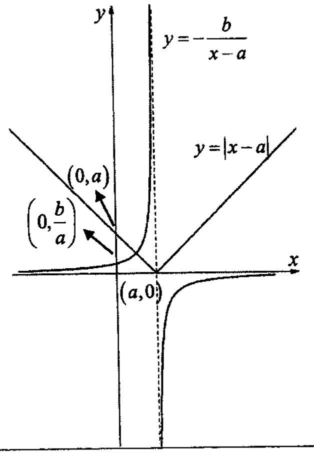
BP~421

2025 NJC Preliminary Examination H2 Mathematics Paper 1 Solutions

1	Suggested Solution
	Let the exchange rate for US Dollar, Japanese Yen and Chinese Yuan be 1 to U, 1 to J and 1 to C Singapore Dollars respectively. $150U + 5500J + 1000C = 419.30$ $250U + 9500J + 2200C = 797.20$ $425U + 1000J + 2000C = 913.10$ By GC, $U=1.2856,\ \ J=0.00862,\ \ C=0.17905$ For Maybelline, $1.2856a+1200(0.17905)=568.40$ $a=275$

2	Suggested Solutions
(i)	$(1+\mathrm{i})z+2w=-2+4\mathrm{i}$ $3z-w=4+2\mathrm{i}$ $\Rightarrow 6z-2w=8+4\mathrm{i}$ Solving simultaneously by elimination: $(6+1+\mathrm{i})z=-2+4\mathrm{i}+8+4\mathrm{i}$ $(7+\mathrm{i})z=6+8\mathrm{i}$ $z=\frac{6+8\mathrm{i}}{7+\mathrm{i}}\times\frac{7-\mathrm{i}}{7-\mathrm{i}}$ $=\frac{42-6\mathrm{i}+56\mathrm{i}+8}{50}$ $=\frac{50+50\mathrm{i}}{50}$ $=1+\mathrm{i}$ w $=3z-4-2\mathrm{i}$ $=3(1+\mathrm{i})-4-2\mathrm{i}$ $=-1+\mathrm{i}$
(ii)	$\frac{w}{z}$ $=\frac{-1+\mathrm{i}}{1+\mathrm{i}}$ $=\frac{-1+\mathrm{i}}{1+\mathrm{i}}\times\frac{1-\mathrm{i}}{1-\mathrm{i}}$ $=\frac{-1+\mathrm{i}+\mathrm{i}+1}{1^2+1^2}$ $=\mathrm{i}$ $w=\mathrm{i}z$ **Alternative** $\frac{w}{z}$ $=\frac{-1+\mathrm{i}}{1+\mathrm{i}}$ $=\frac{\mathrm{i}^2+\mathrm{i}}{1+\mathrm{i}}$ $=\frac{\mathrm{i}(\mathrm{i}+1)}{1+\mathrm{i}}$ $=\mathrm{i}$ $w=\mathrm{i}z$

	Rotating line segment OZ by $\frac{\pi}{2}$ anti-clockwise about the origin will give OW. Or line segment OW is a anti-clockwise rotation about the origin by $\frac{\pi}{2}$ from line segment OZ.

[redacted]	
(i)	
(ii)	$-\frac{b}{x-a}=-(x-a)$ $(x-a)^2=b$ $x=a-\sqrt{b}$ or $x=a+\sqrt{b}$ (rejected since $x<a$) $x<a-\sqrt{b}$ or $x>a$

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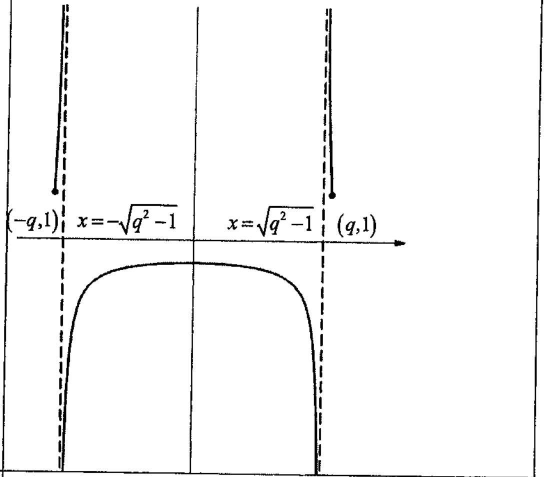
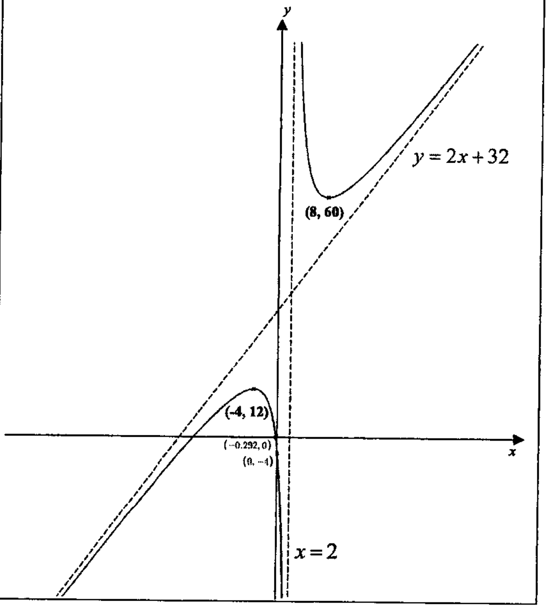
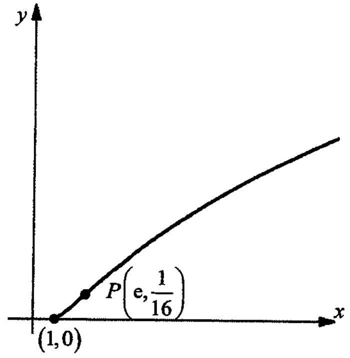
4	Suggested Solutions
	$y=1-\sqrt{q^2-x^2}$, $q>1$ $(y-1)^2=q^2-x^2$ $x^2+(y-1)^2=q^2$ $y=\mathrm{f}(x)$ is a semi-circle.
(i)	
(ii)	$y=1-\sqrt{q^2-x^2}$ Replace y with $y+1$: $y+1=1-\sqrt{q^2-x^2}$ $y=-\sqrt{q^2-x^2}$ Replace x with qx: $y=-\sqrt{q^2-(qx)^2}$ $y=-q\sqrt{1-x^2}$ Replace y with qy: $qy=-q\sqrt{1-x^2}$ $y=-\sqrt{1-x^2}$ Translate 1 unit in the negative y direction. Scale by a factor of $\frac{1}{q}$ parallel to the x-axis. Scale by a factor of $\frac{1}{q}$ parallel to the y-axis.

5	Suggested Solutions
(i)	Given vertical asymptote at $x=2$, $p=-2$ $\therefore y=\frac{2x^2+kx+8}{x-2}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x-2)(4x+k)-(2x^2+kx+8)}{(x-2)^2}$ Given $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=-4}=0$, $((-4)-2)(4(-4)+k)-(2(-4)^2+k(-4)+8)=0$ $(-6)(-16+k)-(32-4k+8)=0$ $\therefore k=28$ By long division or otherwise, $\therefore y=\frac{2x^2+28x+8}{x-2}=2x+32+\frac{72}{x-2}$ Equation of oblique asymptote of C is $\therefore y=2x+32$
(ii)	

6	Suggested Solutions
(a) **(i)**	$S_\infty = \frac{1}{1-r}$ and $S_3 = \frac{(1-r^3)}{1-r}$, $\|r\|<1$ **Method 1** $\frac{1}{1-r} = \left(\frac{1-r^3}{1-r}\right)^2$ $1 = \frac{(1-r^3)^2}{1-r}$, since $\frac{1}{1-r} \neq 0$ $1-r = (1-r^3)^2$ $1-r = 1-2r^3+r^6$ $r^6-2r^3+r=0$ $r^5-2r^2+1=0$, since $r \neq 0$ $(r-1)(r^4+r^3+r^2-r-1)=0$ Since $r \neq 1$, $r^4+r^3+r^2-r-1=0$ **Method 2** $\frac{1}{1-r} = (r^2+r+1)^2$ $1 = (r^2+r+1)^2(1-r)$ $(r^2+(r+1))^2(1-r)-1=0$ $\left[r^4+(1+r)^2+2r^2(1+r)\right](1-r)-1=0$ $r^4(1-r)+(1+r)(1-r^2)+2r^2(1-r^2)-1=0$ $(r^4-r^5)+(1-r^2)+r-r^3+(2r^2-2r^4)-1=0$ $r+r^2-r^3-r^4-r^5=0$ $r^4+r^3+r^2-r-1=0$, since $r \neq 0$
(a) **(ii)**	By G.C., since $\|r\|<1$ $r=-0.661$ (3 s.f.) or $r=0.848$ (3 s.f.)

