RIVER VALLEY HIGH SCHOOL
2025 JC2 Preliminary Examination
Higher 2

NAME	

CLASS						INDEX NUMBER	

MATHEMATICS

9758/01

Paper 1

17 Sep 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

This document consists of **6** printed pages and **2** blank pages.

1 The function f is defined by $f(x)=ax^3+bx^2+cx+d$ where a, b, c and d are real numbers. Given that $4+\mathrm{i}$ and -1 are roots of $f(x)=0$, find b, c and d in terms of a. [4]

2 Without using a calculator, solve the inequality $\dfrac{9}{1-x^2}<\dfrac{x+5}{x+1}$. [3]

Hence solve $\dfrac{9}{1-e^{2x}}<\dfrac{e^x+5}{e^x+1}$. [3]

3 **Do not use a calculator in answering this question.**
Find the roots of the equation $z^2-(1+2\mathrm{i})z+1+7\mathrm{i}=0$, giving your answers in the cartesian form $a+\mathrm{i}b$. [6]

4 It is given that $\displaystyle\sum_{r=1}^{n}\frac{1}{r(r+1)}=1-\frac{1}{n+1}$.

(a) Find $\displaystyle\sum_{r=1}^{N}\frac{1}{(r+1)(r+2)}$ in terms of N. [3]

(b) It is given that

$$8\sum_{r=k+1}^{\infty}\frac{1}{r(r+1)}=\sum_{r=1}^{k}\frac{1}{r(r+1)}.$$

Find the value of k. [3]

5 (a) Find $\displaystyle\int\frac{x}{\sqrt{16-x^4}}\,dx$. [2]

(b) Find $\displaystyle\int\frac{x}{2}\sin^{-1}\left(\frac{x^2}{4}\right)dx$. [5]

6 For any vectors **m** and **n**, explain why $\mathbf{m}\cdot(\mathbf{m}\times\mathbf{n})=0$. [1]

With respect to the origin O, the points A, B and C have position vectors **a**, **b** and **c** respectively. O, A, B and C are non-coplanar. The point M is the mid-point of AC and p denotes the plane OAB.

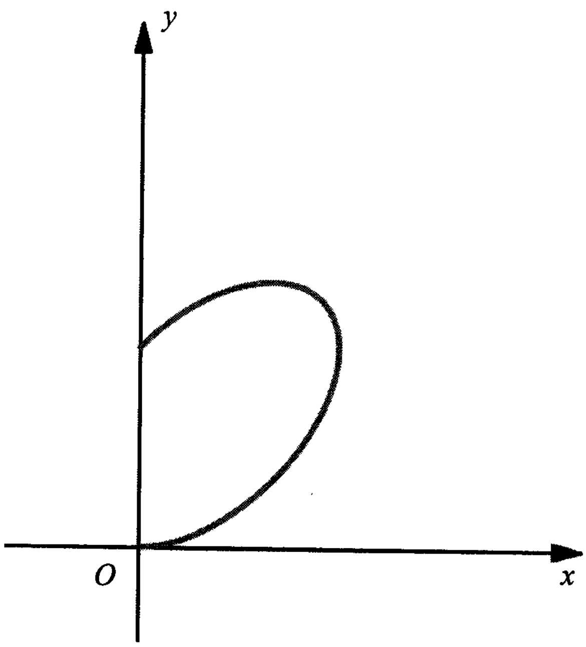
(a) The point R is such that MR is perpendicular to p. Show that R lies on a line ℓ with equation $\mathbf{r}=k(\mathbf{a}+\mathbf{c})+\lambda(\mathbf{a}\times\mathbf{b})$, $\lambda\in\mathbb{R}$, where k is a constant to be determined. [2]

(b) Given that **a** and **b** are unit vectors perpendicular to each other, $\mathbf{a}\cdot\mathbf{c}=-2$ and $\mathbf{b}\cdot\mathbf{c}=4$, find in terms of **a** and **b**, the position vector of the point of intersection between ℓ and p. [3]

7 A curve C has equation $x^2+y^2-4y=xy$, where $x\geq 0$.

(a) Show that $(2y-x-4)\dfrac{\mathrm{d}y}{\mathrm{d}x}=y-2x$. [2]

The diagram below shows the curve C.



M is a point on C with coordinates (x,y) and N is a fixed point $(3,0)$. The area of triangle OMN is denoted by A.

