BP-407

NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 2

NAME	

SUBJECT CLASS	2ma2	REGISTRATION NUMBER	

MATHEMATICS — 9758/01

Paper 1

15 September 2025

3 hours

Candidates answer on the Printed Answer Booklet.

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on the work you hand in.
Write in dark blue or black pen.

Answer **all** the questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

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The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages.

1 Paul is helping his friends to convert their foreign currencies back to Singapore Dollars. The amounts of foreign currencies converted, and the total amount received in Singapore Dollars are shown in the following table.

	Alex	Nicholas	Palmer	Maybelline
US Dollar (USD)	150	250	425	a
Japanese Yen (JPY)	5500	9500	1000	0
Chinese Yuan (CNY)	1000	2200	2000	1200
Total amount: Singapore Dollars (SGD)	419.30	797.20	913.10	568.40

However, he has forgotten the amount of US Dollar that Maybelline has passed to him. Assuming that, for each foreign currency, the exchange rate quoted for each of the friends is the same, calculate the value of a. [4]

2 **Do not use a calculator in answering this question.**

(i) Find the values of z and w that satisfy the equations $(1+\mathrm{i})z+2w=-2+4\mathrm{i}$ and $3z-w=4+2\mathrm{i}$, expressing your answers in the form $c+d\mathrm{i}$, where $c,d\in\mathbf{R}$. [4]

(ii) Points W and Z represent w and z found in part **(i)**. Find $\frac{w}{z}$ in the form $p+\mathrm{i}q$, where $p,q\in\mathbf{R}$. Hence, state the transformation that maps line segment OZ onto line segment OW. [2]

3 **(i)** On the same axes, sketch the graphs of $y=-\frac{b}{x-a}$ and $y=|x-a|$, where a and b are positive constants and $a>b>1$. State, in terms of a and b, the coordinates of the points where the curves cross the x- and y- axes. [3]

(ii) Hence or otherwise, solve the inequality $-\frac{b}{x-a}<|x-a|$. [4]

4 A curve has equation $y=\mathrm{f}(x)$, where $\mathrm{f}(x)=1-\sqrt{q^2-x^2}$ for $q>1$. State the shape of $y=\mathrm{f}(x)$. [1]

(i) Sketch the curve $y=\frac{1}{\mathrm{f}(x)}$, giving the equations of any asymptotes and the coordinates of the end-points. [3]

(ii) Describe the transformations that map the graph of $y=\mathrm{f}(x)$ to $y=-\sqrt{1-x^2}$. [3]

5 The curve C has equation

$$y=\frac{2x^2+kx+8}{x+p},$$

where k and p are constants.

It is given that C has a vertical asymptote $x=2$ and a stationary point at $x=-4$.

(i) Find the equation of the oblique asymptote of C. [5]

(ii) Sketch C, clearly labelling the equations of asymptotes and the coordinates of stationary points. [3]

6 **(a)** An infinite geometric series S has first term 1 and non-zero common ratio r. It is given that the sum to infinity of S is equal to the square of the sum of the first three terms of S.

(i) Show that r satisfies the equation $r^4+ar^3+br^2+cr+d=0$, where $a, b, c,$ and d are constants to be determined. [3]

(ii) Find the possible values of r. [1]

(b) An arithmetic progression with $4n$ terms has first term 7 and common difference d. Every 4^{th} term is removed. Find the sum of the remaining terms in terms of n and d. [4]

7 The curve C is defined parametrically by $x=e^{4t}$, $y=t^2$, where $t \geq 0$.

(i) Find the Cartesian equation of C. [2]

(ii) The tangent at point P has the steepest gradient. Find the exact coordinates of P.

[You do not need to show that the gradient at P is the steepest.] [3]

(iii) Sketch C, indicating the coordinates of P and the point where C crosses the axes clearly. [2]

8 **In this question, you may use expansions from the List of Formulae and Results (MF27).**

It is given that $a>0$.