(b)	Sum of first $4n$ terms $=\frac{4n}{2}\left[2(7)+(4n-1)(d)\right]$ $=28n+8dn^2-2dn$ Terms to be removed, $7+3d, 7+7d, 7+11d, \ldots,$ (First term = $7+3d$; common difference = $4d$; No. of terms removed = n terms. Sum of removed terms $=\frac{n}{2}\left[2(7+3d)+(n-1)(4d)\right]$ $=\frac{n}{2}(14+6d+4dn-4d)$ $=\frac{n}{2}(14+2d+4dn)$ $=7n+nd+2dn^2$ Sum of remaining terms $=28n+8dn^2-2dn-7n-nd-2dn^2$ $=6dn^2-3dn+21n$
	Method 2 Sum every three consecutive terms as a single term i.e. $7+(7+d)+(7+2d)$, $(7+4d)+(7+5d)+(7+6d)$, … $\Rightarrow 21+3d$, $21+15d$, $21+27d$ In the progression above, first term is $21+3d$, common difference is $12d$, and there are n terms. Sum $=\frac{n}{2}(2(21+3d)+(n-1)12d)$ $=21n+3dn+n(n-1)6d$ $=6dn^2-3dn+21n$

7	Suggested Solutions
(i)	$x=\mathrm{e}^{4t}$, $y=t^2$, $t\geq 0$. Sub $t=\dfrac{\ln x}{4}$ into $y=t^2$: $y=\left(\dfrac{\ln x}{4}\right)^2$ $\Rightarrow y=\dfrac{1}{16}(\ln x)^2$
(ii)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{8x}(\ln x)$ Steepest gradient: $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ $=\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$ $=\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{1}{8x}(\ln x)$ $=\dfrac{1}{8}\dfrac{\mathrm{d}}{\mathrm{d}x}x^{-1}(\ln x)$ $=\dfrac{1}{8}\left[-x^{-2}(\ln x)+\left(\dfrac{1}{x}\right)^2\right]$ $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$ $\dfrac{1}{8}\left[-\dfrac{1}{x^2}(\ln x)+\left(\dfrac{1}{x}\right)^2\right]=0$ $\left(\dfrac{1}{x}\right)^2\left[-(\ln x)+1\right]=0$ $-(\ln x)+1=0$ $\Rightarrow x=\mathrm{e}$ $\left(\text{since } \dfrac{1}{x}\neq 0 \text{ for } x\in\mathbf{R}\right)$ $P\left(\mathrm{e},\dfrac{1}{16}\right)$

(iii)	Point $\left(\mathrm{e},\dfrac{1}{16}\right)$ is the point with the greatest gradient. When $t=0$, $x=\mathrm{e}^{4(0)}=1$, $y=0$ Point $(1,0)$ is the initial point. For $t\geq 0$, $x>1$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{8x}(\ln x)>0$. 

8	Suggested Solutions
(i)	$\frac{a}{a-x}-1$ $=a(a-x)^{-1}-1$ $=\frac{a}{a}\left(1-\frac{x}{a}\right)^{-1}-1$ $=\left(1-\frac{x}{a}\right)^{-1}-1$ $=\left(1+(-1)\left(-\frac{x}{a}\right)+\frac{(-1)(-2)}{2!}\left(-\frac{x}{a}\right)^2+\ldots\right)-1$ $=\frac{x}{a}+\frac{x^2}{a^2}+\ldots$ Validity range: $\left\|-\frac{x}{a}\right\|<1$ $\|-1\|\left\|\frac{x}{a}\right\|<1$ $\frac{\|x\|}{\|a\|}<1$ $\|x\|<a$ (since $a>0$) $-a<x<a$
(ii)	$e^{\frac{a}{a-x}-1}=e^{\frac{x}{a}+\frac{x^2}{a^2}+\ldots}$ $=1+\left(\frac{x}{a}+\frac{x^2}{a^2}+\ldots\right)+\frac{1}{2}\left(\frac{x}{a}+\frac{x^2}{a^2}+\ldots\right)^2$ $\approx 1+\left(\frac{x}{a}+\frac{x^2}{a^2}+\ldots\right)+\frac{1}{2}\frac{x^2}{a^2}$ $=1+\frac{x}{a}+\frac{3}{2}\frac{x^2}{a^2}+\ldots$

(iii)	$\int_0^{\frac{a}{2}} e^{\frac{a}{a-x}-1}\,dx$ $\approx \int_0^{\frac{a}{2}} 1+\frac{x}{a}+\frac{3}{2}\frac{x^2}{a^2}\,dx$ $=\left[x+\frac{x^2}{2a}+\frac{1}{2}\frac{x^3}{a^2}\right]_0^{\frac{a}{2}}$ $=\left(\frac{a}{2}\right)+\frac{\left(\frac{a}{2}\right)^2}{2a}+\frac{1}{2}\frac{\left(\frac{a}{2}\right)^3}{a^2}$ $=\frac{11}{16}a$ The neglected terms in the expansion of $e^{\frac{a}{a-x}-1}$ for $0<x<\frac{a}{2}$ are all positive terms. The approximating curve $1+\frac{x}{a}+\frac{3}{2}\frac{x^2}{a^2}+\ldots$ will be below $y=e^{\frac{a}{a-x}-1}$. Hence area under the approximated curve will be smaller than area under $y=e^{\frac{a}{a-x}-1}$ for $0<x<\frac{a}{2}$.

9	Solution
(a)	$\int \cos(3\ln x)\,\mathrm{d}x$ $= x\cos(3\ln x) - \int x\left(-\sin(3\ln x)\frac{3}{x}\right)\mathrm{d}x$ $= x\cos(3\ln x) + 3\int \sin(3\ln x)\,\mathrm{d}x$ $= x\cos(3\ln x) + 3\left[x\sin(3\ln x) - \int x\cos(3\ln x)\frac{3}{x}\,\mathrm{d}x\right]$ $= x\cos(3\ln x) + 3x\sin(3\ln x) - 9\int \cos(3\ln x)\,\mathrm{d}x$ Hence $10\int \cos(3\ln x)\,\mathrm{d}x = x\cos(3\ln x) + 3x\sin(3\ln x) + c$ $\int \cos(3\ln x)\,\mathrm{d}x = \frac{x}{10}\cos(3\ln x) + \frac{3x}{10}\sin(3\ln x) + C$
(b) (i)	$\int \frac{1-x}{1-\sqrt{x}}\,\mathrm{d}x = \int \frac{\left(1-\sqrt{x}\right)\left(1+\sqrt{x}\right)}{1-\sqrt{x}}\,\mathrm{d}x$ $= \int 1+\sqrt{x}\,\mathrm{d}x$ $= x + \frac{2}{3}x^{\frac{3}{2}} + c$ OR $\int \frac{1-x}{1-\sqrt{x}}\,\mathrm{d}x = \int \frac{(1-x)\left(1+\sqrt{x}\right)}{\left(1-\sqrt{x}\right)\left(1+\sqrt{x}\right)}\,\mathrm{d}x$ $= \int \frac{(1-x)\left(1+\sqrt{x}\right)}{1-x}\,\mathrm{d}x$ $= \int 1+\sqrt{x}\,\mathrm{d}x$ $= x + \frac{2}{3}x^{\frac{3}{2}} + c$
(ii)	$u = 1-\sqrt{x} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{1}{2\sqrt{x}} = -\frac{1}{2(1-u)}$ $\int \frac{1}{1-\sqrt{x}}\,\mathrm{d}x$ $= \int \frac{1}{u}\cdot -2(1-u)\,\mathrm{d}u$ $= 2\int 1-\frac{1}{u}\,\mathrm{d}u$ $= 2(u-\ln u) + c$, since $0 < x < 1, u > 0$ $= 2\left(1-\sqrt{x}-\ln\left(1-\sqrt{x}\right)\right) + c$ $= -2\sqrt{x} - 2\ln\left(1-\sqrt{x}\right) + C$