(b) Find A in terms of y. [1]

(c) Show that $\dfrac{\mathrm{d}A}{\mathrm{d}x}=\dfrac{3}{2}\left(\dfrac{y-2x}{2y-x-4}\right)$. [2]

(d) Hence find the exact value of x for which A is a maximum.
(You do not need to show that A is a maximum for the value of x found.) [2]

8 It is given that $(1+9x^2)\frac{dy}{dx}=3y$, and the curve $y=f(x)$ passes through the y-axis at $(0,e^2)$.

(a) Show that $(1+9x^2)\frac{d^2y}{dx^2}+(18x-3)\frac{dy}{dx}=0$. [2]

(b) Find the Maclaurin series for y, up to and including the term in x^3, giving the exact coefficients for each term. [4]

(c) Given that $\ln y=2+\tan^{-1}3x$, find the Maclaurin series for $e^{\tan^{-1}3x}$, up to and including the term in x^3. [2]

9 A curve C has equation $y=ax+b+\frac{b-2a}{x+2}$, where a and b are real constants such that $a>0$, $a\neq\frac{1}{2}b$ and $x\neq-2$.

(a) Given that C has stationary points, use differentiation to find the relationship between a and b. [3]

It is now given that $a=1$ and $b=3$.

(b) Prove algebraically that y cannot lie between -1 and 3. [3]

(c) Sketch C, stating the equations of any asymptotes and the coordinates of any axial intercepts and turning points. [2]

10 The function f is defined by $\mathrm{f}: x \mapsto \dfrac{1}{(x+2)^2}$, $x \in \mathbb{R}, x \neq -2$.

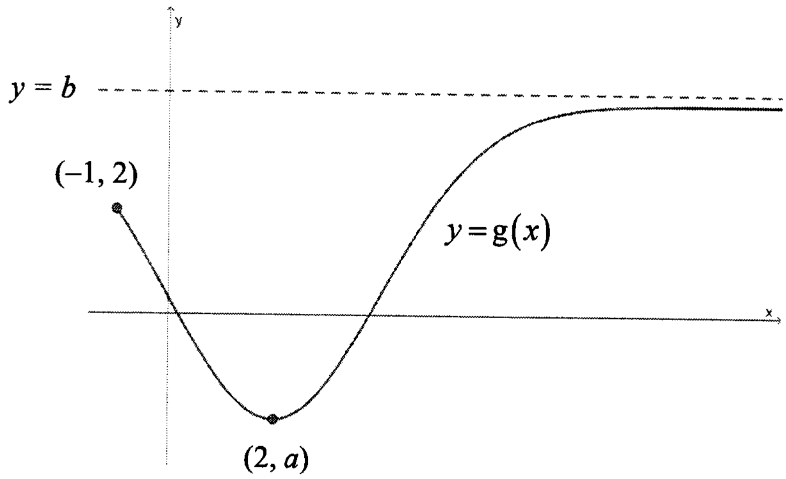
The domain of f is further restricted to be $x > a$, where a is an integer.

(a) State the least value of a such that the function f^{-1} exists. [1]

(b) Hence find $\mathrm{f}^{-1}(x)$ and state its domain. [3]

(c) Sketch the graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$ on the same diagram, giving the equations of any asymptotes and the coordinates of the points where the curves meet the axes. [2]

The function g is defined on the domain $[-1, \infty)$. The graph of $y=\mathrm{g}(x)$ has minimum point at (2, a) and the equation of its asymptote is $y = b$ as shown below.



(d) Determine if gf exist. If it exists, find its range. [3]

11 The curve C has cartesian equation $y=\sqrt{4-2x^2}$.

(a) Sketch the curve C, labelling the exact coordinates of any axial intercepts. [2]

(b) Find the equation of the tangent to C at the point $P\left(1,\sqrt{2}\right)$, leaving your answer in exact form. [2]

(c) Using the substitution $x=\sqrt{2}\sin\theta$, find the exact value of $\int_1^{\sqrt{2}} \sqrt{4-2x^2}\,\mathrm{d}x$. [4]

(d) Hence, find the exact value of the area of the region bounded by C, the tangent to C at P, and the x-axis. [2]

12 The curve C is defined by the parametric equations $x=6t-5$ and $y=2t^2+1$, where $t \geq \frac{5}{6}$.

(a) Sketch the curve C, labelling the exact coordinates of any axial intercept(s). [1]

(b) Find the equation of the normal to C at the point P where $t=1$. [2]

(c) Find the acute angle between the normal to C at the point P where $t=1$ and the tangent to C at the point M where $t=3$. [2]

(d) The curve C is translated 5 units in the positive x-direction and translated 3 units in the negative y-direction, to form the curve D. Find the equation of D in parametric form. [2]

The curve E is defined by the parametric equations $x=7u$ and $y=\frac{9}{u}$, where $u \neq 0$.