(i) Find, in terms of a, the series expansion of $\frac{a}{a-x}-1$, in ascending powers of x, up to and including the term in x^2. State, in terms of a, the range of x for which the expansion is valid. [4]

(ii) Hence, find the Maclaurin expansion of $e^{\frac{a}{a-x}-1}$ in ascending powers of x, up to and including the term in x^2. [2]

(iii) Use the expansion in part **(ii)** to approximate $\int_0^{\frac{a}{2}} e^{\frac{a}{a-x}-1}\,dx$. Explain why this approximation is an under-estimation. [4]

9 **(a)** Find $\int \cos(3\ln x)\,dx$. [4]

(b) Let I be the indefinite integral $\int \frac{P(x)}{1-\sqrt{x}}\,dx$, $0<x<1$, where $P(x)$ is a polynomial in x.

(i) Find I when $P(x)=1-x$. [2]

(ii) By using the substitution $u=1-\sqrt{x}$, find I when $P(x)=1$. [3]

Hence find I when $P(x)=x$. [2]

10 A sequence of numbers $u_1, u_2, u_3, \ldots$ has a sum S_n, where $S_n=\sum_{r=1}^{n} u_r$. It is given that $S_n = A-\frac{2}{(n+1)!}$, where A is a non-zero constant.

(i) Find the value of A if $u_1=1$. [1]

(ii) Show that $u_n=\frac{2}{(n+1)\left[(n-1)!\right]}$ for $n\geq 1$. [3]

(iii) Find a recurrence relation in the form $u_{n+1}=\left[f(n)\right]u_n$. [2]

(iv) Explain why S_n converges as $n\to\infty$. [1]

(v) Hence, find the least value of m such that the sum of the infinite series

$$u_m+u_{m+1}+u_{m+2}+\ldots$$

does not exceed 10^{-10}. [3]

11 The function f is defined by

$$f(x)=\begin{cases}\frac{a}{2}+\frac{4}{3}\left(x-\frac{1}{2}\right)^2 & \text{for } x\in\mathbf{R},\ \frac{1}{2}<x<2,\\ \frac{a}{x} & \text{for } x\in\mathbf{R},\ x\geq 2,\end{cases}$$

where a is a positive constant.

(i) Find the range of f. [3]

(ii) Find $f^{-1}(x)$ and state its domain. [3]

The function g is defined by $g(x)=3+e^x$, for $x\in\mathbf{R}$.

(iii) Show that fg exists. [1]

(iv) Find the exact value of k for which $fg(k)=\frac{a}{7}$. [3]

12 A model of a triangular canopy that provides shade outdoors is shown in **Figure 1** below.

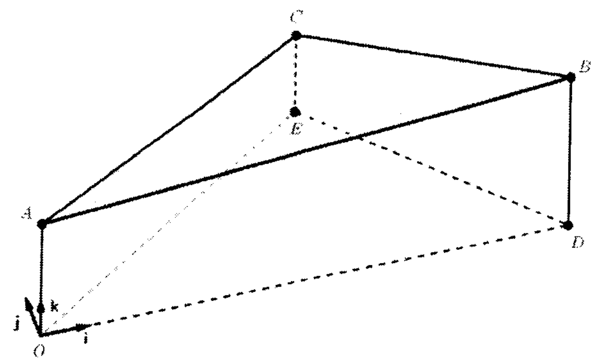

Figure 1

With point O taken as the origin, the canopy ABC is held taut using three vertical columns given by OA, DB and EC. The unit vectors **i** and **k** are defined with **i** along OD, **k** along OA, and unit vector **j** is perpendicular to both. The bases of the vertical columns are anchored to the horizontal ground ODE, which is perpendicular to OA.

(i) State the cartesian equation of plane OAB. [1]

Points A and B have position vectors given by $\overrightarrow{OA} = 3\mathbf{k}$ and $\overrightarrow{OB} = 10\mathbf{i} + 4\mathbf{k}$. A marking, given by point M, is to be placed on the line segment AB.