	$\int \frac{x}{1-\sqrt{x}}\,\mathrm{d}x = -\int \frac{1-x}{1-\sqrt{x}}\,\mathrm{d}x + \int \frac{1}{1-\sqrt{x}}\,\mathrm{d}x$ $= -x - \frac{2}{3}x^{\frac{3}{2}} + 2\left(1-\sqrt{x}-\ln\left(1-\sqrt{x}\right)\right) + c$ $= -x - \frac{2}{3}x^{\frac{3}{2}} + 2 - 2\sqrt{x} - 2\ln\left(1-\sqrt{x}\right) + c$ $= -x - \frac{2}{3}x^{\frac{3}{2}} - 2\sqrt{x} - 2\ln\left(1-\sqrt{x}\right) + C$

10	Suggested Solution
(i)	$u_1 = S_1 = 1$ $1 = A - \frac{2}{2!}$ $A = 2$
(ii)	$u_n = S_n - S_{n-1}$ $= \left[2 - \frac{2}{(n+1)!}\right] - \left[2 - \frac{2}{n!}\right]$ $= \frac{2}{n!} - \frac{2}{(n+1)!}$ $= \frac{2(n+1)}{(n+1)!} - \frac{2}{(n+1)!}$ $= \frac{2n}{(n+1)!}$ $= \frac{2n}{(n+1)(n)\left[(n-1)!\right]}$ $= \frac{2}{(n+1)\left[(n-1)!\right]}$ So $u_n = \frac{2}{(n+1)\left[(n-1)!\right]}$. Check for u_1: $\frac{2}{(1+1)\left[(1-1)!\right]} = \frac{2}{2(0!)} = 1 = u_1$ For $n \geq 1$, $u_n = \frac{2}{(n+1)\left[(n-1)!\right]}$.
(iii)	$u_{n+1} = \frac{2}{(n+2)(n!)}$. $\frac{u_{n+1}}{u_n} = \frac{\frac{2}{(n+2)(n!)}}{\frac{2}{(n+1)\left[(n-1)!\right]}}$ $= \frac{2}{(n+2)(n!)} \times \frac{(n+1)\left[(n-1)!\right]}{2}$ $= \frac{n+1}{n(n+2)}$ So $u_{n+1} = \frac{n+1}{n(n+2)} u_n$

<table>
<tr><td>(iv)</td><td>As $n \to \infty$, $(n+1)! \to \infty$, $\frac{2}{(n+1)!} \to 0$.

Therefore S_n converges to 2 .</td></tr>
<tr><td>(v)</td><td>$u_m + u_{m+1} + u_{m+2} + \ldots$
$= \sum_{r=1}^{\infty} u_r - \sum_{r=1}^{m-1} u_r$
$= 2 - \left[2 - \frac{2}{(m-1+1)!}\right]$
$= \frac{2}{m!} \leq 10^{-10}$

By GC,
<table>
<tr><td>m</td><td>$\frac{2}{m!}$</td></tr>
<tr><td>…</td><td>…</td></tr>
<tr><td>13</td><td>3.21×10^{-10}</td></tr>
<tr><td>14</td><td>2.29×10^{-11}</td></tr>
<tr><td>…</td><td>…</td></tr>
</table>

Least value of m is 14.</td></tr>
</table>

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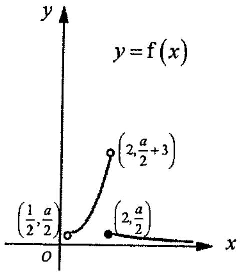
11	Suggested Solution
(i)	 $\mathrm{R_f}=\left(0,\frac{a}{2}+3\right)$
(ii)	For $\frac{1}{2}<x<2$, $y=\frac{a}{2}+\frac{4}{3}\left(x-\frac{1}{2}\right)^2$ $\frac{1}{2}\pm\sqrt{\frac{3}{4}\left(y-\frac{a}{2}\right)}=x$ $x=\frac{1}{2}+\sqrt{\frac{3}{4}\left(y-\frac{a}{2}\right)}$ (since $x>\frac{1}{2}$) $\therefore \mathrm{f}^{-1}(x)=\frac{1}{2}+\sqrt{\frac{3}{4}\left(x-\frac{a}{2}\right)}$ For $x\geq 2$, $y=\frac{a}{x}$ $x=\frac{a}{y}$ $\therefore \mathrm{f}^{-1}(x)=\frac{a}{x}$ $\mathrm{f}^{-1}(x)=\begin{cases}\frac{a}{x} & \text{for } x\in\mathbf{R},\ 0<x\leq\frac{a}{2}\\ \frac{1}{2}+\sqrt{\frac{3}{4}\left(x-\frac{a}{2}\right)} & \text{for } x\in\mathbf{R},\ \frac{a}{2}<x<\frac{a}{2}+3\end{cases}$
(iii)	Since $\mathrm{R_g}=(3,\infty)\subseteq\left(\frac{1}{2},\infty\right)=\mathrm{D_f}$, fg exists

(iv)	**Method 1:** $\mathrm{fg}(k)=\frac{a}{7}$ $\mathrm{g}(k)=\mathrm{f}^{-1}\left(\frac{a}{7}\right)$ $3+\mathrm{e}^k=\frac{a}{\left(\frac{a}{7}\right)}\left(\because 0<\frac{a}{7}\leq\frac{a}{2}\right)$ $\mathrm{e}^k=4$ $k=\ln 4$ **Method 2:** $\mathrm{fg}(k)=\mathrm{f}\left(3+\mathrm{e}^k\right)$ $=\frac{a}{3+\mathrm{e}^k}\ \left(\because 3+\mathrm{e}^k>3>2\right)$ $\frac{a}{3+\mathrm{e}^k}=\frac{a}{7}$ $7=3+\mathrm{e}^k$ $4=\mathrm{e}^k$ $k=\ln 4$

12 Suggested Solutions [illegible]

(i) $y=0$

(ii)

$$\overrightarrow{OM}=\overrightarrow{OA}+\lambda\left(\overrightarrow{OB}-\overrightarrow{OA}\right),0\le\lambda\le1$$

$$\overrightarrow{OM}=\begin{pmatrix}0\\0\\3\end{pmatrix}+\lambda\left(\begin{pmatrix}10\\0\\4\end{pmatrix}-\begin{pmatrix}0\\0\\3\end{pmatrix}\right),0\le\lambda\le1$$

$$\overrightarrow{OM}=\begin{pmatrix}10\lambda\\0\\3+\lambda\end{pmatrix},0\le\lambda\le1$$

OR

$$\overrightarrow{OM}=\begin{pmatrix}10-10\mu\\0\\4-\mu\end{pmatrix},0\le\mu\le1$$

(iii)

$$\overrightarrow{MC}=\overrightarrow{OC}-\overrightarrow{OM}=\begin{pmatrix}6\\4\\2\end{pmatrix}-\begin{pmatrix}10\lambda\\0\\3+\lambda\end{pmatrix}=\begin{pmatrix}6-10\lambda\\4\\-1-\lambda\end{pmatrix}$$

<u>Method 1:</u>

$$\overrightarrow{MC}\bullet\overrightarrow{EC}=0\ \Rightarrow\ \begin{pmatrix}6-10\lambda\\4\\-1-\lambda\end{pmatrix}\bullet\begin{pmatrix}0\\0\\1\end{pmatrix}=0$$

$-1-\lambda=0\ \Rightarrow\ \lambda=-1$

Hence, since $0\le\lambda\le1$ for a point on line segment AB, cables will not be perpendicular.