(e) Show that at the point of intersection of the curves C and E, $6t^3-5t^2+3t-34=0$. Deduce that there is only one point of intersection and find the coordinates of this point. [4]

13 A cafe roasts its own coffee beans and packages them to be sold. The amount of roasted coffee beans remaining in the cafe at time t days is denoted by x kg. The cafe produces roasted coffee beans at a fixed rate of 10 kg/day, and sells the roasted coffee beans at a rate proportional to x^2 kg/day.

(a) Given that there is no change in the amount of roasted coffee beans remaining in the cafe when there is 5 kg of beans remaining, show that $\frac{dx}{dt}=10-\frac{2}{5}x^2$. Find the general solution of this differential equation. (You do not have to make x the subject.) [3]

Due to improvement in roasting capabilities of the cafe, it is now able to produce roasted coffee beans at the rate of $3x$ kg/day, and sells the roasted coffee beans at a rate proportional to x^2 kg/day.

(b) Given that amount of roasted coffee beans is increasing at the rate of 6 kg/day when the amount of roasted coffee beans remaining in the cafe is 6 kg, write down the modified differential equation relating $\frac{dx}{dt}$ and x. [2]

(c) Given that the initial amount of roasted coffee beans in the cafe is 13.5 kg, solve the differential equation in part (b), expressing x in terms of t.
Deduce the amount of roasted coffee beans remaining in the cafe in the long run, and sketch the curve of x against t. [6]

(d) Based on the curve in part (c), explain in context why this might be favourable for the cafe owner. [1]

END OF PAPER

BLANK PAGE

BLANK PAGE

RIVER VALLEY HIGH SCHOOL
2025 JC2 Preliminary Examination
Higher 2

NAME	

CLASS					

INDEX NUMBER	

MATHEMATICS

9758/02

Paper 2

19 Sep 2025

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

3 hours

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

This document consists of **8** printed pages and **0** blank page.

Section A: Pure Mathematics [40 marks]

1 A sequence is such that $u_1 = N$ where N is a constant, and $u_{n+1} = 3u_n - 8$, for $n \geq 1$.

(a) Describe how the sequence behaves when

(i) $N = 4$, [1]

(ii) $N = 3$. [1]

(b) Find the value of N for which $u_5 = 409$. [2]

(c) The sequence $v_n = (-1)^n (u_n + k)$ for $n \geq 1$ where k is a constant, is convergent when $u_1 = 4$. State the value of k. [1]

2 The points A, B and C represent the complex numbers $z_A = 5 + 6\text{i}$, $z_B = 9 + 3\text{i}$ and z_C respectively. ABC is an isosceles triangle labelled in a clockwise direction where $\angle CAB = 90°$.

(a) Find z_C. [3]

(b) The point D representing the complex number z_D, is such that $ABDC$ is a parallelogram. Find z_D. [2]

3 (a) The region A is bounded by the curves $x^2 + y^2 = 5$ and $y = x^2 + 1$ for $y \geq 1$. Find the volume when A is rotated π radians about the y-axis. [2]

(b) The region B is bounded by the curve $y = \dfrac{2}{\sqrt{x^2 - 4x + 7}}$, the line $y = x - 2$, the line $x = 1$ and the x-axis. Find the exact volume when B is rotated 2π radians about the x-axis. Give your answer in the form $a\sqrt{3}\pi^2 + b\pi$, where a and b are constants to be determined. [6]

4 The dosage of medicine given to a patient needs to be carefully managed in order to achieve the desired result.

(a) To give the patient time to adapt to a particular medicine, the dosage is slowly increased over time. The patient is initially given a dosage of 0.75 units. The dosage is increased by 0.5 units each day until a total of at least 10 units of medicine has been taken. Find the least number of days it takes to achieve this, and the dosage on that day. [4]

(b) The doctor prescribed a new medication to another patient. After n days, the total dosage of the new medicine administered to the patient is given by $\frac{3}{20}\left(\frac{5^n - 4^n}{5^{n-1}}\right)$ units.

(i) Show that the increase in dosage of the new medicine administered per day follows a geometric progression. [3]

(ii) Find the total dosage of the new medicine administered to the patient after a long period of treatment. [2]

(c) Doctors can manage the medication so that the daily dosage of medicine increases as an arithmetic progression. Explain in context why the arithmetic progression is not a preferred model for the dosage of medication. [1]

5 The Planetary Defense Coordination Office at NASA observes and tracks Near Earth Objects (NEOs) that could be potentially hazardous in a collision with Earth. Over short periods of time, vectors can be used to model the trajectories of the Earth and NEOs. They use coordinates (x, y, z) with units in millions of kilometers relative to the sun which is at position (0, 0, 0).