(ii) Find $\overrightarrow{OM}$ in terms of a parameter λ, stating the range of λ. [2]

Point C has position vector given by $\overrightarrow{OC} = 6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$. The plan is to lay cables to connect M to C and then C to E. All cables are laid in straight lines and have negligible thickness.

(iii) Explain why it is not possible for angle MCE to be 90°. [1]

With reference to **Figure 2** below, points F and G lie on lines AB and OD respectively. Party streamers, with negligible thickness, are laid in straight lines to connect E, C, F and G. The quadrilateral formed lies on a plane with cartesian equation $x - y = 2$.

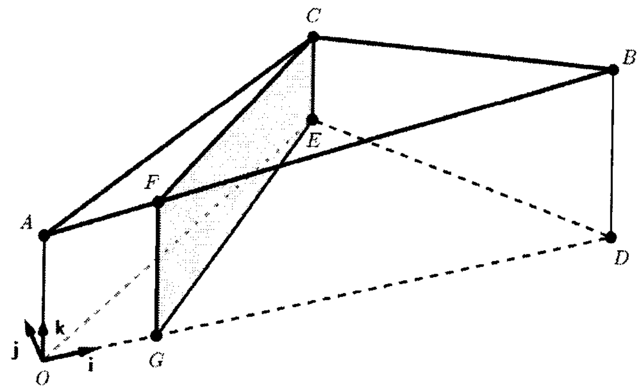

Figure 2

(iv) Show that quadrilateral $ECFG$ is a trapezium. [3]

(v) Find the shortest distance between F and the line CE. Hence or otherwise, find the area enclosed by the streamers. [5]

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NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 2

NAME

CLASS 2ma2

REGISTRATION NUMBER

MATHEMATICS

9758/02

Paper 2

22 September 2025

3 hours

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Section A: Pure Mathematics [40 marks]

1 **Do not use a calculator in answering this question.**

The complex number w is such that $w = \mathrm{i} - \sqrt{3}$.

(i) Find $|w|$ and $\arg(w)$ in exact form. [2]

(ii) Represent w, $-w$ and $2-w$ in the same Argand diagram, illustrating clearly the geometrical relationship between them. [3]

(iii) Hence, find $\arg(2-w)$ exactly. [2]

2 The curve E has equation

$$(x-a)^2 + 4y^2 = 4, \text{ where } 0 < a < 2.$$

(i) Sketch E. [2]

You are now given that $a = 1$.

(ii) Show that the equation of a normal to E at $x = 0$ is $y = 2\sqrt{3}x - \frac{\sqrt{3}}{2}$. [4]

(iii) The region R is bounded by E, the normal in part **(ii)** and the y-axis. Find the volume of the solid generated when R is rotated through 2π radians about the y-axis. [5]

3 With respect to an origin O, points P and Q have variable position vectors $\mathbf{p}$ and $\mathbf{q}$ respectively, given by $\mathbf{p} = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} - \mathbf{k}$, $\mathbf{q} = (\cos 2t)\mathbf{i} - (\sin 2t)\mathbf{j} + \alpha\mathbf{k}$, where α is a real parameter and $0 < t < \pi$.

(i) By considering the cross product or otherwise, find the exact values of t and α if O, P and Q are collinear. [5]

It is given that $\alpha = 0.5$.

(ii) Show that $\mathbf{p} \cdot \mathbf{q} = \cos 3t - 0.5$. Find the greatest length of projection of $\mathbf{q}$ onto $\mathbf{p}$. [4]

4 The rate of decrease of atmospheric pressure P Pascals (Pa) with respect to altitude h kilometres above sea level, is proportional to $\frac{P}{T}$, where T is the temperature measured in Kelvin (K). The temperature T decreases linearly by 6.5 K for every one-kilometre increase in the altitude above sea level. It is given that the temperature is 293 K at sea level.