<u>Method 2:</u>

$$\overrightarrow{OM}=\begin{pmatrix}0\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}10\\0\\1\end{pmatrix}=\begin{pmatrix}10\lambda\\0\\3+\lambda\end{pmatrix}$$

$$\overrightarrow{MC}=\overrightarrow{OC}-\overrightarrow{OM}=\begin{pmatrix}6\\4\\2\end{pmatrix}-\begin{pmatrix}10\lambda\\0\\3+\lambda\end{pmatrix}=\begin{pmatrix}6-10\lambda\\4\\-1-\lambda\end{pmatrix}$$

$$\cos\angle MCE=\frac{\overrightarrow{MC}\bullet\overrightarrow{EC}}{\left|\overrightarrow{MC}\right|\left|\overrightarrow{EC}\right|}$$

$$=\frac{\begin{pmatrix}6-10\lambda\\4\\-1-\lambda\end{pmatrix}\bullet\begin{pmatrix}0\\0\\1\end{pmatrix}}{\sqrt{(6-10\lambda)^2+4^2+(-1-\lambda)^2}}$$

$$=-\frac{1+\lambda}{\sqrt{(6-10\lambda)^2+4^2+(1+\lambda)^2}}$$

From GC, for $0\le\lambda\le1$, $\angle MCE$ is always greater than $\frac{\pi}{2}$. (i.e. between 97.9° and 109.4°). Hence it is impossible for MC and EC to be perpendicular.

<u>Method 3:</u>

If both cables are perpendicular, it is necessary that M is a point that is vertically 2 units from the horizontal ground.

z-component of $\overrightarrow{OM}$ is $3+\lambda$, where $0\le\lambda\le1$. This means that the z-component is always greater than 2.

<u>Both cables are never perpendicular to each other.</u>

(iv) Solving $ECFG$: $x-y=2$ and OAB: $y=0$:

Method 1a: Intersection of planes (using GC)

$$\begin{pmatrix}x\\y\\z\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\0\end{pmatrix}=2\text{ and }\begin{pmatrix}x\\y\\z\end{pmatrix}\bullet\begin{pmatrix}0\\1\\0\end{pmatrix}=0$$