Earth's orbit is contained by the plane Π with equation $\mathbf{r} \bullet \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} = 0$.

(a) A satellite moves along a path with equation $\mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$, $\lambda \in \mathbb{R}$. Determine whether the satellite crosses Earth's orbital plane. [3]

Over a short period of time, Earth's motion can be modelled by the equation

$$\mathbf{r} = \begin{pmatrix} 15 \\ 20 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix}, \mu \in \mathbb{R}.$$

An asteroid moves along a path with equation

$$\frac{x-10}{2} = y+2 = \frac{0.25-z}{k}$$

where k is a positive constant.

(b) Determine the possible values of k if the paths of the Earth and the asteroid do not intersect. [3]

It is now given that $k = 3$.

(c) Find the acute angle between the paths of the Earth and the asteroid. [2]

The asteroid is closest to the Earth when they are at points $(12, 18, -3)$ and $(12, -1, -2.75)$ on their respective paths.

(d) (i) Find the distance between the Earth and the asteroid at this instant. [2]

(ii) By considering the vector between the asteroid and the Earth at this instant, determine whether the distance found in part (d)(i) is the shortest distance between the two paths. [2]

Section B: Statistics [60 marks]

6 A Pizzeria sells pizza by individual slices. 8 slices forms a full pizza.

(a) A group of three friends bought 4 slices of Hawaiian, 1 slice of pepperoni, 1 slice of seafood, 1 slice of cheese, and 1 slice of vegetarian pizzas. Find the number of ways the pizza slices bought could be arranged in a circle such that the 4 slices of Hawaiian pizzas are placed together. [1]

(b) The three friends each eat a slice of Hawaiian pizza. They then want to distribute the remaining 5 slices amongst the three of them. Find the number of ways this can be done such that everyone gets at least one more slice of pizza. [3]

7 In a game, a fair six-sided die is rolled. If a 1 or a 2 is rolled, then the die is rerolled and the score is the result of the reroll. Otherwise, the score is the number rolled originally.

(a) Give the probability distribution function of the scores in the form of a table. [2]

(b) Show that the expected score is $\frac{25}{6}$. Explain the meaning of this value in this context. [2]

(c) Find the exact variance of the scores. [1]

8 A game at a funfair involves pulling balls of different colours out of a bag. A bag contains 1 white ball, 2 blue balls, 3 green balls and 4 red balls. The aim of the game is to pull the white ball from the bag as many times as possible. Players get three attempts. If a red ball is drawn, it is discarded, but any other ball is replaced in the bag.

(a) Find the probability that a player draws a white ball and two green balls on their three attempts. [2]

(b) Find the probability that exactly one white ball was drawn, given that a red ball was drawn on the second attempt. [3]

Events A and B are defined as

A: A white ball is drawn on the first attempt
B: A white ball is drawn on the second attempt

(c) Determine with reason whether events A and B are independent. [2]

9 A machine produces a large number of camera lenses daily. During the process, lenses could get scratched and become defective.

(a) State, in the context of the question, one assumption needed for the number of defective lenses made to be modelled by a binomial distribution and explain why the assumption may not hold in this context. [2]

It is assumed that the number of defective lenses produced daily has a binomial distribution and that on average, 1 in 20 lenses produced are defective.

(b) 30 independent samples of 100 lenses each are taken. Find the probability that there are more than 10 samples in which there are exactly 3 defective lenses in each sample. [2]

(c) 2 independent samples of 100 lenses are taken. It is given that exactly one of the samples was found to have exactly 3 defective lenses. Find the probability that there are a total of 10 defective lenses in the 2 samples. [2]

(d) Every day, lenses are randomly selected for quality checks on the production. An initial sample of 100 lenses is obtained.

- The entire production of lenses for that day would be deemed to have passed the quality check when there are at most 2 defective lenses in the initial sample.
- If there are more than 3 defective lenses found in the initial sample, then the entire production of lenses for that day will be rejected.
- If the initial sample contains exactly 3 defective lenses, another sample of 100 lenses will be taken. If the 2 samples contain an average of at most 2 defective lenses, then the entire production of lenses for that day would be deemed to have passed the quality check.

Find the probability that the production of lenses of any particular day would have passed the quality check. [3]

10 As part of the sound calibration for the National Day Parade at the Padang, sound engineers test how sound intensity changes with distance from the main stage speakers during rehearsals. This is helpful for calibrating the speaker system to avoid sound distortion or echo across seating zones.