(i) Find a differential equation for $\frac{\mathrm{d}P}{\mathrm{d}h}$ in terms of P and h. [2]

(ii) Given that $P > 0$, solve this model in part (i) to show that

$$P = A|293 - 6.5h|^b \text{, where } A \text{ and } b \text{ are constants.}$$

Explain whether b is positive or negative. [4]

It is given that the atmospheric pressures are 101 300 Pa at sea level and 80 000 Pa at $h = 2$.

(iii) Find the atmospheric pressure at the top of Mount Everest with an altitude of 8848 metres. [4]

(iv) For $h \geq 0$, sketch the graph of P against h. Explain a limitation of this model. [3]

Section B: Probability and Statistics [60 marks]

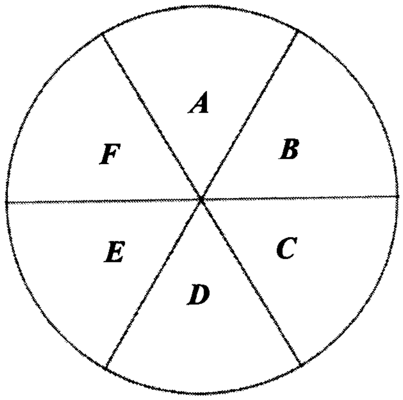
5 The random variable Y has distribution $\mathrm{B}(12, p)$ for $0 < p < 1$.

(i) Show that $\dfrac{\mathrm{P}(Y=k)}{\mathrm{P}(Y=k-1)} = \dfrac{(13-k)p}{k(1-p)}$. [2]

It is given that the mode of the distribution is 4.

(ii) Find the exact range of values of p. [3]

6



A circular board has 6 sectors labelled A, B, C, D, E and F. A game is played by placing a pawn on sector A. It is then moved clockwise around the board according to the number shown on the top face of a fair die when it is tossed. The die has faces labelled 2, 2, 3, 4, 4, 5. For example, an initial toss of 4 would move the pawn from sector A to sector E. The player continues tossing the die and moving the pawn accordingly. When the pawn lands on sector F, the game ends.

Let T be the number of tosses required to move the pawn until the game ends.

(a) Show that $\mathrm{P}(T=3) = \dfrac{5}{36}$. Hence, copy and complete the probability distribution table for T.

t	1	2	3	4 or more
$\mathrm{P}(T=t)$				

[4]

The score S for each game is given by $S = 5T$, for $t = 1, 2, 3$ and $s = 0$ for $t \geq 4$.

(b) Find $\mathrm{E}(S)$ and $\mathrm{Var}(S)$. [3]

7 A toy train is made up of 3 types of train units: engine, carriage, and wagon. There are 4 identical engines (E), 3 identical carriages (C), and 2 identical wagons (W).

(a) All the train units are placed in a line such that either the first unit is an engine or the last two units are engines, or both. Find the number of ways to form the toy train. [4]

(b) For another kind of toy train, all the train units are placed in a line such that the first train unit is an engine and all the carriages are in front of the wagons. An example of such a toy train would be:

$$E\,C\,C\,E\,E\,C\,W\,E\,W.$$

Find the number of ways to form the toy train. [3]

8 The Ministry of Health declared "War on Diabetes" in 2016 to rally a whole-of-nation effort to tackle diabetes. A study was conducted to investigate the effects of diabetes on one's health, specifically to determine if the blood pressure of an adult aged between 50 and 60 years old is dependent on one's haemoglobin level. Data from six patients in this age group from a hospital was collected. Their haemoglobin level, s mmol/mol, and blood pressure, p mmHg, are shown in the table.

s	32	37	41	46	50	54
p	120	150	168	172	179	183

(i) Draw a scatter diagram of these data. [1]

It is thought that blood pressure p can be modelled by one of the formulae

$$p = a + bs^2 \quad \text{or} \quad p = c + \frac{d}{s}$$

where a, b and c are positive real numbers and d is a negative real number.