From G.C.:

```
NORMAL FLOAT AUTO REAL DEGREE MP
PLYSMLT2 APP
        SOLUTION SET
x1=2
x2=0
x3=x3
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$$\mathrm{r}=\begin{pmatrix}2\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}0\\0\\1\end{pmatrix},\lambda\in\mathrm{R}$$

Method 1b: Intersection of planes (w/o GC)

Solving simultaneously, we get $x=2$, $y=0$ and $z\in\mathbb{R}$.

Equation of line of intersection l_{FG}:

$$\mathbf{r}=\begin{pmatrix}2\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}0\\0\\1\end{pmatrix},\lambda\in\mathbb{R}$$

Points F and G lie on l so $\overrightarrow{FG}$ is parallel to $\begin{pmatrix}0\\0\\1\end{pmatrix}$ and is parallel to $\overrightarrow{CE}$.

Hence, $ECFG$ is a trapezium.

Method 2: Solve for points F and G

To find F:
Solve $ECFG$ with $x-y=2$ and line AB with $\mathbf{r}=\begin{pmatrix}10\lambda\\0\\3+\lambda\end{pmatrix}$:

$10\lambda-0=2\Rightarrow\lambda=\frac{1}{5}$

$$\overrightarrow{OF}=\begin{pmatrix}10\left(\frac{1}{5}\right)\\0\\3+\frac{1}{5}\end{pmatrix}=\begin{pmatrix}2\\0\\16/5\end{pmatrix}$$

To find G:
Solve $ECFG$ with $x-y=2$ and line OD with $r=\mu\begin{pmatrix}1\\0\\0\end{pmatrix}$:

$\mu-0=2\Rightarrow\mu=2$

$$\overrightarrow{OG}=\begin{pmatrix}2\\0\\0\end{pmatrix}$$

$$\overrightarrow{GF}=\overrightarrow{OF}-\overrightarrow{OG}=\begin{pmatrix}2\\0\\16/5\end{pmatrix}-\begin{pmatrix}2\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\16/5\end{pmatrix}=k\begin{pmatrix}0\\0\\1\end{pmatrix}$$, for non-zero k.

$\overrightarrow{GF}$ is parallel to $\begin{pmatrix}0\\0\\1\end{pmatrix}$ and is parallel to $\overrightarrow{CE}$.

(v) **Method 1:** Apply $\overrightarrow{EG}\perp\overrightarrow{EC}$

Since $\overrightarrow{EC}$ is perpendicular to $z=0$, $\overrightarrow{EC}\perp\overrightarrow{EG}$.

$\overrightarrow{OG}=\begin{pmatrix}2\\0\\0\end{pmatrix},\overrightarrow{OE}=\begin{pmatrix}6\\4\\0\end{pmatrix}$ (FG and CE are vertical poles)

Perpendicular height of trapezium $ECFG$

$$\left|\overrightarrow{EG}\right|=\left|\begin{pmatrix}2\\0\\0\end{pmatrix}-\begin{pmatrix}6\\4\\0\end{pmatrix}\right|=\left|\begin{pmatrix}-4\\-4\\0\end{pmatrix}\right|=4\sqrt{2}\text{ (or }\sqrt{32}\text{)}$$

Method 2: Use of $\left|\mathbf{a}\times\frac{\mathbf{b}}{|\mathbf{b}|}\right|$

To find $\overrightarrow{OF}$:
Solve equation of l_{AB} and l_{FG} simultaneously:

$$\begin{pmatrix}0\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}10\\0\\1\end{pmatrix}=\begin{pmatrix}2\\0\\0\end{pmatrix}+\mu\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$\begin{pmatrix}10\lambda\\0\\3+\lambda\end{pmatrix}=\begin{pmatrix}2\\0\\\mu\end{pmatrix}\Rightarrow\lambda=0.2,\mu=3.2$$

Sub $\mu=3.2$:

$$\overrightarrow{OF}=\begin{pmatrix}2\\0\\0\end{pmatrix}+3.2\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}2\\0\\3.2\end{pmatrix}$$

Hence shortest distance from C to line FG is

$$\left|\overrightarrow{CF}\times\frac{\begin{pmatrix}0\\0\\1\end{pmatrix}}{\left|\begin{pmatrix}0\\0\\1\end{pmatrix}\right|}\right|=\left|\left[\begin{pmatrix}2\\0\\3.2\end{pmatrix}-\begin{pmatrix}6\\4\\2\end{pmatrix}\right]\times\begin{pmatrix}0\\0\\1\end{pmatrix}\right|=\left|\begin{pmatrix}-4\\-4\\1.2\end{pmatrix}\times\begin{pmatrix}0\\0\\1\end{pmatrix}\right|$$

$$=\sqrt{32}\ \left(\text{or } 4\sqrt{2} \text{ or } 5.66\ (5.65685)\right)$$

Method 1: Area of trapezium formula

$EC=2, FG=3.2$

Area of $ECFG$

$$=\frac{1}{2}(3.2+2)\left(\sqrt{32}\right)$$

$$=\frac{1}{2}(2+3.2)\left(4\sqrt{2}\right)$$

$$=(10.4)\sqrt{2}$$

$$=\frac{52}{5}\sqrt{2}\text{ sq units}$$

Method 2: Use of area of triangles

Area of triangle ECG

$$=\frac{1}{2}\left|\overrightarrow{EC}\times\overrightarrow{EG}\right|=\frac{1}{2}\left|\begin{pmatrix}0\\0\\2\end{pmatrix}\times\begin{pmatrix}-4\\-4\\0\end{pmatrix}\right|$$

$$=\frac{1}{2}\left|\begin{pmatrix}0\\0\\2\end{pmatrix}\times\begin{pmatrix}-4\\-4\\0\end{pmatrix}\right|=\frac{1}{2}|2||-4|\left|\begin{pmatrix}0\\0\\1\end{pmatrix}\times\begin{pmatrix}1\\1\\0\end{pmatrix}\right|$$

$$=4\left|\begin{pmatrix}0\\0\\1\end{pmatrix}\times\begin{pmatrix}1\\1\\0\end{pmatrix}\right|=4\sqrt{2}$$

Area of triangle CFG

$$=\frac{1}{2}\left|\overrightarrow{CF}\times\overrightarrow{CG}\right|$$

$$=\frac{1}{2}\left|\begin{pmatrix}-4\\-4\\1.2\end{pmatrix}\times\begin{pmatrix}-4\\-4\\-2\end{pmatrix}\right|=\frac{1}{2}|-2|\left|\begin{pmatrix}-4\\-4\\1.2\end{pmatrix}\times\begin{pmatrix}2\\2\\1\end{pmatrix}\right|$$

$$=\frac{1}{2}|-2|\left|\begin{pmatrix}-6.4\\-6.4\\0\end{pmatrix}\right|=\sqrt{2(6.4)^2}=6.4\sqrt{2}$$

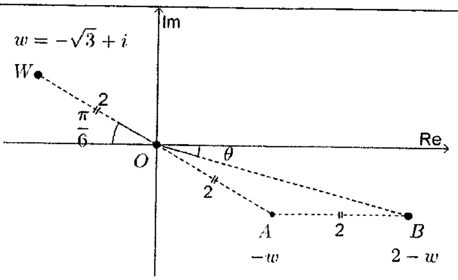
Area of $ECFG$

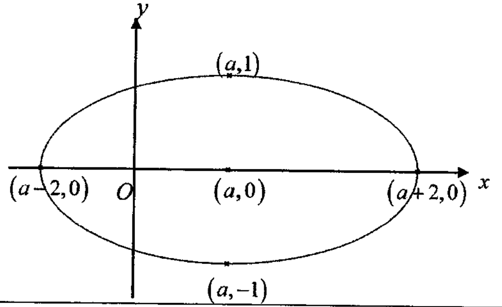
$$=4\sqrt{2}+6.4\sqrt{2}$$

$$=10.4\sqrt{2}$$

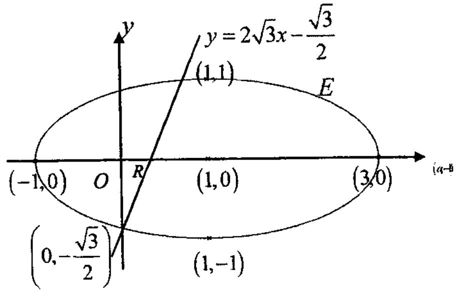
$$=\frac{52}{5}\sqrt{2}\text{ sq units}$$

2025 NJC Preliminary Examination H2 Mathematics Paper 2 Solutions

1	Suggested Solution
(i)	$\lvert w\rvert=\lvert i-\sqrt{3}\rvert$ $=\sqrt{\left(-\sqrt{3}\right)^2+1^2}$ $=2$ $\arg(w)=\pi-\tan^{-1}\frac{1}{\sqrt{3}}$ $=\pi-\frac{\pi}{6}$ $=\frac{5\pi}{6}$
(ii)	 The point W represents w and the point B represents $2-w$.
(iii)	Since $\lvert w\rvert=2$, the triangle OAB is an isosceles triangle. AB is parallel to the real-axis, so $\angle AOB=\angle ABO=\theta$. $2\theta=\frac{\pi}{6}$ $\theta=\frac{\pi}{12}$ $\arg(2-w)=-\frac{\pi}{12}$.

2	Suggested Solution
(i)	$(x-a)^2+4y^2=4$ $\frac{(x-a)^2}{2^2}+y^2=1$ 
(ii)	Given $(x-1)^2+4y^2=4$ When $x=0$, $(0-1)^2+4y^2=4$ $y=\pm\frac{\sqrt{3}}{2}$ Differentiate w.r.t. x, $(x-1)^2+4y^2=4$ $2(x-1)+8y\frac{dy}{dx}=0$ $\frac{dy}{dx}=-\frac{x-1}{4y}$ grad. of normal $=\frac{4y}{x-1}$ Since gradient of $y=2\sqrt{3}x-\frac{\sqrt{3}}{2}$ is positive, from the diagram, the normal should have the y-intercept below the O i.e. $-\frac{\sqrt{3}}{2}$. For the point $\left(0,-\frac{\sqrt{3}}{2}\right)$, grad of normal $=\frac{4\left(-\frac{\sqrt{3}}{2}\right)}{0-1}=2\sqrt{3}$. Equation of line: $y=2\sqrt{3}x-\frac{\sqrt{3}}{2}$ (since y-intercept is $-\frac{\sqrt{3}}{2}$)

2	Suggested Solutions
(iii)	Equation of normal: $y=2\sqrt{3}x-\frac{\sqrt{3}}{2}$

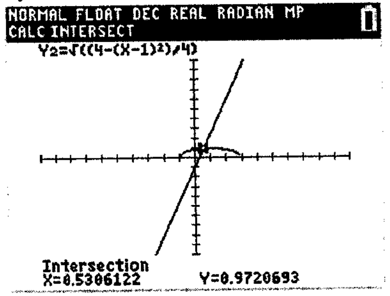


To find the intersection between normal and E, consider the positive square root of y.

$(x-1)^2+4y^2=4$

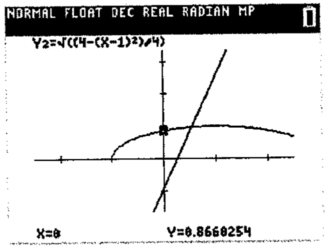
$y=\sqrt{\frac{4-(x-1)^2}{4}}$

By G.C.,



```
NORMAL FLOAT DEC REAL RADIAN MP
CALC INTERSECT
Y2=√((4-(X-1)²)/4)
Intersection
X=0.5306122        Y=0.9720693
```

The coordinates of point of intersection are $(0.530612, 0.972069)$.