At a particular rehearsal, the engineers record the sound intensity, y, measured in decibels, at various distances, x, measured in metres from the stage speakers and recorded the data in the table below.

x	0	2	4	6	8	10
y	100	96	84	65	36	1

(a) Sketch a scatter diagram for these values. Explain, using your diagram, which of the following equations provides the most accurate model of the relationship between x and y.

(A) $y = a - bx^2$,

(B) $y = a + be^{-x}$,

(C) $y = a + b\ln(10 - x)$,

where a and b are constants such that $a > 0$ and $b > 0$. [3]

(b) Using the model you chose in part (a), find the equation of the least squares regression line. State the corresponding value of the product moment correlation coefficient, correct to 5 decimal places. [2]

(c) At a particular spectator seat in the Padang, the sound intensity was recorded to be 20 decibels. Estimate the distance of this spectator seat from the stage speakers. Explain whether your estimate is reliable. [2]

As another part of sound calibration for the National Day Parade, engineers also measured the time that it takes for sound to reach the spectator stands, t, measured in milliseconds, at various distances, x, measured in metres from the stage speakers.

x	10	20	30	40	50	60
t	29.1	58.3	87.5	116.6	145.8	174.9

(d) The engineers would like to investigate if the model $t = 2.916x$ is a good fit of the above data. For a model $t = f(x)$, the residual for a point (p, q) is $q - f(p)$.

(i) Explain why they should use the sum of the squares of the residuals rather than the sum of the residuals when assessing the fit of the model. [1]

(ii) Calculate the sum of the squares of the residuals for the line $t = 2.916x$. [2]

(iii) It is further found that another line $t = 2.916x + D$ where $D \neq 0$ is a better fit for the above data. Explain how the sum of the squares of the residuals for the line $t = 2.916x + D$ differs from the sum of the squares of the residuals of $t = 2.916x$. [1]

11 A family business sells strings which are manually cut into stipulated lengths. A client placed an order for 10 000 strings of length 100 centimetres. The business owner got the workers to cut out the 10 000 strings. To ensure quality, a random sample of 40 strings is chosen from the 10 000 strings for measurement and a constant a was subtracted from the lengths in centimetres to produce the following summary data.

$$\sum(x-a)=10.316 \quad \sum(x-a)^2=41.7$$

(a) Explain in the context of the question what it means for the sample to be random. [1]

(b) Find the unbiased estimates of the population mean and variance. [2]

(c) Using the data, a test was conducted at 10% level of significance and found insufficient evidence to conclude that the mean is not 100 centimetres.
State the hypotheses for the test, defining any parameters that you use. State the critical region and use it to find the range of possible values for the sample mean, giving your answer correct to 2 decimal places. [6]

(d) Explain if the business owner needs to make any assumption about the distribution of the length of strings for the calculations in part (c) to be valid. [1]

(e) The p-value found from the test in part (c) is 0.10286. Write down the p-value if the test was conducted at 6% level of significance on whether the strings are shorter than 100 centimetres. State the conclusion of this test. [2]

12 A factory produces metal discs and metal bars for constructing machinery parts. The mass of a randomly chosen metal disc follows a normal distribution with mean 18 grams and standard deviation 0.6 grams. The mass of a randomly chosen metal bar has an independent normal distribution with mean 20 grams and standard deviation σ grams.

(a) Sketch the distribution for the masses of the metal disc between 14 grams and 22 grams. [1]

(b) 2 metal discs are randomly chosen. Find the probability that the mass of one of the discs is less than 19 grams and the other metal disc has mass more than 19 grams. [2]

(c) Find the range of values of k such that less than 90% of the metal discs have masses within k grams of the mean mass of the metal discs. [2]

(d) It is known that 7.65% of the metal bars have masses more than 21 grams. Find the value of σ. [2]

It is given that $\sigma = 0.5$ for the rest of the question.

(e) A client places an order for a new kind of metal strips. The metal strips are modelled as similar solids to the metal bars with a 20% reduction in mass.
A production worker selects 8 of the metal strips for inspection. Find the probability that at least 5 of the metal strips have masses between 15.8 grams and 17.2 grams. [3]

(f) A protective case is designed for the safe transportation of the metal bar. Each metal bar is placed inside a protective case during transportation. The masses of the protective cases are modelled as 30% of the masses of the metal bars.
Let T denote the total mass of 4 of the metal bars and their protective cases.
Find the variance of T. [2]

END OF PAPER