
(b)(ii)	$A-B \sim N(-8,74)$ $P(A-B>0)=0.17619$ Let C be number of days James will arrive at the company later than John out of 5 days $C \sim B(5, 0.17619)$ $P(C\le 1)=0.785$
(c)	Let $X=A+5$ $E(X)=45+5=50$; $\text{Var}(X)=7^2$ $X \sim N(50,7^2)$ Let $Y=1.08B$ $E(Y)=1.08(53)=57.24$; $\text{Var}(Y)=1.08^2 5^2=29.16$ $Y \sim N(57.24,29.16)$ $W=\frac{X_1+X_2+X_3+Y}{4}$ $E(W)=\frac{1}{4}(150+57.24)=51.81$ $\text{Var}(W)=\frac{1}{16}(3\times 7^2+29.16)=11.01$ $W \sim N(51.81,11.01)$ Required Probability = $P(50<W<60)=0.701$

10	**Solution**
(a)	No, because correlation does not necessarily imply causation.
(b)	 The scatter diagram shows that as x increases, y increases but at a decreasing rate. Therefore, x and y should not be modelled by an equation of the form $y = a + bx$.
(c)	For $\sqrt{x}$ and y, $r = 0.97334$ For $\ln x$ and y, $r = 0.98080$ The value of $\lvert r\rvert$ for $y = a + b\ln x$ is closer than to 1 than the value of $\lvert r\rvert$ for $y = a + b\sqrt{x}$. Therefore, $y = a + b\ln x$ is a better model.
(d)(i)	The regression line y on $\ln x$: $y = -179.83 + 128.49\ln x$ $x = 20$, $y = -179.83 + 128.49(\ln 20) = 205$ (3s.f.)
(d)(ii)	Sub y=230 into $y = -179.83 + 128.49\ln x$ $230 = -179.83 + 128.49\ln x$ $\ln x = 3.18959$ $x = 24.3$ (to 3 sf)
(e)	**Estimate for d(i):** Since $x = 20$ is within the data range $17.5 \le x \le 32.8$ (i.e. estimate is obtained via interpolation) and $r = 0.98080$ suggest a strong positive linear correlation between BMI and total cholesterol , therefore the estimation is reliable. **Estimate for d(ii):** Since neither variable is independent, we should be using the regression line of $\ln x$ on y to estimate x given y. Thus the estimate is not reliable. [Alternative addition] Since r-value is 0.98080, both regression lines should be close to each other and thus the estimate is reliable.

11	**Solution**
(a)	1-tail test. Because "overstated" suggests that the heel stack is less than what is claimed. Let μ be the population mean heel stack of the SuperFly. $H_0 : \mu = 42.7, \quad H_1 : \mu < 42.7$
(b)	Every shoe has an equal probability of being selected, and every shoe is selected independently of other shoes.
(c)	Need to assume that the population heel stack is normally distributed in order for sample mean to also be normally distributed, as sample size is too small to use Central Limit Theorem approximation.
(d)	$\bar{x} = \frac{\sum(x-42.7)}{16} + 42.7 = 42.3$ $s^2 = \frac{1}{15}\left(\sum(x-42.7)^2 - \frac{\left(\sum(x-42.7)\right)^2}{16}\right) = \frac{1}{15}\left(13.2 - \frac{(-6.4)^2}{16}\right) = 0.70933 \approx 0.709$ (3 s.f.) Let $\bar{X}$ be the mean heel stack in a sample of 16. Under H_0, $\bar{X} \sim N\left(42.7, \frac{0.70933}{16}\right)$. Test at 2.5% level of significance: Reject if p-value < 0.025; p-value $= 0.0287 > 0.025$ | Reject if $\bar{x} < 42.287$; $\bar{x} = 42.3 > 42.287$ | Reject if $z < -1.95996$; $z = \frac{42.3-42.7}{\sqrt{0.70933/16}} = -1.8997 > -1.95996$ So we do not reject H_0 and conclude at 2.5% level of significance that there is insufficient evidence that the heel stack is less than 42.7 millimetres.
(e)	To test $H_0 : \mu = 42.7, \quad H_1 : \mu \neq 42.7$ at 2.5% level of significance. Under H_0, $\bar{X} \sim N\left(42.7, \frac{0.987}{20}\right)$. We are given that H_0 is rejected. From GC, critical region is given by: $\bar{x} < 42.2$ or $\bar{x} > 43.2$ <u>Alternative Method (Standardisation)</u> To reject H_0, $P(\bar{X} < \bar{x}) < 0.0125$ or $P(\bar{X} > \bar{x}) < 0.0125$ $P\left(Z < \frac{\bar{x}-42.7}{\sqrt{0.987/20}}\right) < 0.0125$ or $P\left(Z > \frac{\bar{x}-42.7}{\sqrt{0.987/20}}\right) < 0.0125$ $\frac{\bar{x}-42.7}{\sqrt{0.987/20}} < -2.2414$ or $\frac{\bar{x}-42.7}{\sqrt{0.987/20}} > 2.2414$ $\bar{x} < 42.2$ or $\bar{x} > 43.2$.