```
NORMAL FLOAT DEC REAL RADIAN MP
Y2=√((4-(X-1)²)/4)
X=0               Y=0.8660254
```

The y-intercept of E is 0.866025.

To find solid R, make x the subject of the formula,

$x=1\pm\sqrt{4-4y^2}$

Volume of solid R

$$=\frac{1}{3}\pi(0.530612)^2\left(\frac{\sqrt{3}}{2}+0.972069\right)$$
$$-\pi\int_{0.8660254}^{0.972069}\left(1-\sqrt{4-4y^2}\right)^2\,\mathrm{d}y$$
$$=0.517\text{ units}^3\ (3\text{ s.f.})$$

3 **Suggested Solutions**

(i)

$$\mathbf{p}=\begin{pmatrix}\cos t\\ \sin t\\ -1\end{pmatrix},\ \mathbf{q}=\begin{pmatrix}\cos 2t\\ -\sin 2t\\ \alpha\end{pmatrix}$$

$$\mathbf{p}\times\mathbf{q}$$

$$=\begin{pmatrix}\cos t\\ \sin t\\ -1\end{pmatrix}\times\begin{pmatrix}\cos 2t\\ -\sin 2t\\ \alpha\end{pmatrix}$$

$$=\begin{pmatrix}\alpha\sin t-\sin 2t\\ -\alpha\cos t-\cos 2t\\ -\sin 2t\cos t-\cos 2t\sin t\end{pmatrix}$$

$$=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$$

<u>Method 1: Equate **k** -component to 0</u>

$$-\sin 2t\cos t-\cos 2t\sin t=0$$

$$-(\sin 2t\cos t+\cos 2t\sin t)=0$$

$$-\sin 3t=0$$

For $-\sin 3t=0\Rightarrow t=\frac{\pi}{3},\frac{2\pi}{3}$.

When $t=\frac{\pi}{3}$:

$$-\left(\alpha\cos\left(\frac{\pi}{3}\right)+\cos 2\left(\frac{\pi}{3}\right)\right)=0$$

$$\left(\alpha\left(\frac{1}{2}\right)+\left(-\frac{1}{2}\right)\right)=0$$

$\alpha=1$

When $t=\frac{2\pi}{3}$

$$-\left(\alpha\cos\left(\frac{2\pi}{3}\right)+\cos 2\left(\frac{2\pi}{3}\right)\right)=0$$

$$\left(\alpha\left(-\frac{1}{2}\right)+\left(-\frac{1}{2}\right)\right)=0$$

$\alpha=-1$

Final answers: $t=\frac{\pi}{3},\alpha=1;\ t=\frac{2\pi}{3},\alpha=-1$

<u>Method 2: **q** is a scalar multiple of **p**</u>

$$\mathbf{p}=k\mathbf{q}\Rightarrow\begin{pmatrix}\cos 2t\\ -\sin 2t\\ \alpha\end{pmatrix}=k\begin{pmatrix}\cos t\\ \sin t\\ -1\end{pmatrix}$$

$$\begin{pmatrix}\cos 2t\\ -\sin 2t\\ \alpha\end{pmatrix}=\begin{pmatrix}k\cos t\\ k\sin t\\ -k\end{pmatrix}$$

From **j**-component:

$$-\sin 2t=k\sin t$$

$$-2\cos t\sin t=k\sin t$$

$$-2\cos t\sin t-k\sin t=0$$

$$\sin t(-2\cos t-k)=0$$

$\sin t=0$ or $-2\cos t-k=0$

$t=0$ (rejected $\because 0<t<\pi$) or $\cos t=-\frac{k}{2}$

From **i**-component:

$$\cos 2t=k\cos t$$

$$2\cos^2 t-1-k\cos t=0$$

$$2\cos^2 t-k\cos t-1=0$$

Sub $\cos t=-\frac{k}{2}$ into equation $2\cos^2 t-k\cos t-1=0$:

$$2\left(-\frac{k}{2}\right)^2-k\left(-\frac{k}{2}\right)-1=0$$

$$\frac{k^2}{2}+\frac{k^2}{2}=1$$

$k=\pm 1$

Since $\alpha=k$, $\alpha=-1$ or $\alpha=1$.

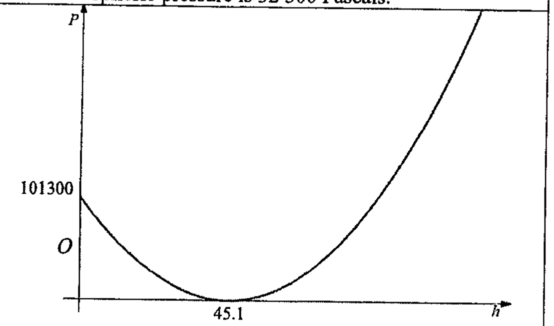
When $\alpha=k=1$, $t=\cos^{-1}\left(-\frac{k}{2}\right)=\frac{2\pi}{3}$

When $\alpha=k=-1$, $t=\cos^{-1}\left(-\frac{k}{2}\right)=\frac{\pi}{3}$

(ii)	$\mathbf{p}\cdot\mathbf{q}$ $=\begin{pmatrix}\cos t\\ \sin t\\ -1\end{pmatrix}\cdot\begin{pmatrix}\cos 2t\\ -\sin 2t\\ 0.5\end{pmatrix}$ $= \cos t\cos 2t-\sin t\sin 2t-0.5$ $=\cos(2t+t)-0.5$ $=\cos 3t-0.5$ $\dfrac{\lvert\mathbf{p}\cdot\mathbf{q}\rvert}{\lvert\mathbf{p}\rvert}$ $=\dfrac{\lvert\cos 3t-0.5\rvert}{\sqrt{2}}$ $=\dfrac{1}{\sqrt{2}}\lvert\cos 3t-0.5\rvert$ Greatest length $=\dfrac{1}{\sqrt{2}}\lvert-1-0.5\rvert$ $=\dfrac{3}{2\sqrt{2}}$ or 1.06 (3 s.f.)

[redacted]

(i)	$\dfrac{dP}{dh}=-k\left(\dfrac{P}{T}\right)$ $T=293-6.5h$ $\dfrac{dP}{dh}=\dfrac{-kP}{293-6.5\,h}$, where $k>0$ or $\dfrac{dP}{dh}=\dfrac{kP}{293-6.5\,h}$, where $k<0$
(ii)	$\dfrac{1}{P}\dfrac{dP}{dh}=-\dfrac{k}{293-6.5\,h}$ $\displaystyle\int\frac{1}{P}\,dP=\int-\frac{k}{293-6.5\,h}\,dh$ Since $P>0$, $\ln P=\dfrac{k}{6.5}\ln\lvert 293-6.5\,h\rvert+C$ $P=e^{\frac{k}{6.5}\ln\lvert 293-6.5\,h\rvert+C}$ $=A\left[e^{\ln\lvert 293-6.5\,h\rvert}\right]^{\frac{k}{6.5}}$, where $A=e^{C}$ $=A\lvert 293-6.5h\rvert^{\frac{k}{6.5}}$ $=A\lvert 293-6.5h\rvert^{b}$ Since $k>0$, $b=\dfrac{k}{6.5}>0$.
(iii)	When $h=0$, $P=101300$, $101300=A\left(293^{b}\right)$ ...(1) When $h=2$, $P=80000$, $80000=A(293-6.5\times 2)^{b}$ $80000=A\left(280^{b}\right)$ ...(2) (1) / (2), $\dfrac{101300}{80000}=\left(\dfrac{293}{280}\right)^{b}$ $b=\dfrac{\ln\frac{101300}{80000}}{\ln\frac{293}{280}}$ $=5.2015$ (5 s.f.)