(ii) By using your diagram, explain which of $p = a + bs^2$ or $p = c + \frac{d}{s}$ is the better model. [2]

(iii) Using the model you chose in part **(ii)**, write down the equation for the relationship between s and p, giving the numerical values of the coefficients. State the product moment correlation coefficient for this model. [2]

(iv) Estimate the value of p when $s = 45$. Determine whether this estimate is reliable. [3]

9 In a sales campaign, a supermarket gives each shopper a receipt printed with a cartoon character when a shopper pays for his or her purchase. There are 10 different cartoon characters available, character A, B, C, ..., J. The printed cartoon character is equally likely to be any one of the 10 different cartoon characters.

A shopper received three receipts after having made three purchases.

(i) Find the probability that only one receipt has the character A printed on it. [2]

(ii) Given that exactly one of the cartoon characters printed is A, find the probability that the other two characters printed are B and C. [3]

To redeem a gift, exactly 8 distinct cartoon characters need to be collected. After some time, Tom collected 7 distinct characters. Let p_n be the probability that Tom can redeem the gift on exactly the n^{th} additional purchase.

(iii) Find p_1 and the recurrence relation in the form $p_n = \mathrm{f}\left(p_{n-1}\right)$, for $n \geq 2$. [2]

(iv) Find the probability, in terms of n, that at most n additional purchases are needed to redeem the gift. [2]

10 The production manager of a food container manufacturing company wishes to take a random sample of a certain type of lunch box from the thousands produced one day at his factory, for quality control purposes. He wishes to check that the mean height of the lunch boxes is 12.8 cm.

(i) State what it means for a sample to be random in this context. [1]

(ii) Explain why the manager should take a sample of at least 30 lunch boxes to carry out a hypothesis test. [1]

The heights, x cm, of a random sample of 48 lunch boxes are summarised as follows.

$$\sum(x-13) = -3.6 \text{ and } \sum(x-13)^2 = 12.02$$

(iii) Calculate the unbiased estimates of the population mean and variance of the height of the lunch boxes. [2]

The production manager claims that the mean height of the lunch boxes is 12.8 cm.

(iv) Test his claim at the 10% level of significance, stating the hypotheses clearly. [4]

In the hope of keeping the mean height of the lunch boxes not to exceed 12.8 cm, the production manager decides to replace the production line such that the population variance is reduced to 0.04 cm^2. He then carries out a hypothesis test at 2% level of significance using another random sample of n lunch boxes.

(v) Assuming that n is sufficiently large, find the critical region for this test in terms of n. [4]

11 The fasting glucose concentration in millimoles per litre (mmol/L) of a randomly chosen man and a randomly chosen woman is normally distributed with mean and standard deviation as given in the table below.

	Mean	Standard deviation
Man	5.4	0.5
Woman	5.0	0.3

(a) If one man and one woman are chosen randomly, find the probability that the man's fasting glucose concentration exceeds the woman's. State an assumption you make in your calculation. [3]

(b) The fasting glucose concentration of at most 3% of men exceeds $5.4\ \text{mmol}/\text{L}$ by $c\ \text{mmol}/\text{L}$. Find the range of c. [2]

(c) For $k > 5.4$, explain which of the following has a larger value:

p_1 : the probability that the fasting glucose concentration of a randomly chosen man is greater than k, or

p_2 : the probability that the mean fasting glucose concentration of four randomly chosen women is greater than k. [3]

Fasting glucose concentration can also be measured using milligram per decilitre (mg/dL). The formula to convert mmol/L to mg/dL is

$$1\ \text{mmol}/\text{L} = 18.0\ \text{mg}/\text{dL}.$$

The random variable G denotes the fasting glucose concentration, in mg/dL, of a randomly chosen man.

(d) Draw a sketch to show the distribution of G, including the main features of the curve. [2]

(e) On your sketch, shade the region represented by $\text{P}(79 < G < 106)$ and state its value. [2]