	$A=\frac{101300}{293^{5.2015}}$ $=1.4935\times10^{-8}$ $P=1.4935\times10^{-8}\left\|293-6.5h\right\|^{5.2015}$ When $h=8.848$, $P=1.4935\times10^{-8}\left\|293-6.5(8.848)\right\|^{5.2015}$ $=32500$ (to 3 s.f.) The atmospheric pressure is 32 500 Pascals.
(iv)	
	Either The model suggests that when altitude is greater than 45.1 km, atompheric pressure increases (to infinity) as altitude increases, which is not possible. **Or** As the temperature T measured in Kelvin can only be positive, the model is invalid if $T=293-6.5h<0$.

[redacted]	
	$\frac{\mathrm{P}(Y=k)}{\mathrm{P}(Y=k-1)}$ $=\frac{\binom{12}{k}p^{k}(1-p)^{12-k}}{\binom{12}{k-1}p^{k-1}(1-p)^{12-k+1}}$ $=\frac{\frac{12!}{k!(12-k)!}(p)}{\frac{12!}{(k-1)!(13-k)!}(1-p)}$ $=\frac{\frac{1}{k(k-1)!(12-k)!}(p)}{\frac{1}{(k-1)!(13-k)(12-k)!}(1-p)}$ $=\frac{(13-k)p}{k(1-p)}$ (Shown) Given mode is 4, $\mathrm{P}(Y=4)>\mathrm{P}(Y=3)$ $\frac{\mathrm{P}(Y=4)}{\mathrm{P}(Y=3)}>1$ $\frac{9p}{4(1-p)}>1$ $9p>4-4p$ $p>\frac{4}{13}$ and $\mathrm{P}(Y=4)>\mathrm{P}(Y=5)$ $\frac{\mathrm{P}(Y=5)}{\mathrm{P}(Y=4)}<1$ $\frac{8p}{5(1-p)}<1$ $8p<5-5p$ $p<\frac{5}{13}$ Hence $\frac{4}{13}<p<\frac{5}{13}$.

6

(a)

$P(T=3)$

$=P(\text{getting a 2, 4 and 5 from three tosses})$

$+P(\text{getting a 3, 3 and 5 from three tosses})$

$+P(\text{getting a 3, 4 and 4 from three tosses})$

$$=\left(\frac{2}{6}\right)^2\left(\frac{1}{6}\right)\times(3!-2)+\left(\frac{1}{6}\right)^3\times\left(\frac{3!}{2!}-1\right)+\left(\frac{1}{6}\right)\left(\frac{2}{6}\right)^2\times\frac{3!}{2!}$$

$=\frac{5}{36}$ (Shown)

$P(T=1)=P(\text{getting a 5 from one toss})$

$=\frac{1}{6}$

$P(T=2)=P(\text{getting a 2 and 3 from two tosses})$

$=\left(\frac{2}{6}\right)\left(\frac{1}{6}\right)\times 2!$

$=\frac{1}{9}$

$P(T\geq 4)=1-\frac{1}{6}-\frac{1}{9}-\frac{5}{36}$

$=\frac{7}{12}$

(b)

s	5	10	15	0
$P(S=s)$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{7}{12}$

$E(S)=5\left(\frac{1}{6}\right)+10\left(\frac{1}{9}\right)+15\left(\frac{5}{36}\right)$

$=\frac{145}{36}$ or 4.03 (3 s.f.)

$\text{Var}(S)=E(S^2)-\left[E(S)\right]^2$

$=5^2\left(\frac{1}{6}\right)+10^2\left(\frac{1}{9}\right)+15^2\left(\frac{5}{36}\right)-\left(\frac{145}{36}\right)^2$

$=\frac{39275}{1296}$ or 30.3 (3 s.f.)

(a)

Method 1

Number of ways with 1 engine at the front

$=\frac{8!}{3!3!2!}$

$=560$

Number of ways with 2 engines at the back

$=\frac{7!}{2!3!2!}$

$=210$

Number of ways with 1 engine at the front and 2 engine at the back

$=\frac{6!}{3!2!}$

$=60$

Total number of ways to form the rail train

$=560+210-60$

$=710$

Method 2

Number of cases where the first unit is E and the last two units are not EE *(E_ _ _ _ _ _ _E'E')*

$=\binom{6}{3}\binom{5}{3}$ $+\binom{6}{2}\binom{5}{3}$

(E_ _ _ _ _ _ _E'E') *(E_ _ _ _ _ _ _E'E)*

$+\binom{6}{2}\binom{5}{3}$

(E_ _ _ _ _ _ _EE')

$=500$

Number of cases where the first unit is not E and the last two units are EE *(E'_ _ _ _ _ _ _EE)*

$=\binom{6}{2}\binom{5}{3}$

$=150$

Number of cases where the first unit is E and the last two units are EE *(E_ _ _ _ _ _ _EE)*

$=\binom{6}{1}\binom{5}{2}$

$=60$

Total number of ways

$=500+150+60=710$

BP~438

(b) **Method 1**

Let E, C and W represent a engine, carriage and wagon respectively.

Consider the arrangement:
E_C_C_C_W_W_

Number of ways to have the remaining 3 engines together in one of the six slots
$=6$

Number of ways to have the remaining 3 engines separated into E and EE
$=\binom{6}{2}\times 2$
$=30$

Number of ways to have the remaining 3 engines all separated
$=\binom{6}{3}$
$=20$

Total number of ways to form the toy train
$=6+30+20$
$=56$

Method 2

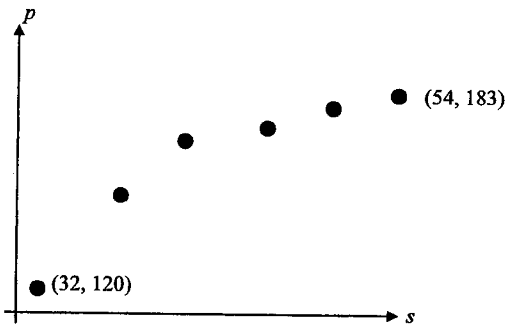
No. of ways to choose 5 slots out of 8 for three carriages and two wagons $=\binom{8}{5}$

No of ways to order three carriages and two wagons
$=\binom{8}{5}\times 1$

Total number of ways
$=\binom{8}{5}\times 1\times\binom{3}{3}$ (remaining three slots are to have three engines)
$=56$

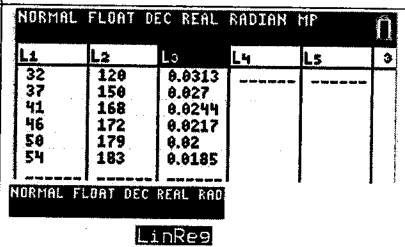
Suggested Solutions

(i)



(ii) The equation (B) $p=c+\frac{d}{s}$ is a better model because from the scatter diagram, as <u>s increases, p increases at a decreasing rate.</u>

(iii)



```
NORMAL FLOAT DEC REAL RADIAN MP
L1    L2    L3      L4      L5
32    120   0.0313  ------  ------
37    150   0.027
41    168   0.0244
46    172   0.0217
50    179   0.02
54    183   0.0185
------ ------ ------
NORMAL FLOAT DEC REAL RAD
LinReg
y=a+bx
a=276.8793319
b=-4822.643889
r²=0.9461051441
r=-0.9726793635
```

$p=276.88-\frac{4822.6}{s}$ (5 s.f.)

$p=277-\frac{4820}{s}$ (3 s.f.)

product moment correlation coefficient, $r=-0.973$ (3 s.f.)

(iv) when $s=45$,

$p=276.88-\frac{4822.6}{45}$

$p=169.71$ (5 s.f.)

$p=170$ (3 s.f.)

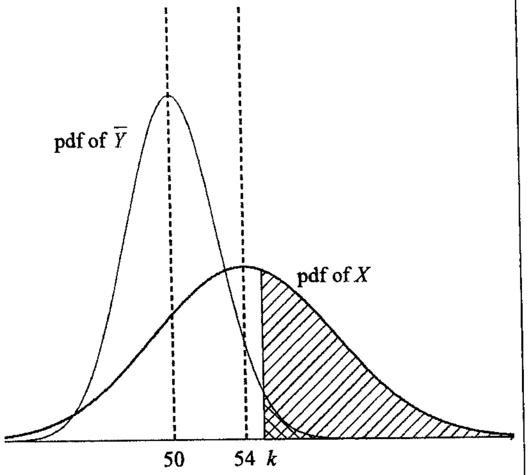
Since $s=45$ lies within the range of values of s [32,54] and the product moment correlation coefficient, $r=-0.973$ is close to -1, which indicates a strong negative <u>linear</u> correlation between <u>p and $\frac{1}{s}$</u>, this estimate is reliable.

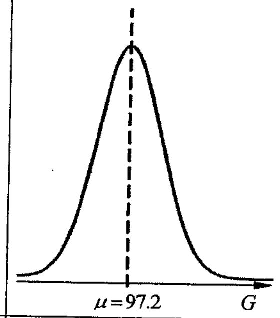
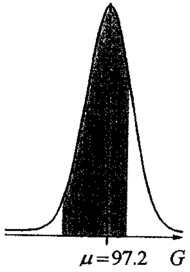
9	Suggested Solution
(i)	$\text{P}(\text{exactly one picture of A})$ $=\text{P}(AA'A')+\text{P}(A'AA')+\text{P}(A'A'A)$ $=(0.1)(1-0.1)^2+(1-0.1)(0.1)(1-0.1)+(1-0.1)^2(0.1)$ $=0.243$
(ii)	**<u>Method 1</u>** $\text{P}\begin{pmatrix}\text{the other two characters are } B \text{ and } C\\ \text{ \textbar exactly one character of } A\end{pmatrix}$ $=\dfrac{\text{P}(\text{exactly one character of } A, B \text{ and } C \text{ respectively})}{\text{P}(\text{exactly one character of } A)}$ $\text{P}(\text{exactly one character of } A, B \text{ and } C \text{ respectively})$ $=\text{P}(ABC)+\text{P}(ACB)+\text{P}(BAC)$ $+\text{P}(BCA)+\text{P}(CAB)+\text{P}(CBA)$ $=6(0.1)^3$ $=0.006$ $\dfrac{\text{P}(\text{exactly one character of } A, B \text{ and } C \text{ respectively})}{\text{P}(\text{exactly one character of } A)}$ $=\dfrac{0.006}{0.243}$ $=\dfrac{2}{81}$ $=0.024691$ **Method 2: Counting** Required probability $=\dfrac{n(A,B,C \text{ out of three coupons})}{n(\text{exactly one 'A' out of three coupons})}$ $=\dfrac{3(2)}{3(1\times 9\times 9)}$ $=\dfrac{2}{81}$ $=0.024691$
(iii)	$p_1=\dfrac{3}{10}\quad p_n=\dfrac{7}{10}p_{n-1}$

(iv)	**Method 1: Geometric Series** $\dfrac{3}{10}+\left(\dfrac{7}{10}\right)\dfrac{3}{10}+\left(\dfrac{7}{10}\right)^2\dfrac{3}{10}+\ldots+\left(\dfrac{7}{10}\right)^{n-1}\dfrac{3}{10}$ $=\dfrac{\dfrac{3}{10}\left(1-\left(\dfrac{7}{10}\right)^n\right)}{1-\dfrac{7}{10}}=1-\left(\dfrac{7}{10}\right)^n$ **Method 2: Considering Complement** $\text{P}(\text{not getting final character})=0.7$ After n additional purchases, $\text{P}(\text{not getting final character})=(0.7)^n$ After n additional purchases, $\text{P}(\text{getting final character})=1-(0.7)^n$	

10	Suggested Solution
(i)	Each lunch box produced on that day has an equal chance (the same probability) of being selected. The event of a lunch box being selected is independent of another lunch box being selected.
(ii)	As the distribution of the heights of the lunch boxes is unknown, a large sample size (of at least 30) is needed to apply the Central Limit Theorem so that the sample mean follows a normal distribution approximately.
(iii)	Unbiased estimate of population mean, $\bar{x} = 13 + \frac{-3.6}{48} = 12.925$ Unbiased estimate of population variance, $s^2 = \frac{1}{47}\left(12.02 - \frac{(-3.6)^2}{48}\right) = 0.25$
(iv)	Let the population mean height of the lunch boxes be μ cm. To test: $H_0 : \mu = 12.8$ against $H_1 : \mu \neq 12.8$ at 10% level of significance Under H_0, $\overline{X} \sim N\left(12.8, \frac{0.25}{48}\right)$ approximately by the Central Limit Theorem, since the sample size $n = 48 > 30$ is large. Hence $Z = \frac{\overline{X} - 12.8}{\sqrt{\frac{0.25}{48}}} \sim N(0,1)$. By GC, p-value =0.0833 < 0.10 (or $\lvert z_{stat} \rvert = 1.732 < 1.645 = z_{crit}$) We reject H_0 and conclude that there is sufficient evidence at the 10% significance level to claim that the population <u>mean height</u> of the lunch boxes <u>is not 12.8</u> cm.
(v)	To test: $H_0 : \mu = 12.8$ against $H_1 : \mu > 12.8$ at 2% level of significance Under H_0, $\overline{X} \sim N\left(12.8, \frac{0.04}{n}\right)$ approximately by the Central Limit Theorem since the sample size n is large Hence $Z = \frac{\overline{X} - 12.8}{\sqrt{\frac{0.04}{n}}} \sim N(0,1)$. Let the critical region be $\bar{x} \geq a$. $P\left(\overline{X} \geq a\right) = 0.02$ $P\left(\frac{\overline{X} - 12.8}{\sqrt{\frac{0.04}{n}}} \geq \frac{a - 12.8}{\sqrt{\frac{0.04}{n}}}\right) = 0.02$ $P\left(Z \geq \frac{a - 12.8}{\frac{0.2}{\sqrt{n}}}\right) = 0.02$ $\frac{(a - 12.8)\sqrt{n}}{0.2} = 2.0537489$ $a = 12.8 + \frac{0.411}{\sqrt{n}}$ The critical region is $\bar{x} \geq 12.8 + \frac{0.411}{\sqrt{n}}$

11	Suggested Solutions
(a)	Let X and Y be the random variable denoting the fasting glucose concentration of a man and woman in mmol / L respectively. $X-Y \sim \mathrm{N}\left(5.4-5.0, 0.5^2+0.3^2\right)$ $X-Y \sim \mathrm{N}(0.4, 0.34)$ $\mathrm{P}(X>Y)$ $=\mathrm{P}(X-Y>0)$ $=0.754(3 \text{ s.f.})$ The assumption is that the fasting glucose concentration of a man is independent of that of a woman. OR The assumption is that the fasting glucose concentration of ALL men and women are independent.
(b)	$X \sim \mathrm{N}\left(5.4, 0.5^2\right)$ $\mathrm{P}(X-5.4 \geq c) \leq 0.03$ $\mathrm{P}\left(\frac{X-5.4}{0.5} \geq \frac{c}{0.5}\right) \leq 0.03$ $\mathrm{P}\left(Z \geq \frac{c}{0.5}\right) \leq 0.03$ $\frac{c}{0.5} \geq 1.880794$ $c \geq 0.940$ (3 s.f.)

(c)	$X \sim \mathrm{N}\left(5.4, 0.5^2\right)$ $\overline{Y}=\frac{Y_1+Y_2+Y_3+Y_4}{4} \sim \mathrm{N}\left(5.0, \frac{0.3^2}{4}\right)$ **Method 1** $p_1=\mathrm{P}(X>k)=\mathrm{P}\left(Z>\frac{k-5.4}{0.5}\right)$ $p_2=\mathrm{P}\left(\overline{Y}>k\right)$ $=\mathrm{P}\left(Z>\frac{k-5.0}{\frac{0.3}{2}}\right)$ $=\mathrm{P}\left(Z>\frac{k-5.0}{0.15}\right)$ For $k>5.4$, $k-5.4<k-5.0$ and $0.5>0.15>0$, we have $\frac{k-5.0}{0.15}>\frac{k-5.4}{0.5}$. Hence, $p_1>p_2$. **Method 2**  By comparing the areas of the shaded region in the diagram, $p_1>p_2$..

(d)	$X \sim \mathrm{N}(5.4, 0.5^2)$ $\mathrm{E}(G) = \mathrm{E}(18.0X) = 18.0\mathrm{E}(X) = 18.0 \times 5.4 = 97.2$ $\mathrm{Var}(G) = \mathrm{Var}(18.0X) = 18.0^2 \times 0.5^2 = 81.0$ $G \sim \mathrm{N}(97.2, 81.0)$ 
(e)	$G \sim \mathrm{N}(97.2, 81.0)$  $\mathrm{P}(79 < G < 106) = 0.814$ (3 s.f)