BP~623

TAMPINES MERIDIAN JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

CANDIDATE NAME: ____________________

CIVICS GROUP: ____________________

H2 MATHEMATICS

9758/01

Paper 1

16 SEPTEMBER 2025

3 hours

Additional materials: Printed Answer Booklet

List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** the questions.

Write your answers in the spaces provided in the Printed Answer Booklet. Follow the instructions on the front cover of the Printed Answer Booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

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The number of marks is given in brackets [ ] at the end of each question or part question.

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[Turn Over

1 **(a)** On the same diagram, sketch the graphs of $y=|x-a|$ and $y=\frac{1}{|x-a|}$, where a is a real constant such that $a>1$. You should show clearly the equations of any asymptotes and axial intercepts of both graphs. [3]

(b) Hence, or otherwise, solve the inequality $|x-a|>\frac{1}{|x-a|}$. [3]

2 A curve C has equation $y^2=x+e^{2x}y$.

(a) Show that $\left(2y-e^{2x}\right)\frac{dy}{dx}=1+2ye^{2x}$. [2]

(b) Find the equations of the tangents to C at the points where $x=0$. [4]

3 Find the following integrals.

(a) $\int \frac{x^2-3x-1}{x^2-4x+1}\,dx$, [3]

(b) $\int x\ln(kx)\,dx$, where k is a real constant, [2]

(c) $\int \frac{\sin\theta}{2\cos 2\theta+1}\,d\theta$, using the substitution $x=\cos\theta$. [4]

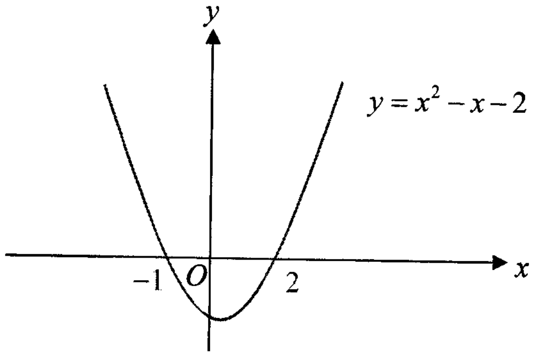
4 An arithmetic series has first term a and common difference d, where a and d are non-zero real constants. A convergent geometric series has first term b and common ratio r, where $b>0$ and r is a non-zero real constant. The sum of the first n terms of the geometric series is denoted by S.

It is given that the 7th, 10th and 11th terms of the arithmetic series are equal to the 3rd, 6th and 10th terms of the geometric series respectively.

(a) Show that r satisfies the equation $3r^7-4r^3+1=0$. Solve this equation, giving your answer correct to 4 decimal places. [4]

(b) A new series is formed by taking the even numbered terms of the geometric series. Find the smallest value of n for which S differs from the sum to infinity of the new series by at least $1.78b$. [4]

5



The diagram shows the graph of $y=x^2-x-2$. The two roots of the equation $x^2-x-2=0$ are -1 and 2.

A sequence of real numbers $x_1, x_2, x_3, \ldots$ satisfies the recurrence relation

$$x_{n+1}=x_n^{\ 2}-2$$

for $n \geq 1$.

(a) It is given that as $n \to \infty$, $x_n \to L$. Show that L can take values -1 or 2. [2]

(b) Hence or otherwise, determine the behaviour of the sequence when $x_1=-1$. [1]

(c) By considering $x_{n+1}-x_n$, prove that $x_{n+1}<x_n$ if $-1<x_n<2$. [2]

6 [It is given that the volume of a circular cone with base radius r and height h is $\frac{1}{3}\pi r^2 h$.]

A container is made in the shape of an inverted open right circular cone. The height of the container is 6 metres, and the radius is 2 metres. The container is initially empty. Water is poured into the container while, at the same time, water is leaking from a hole at the bottom of the container at a rate of $0.03\ \text{m}^3/\text{s}$, resulting in the depth of water in the container to be increasing at a rate of 0.1 m/s. At time t seconds after the start, the depth of water in the container is h metres.

(a) Find, in terms of h, an expression for the volume of water in the container at time t seconds after the start. [2]

(b) Find the rate at which water is being poured into the container when the depth of water in the container is 3 metres. [4]

[Turn Over

7 The function f is such that

$$f : x \mapsto \frac{1}{2}\sqrt{36-(x-3)^2} \quad \text{for} \quad x \in \mathbb{R}, \quad k \le x < 9,$$

where k is a real constant.

(a) State the least value of k for which the function f^{-1} exists. [1]

For the rest of the question, take the value of k as the value found in part **(a)**.

(b) Find $f^{-1}(x)$. [3]

(c) Sketch, on the same diagram, the graphs of $y = f(x)$, $y = f^{-1}(x)$ and $y = f^{-1}f(x)$, showing clearly the relationship between the three graphs. [3]

The function g is such that

$$g : x \mapsto |4x-1|(2x-5) \quad \text{for} \quad x \in \mathbb{R}, x > 0.$$

(d) Show that the composite function gf exists. [2]

(e) Find the range of gf. [2]

8 A curve C has parametric equations

$$x = 2t+3,\ y = 5-4t^2,$$

for all real values of t.

The line N is the normal to C at the point where $t = -\frac{1}{2}$.

(a) Find the cartesian equation of N. [3]

(b) Find the cartesian equation of C. [2]

(c) The region R is bounded by C, N and the y-axis. Without using a calculator, find the exact volume of the solid generated when the region R is rotated through 2π radians about the y-axis. [5]

9 It is given that $-2+2\text{i}$ is a root of the equation

$$z^3+az^2+bz-16\sqrt{2}=0,$$

where a and b are real numbers.

(a) Find the values of a and b and the other two roots. Leave your answers in the exact form. [5]

(b) In an Argand diagram with origin O, the three roots are represented by points A, B and C where A represents $-2+2\text{i}$ and C represents the real root. Label these points on an Argand diagram, indicating clearly the modulus and argument of each root. State also a geometrical relationship between A and B. [3]

(c) Hence, prove that

$$\tan\frac{3\pi}{8}=1+\sqrt{2}.$$ [3]

10 A drone flies in a straight line towards a rooftop for inspection. The drone's flight path is modelled by the line $l:\mathbf{r}=\begin{pmatrix}2\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}3\\-2\\1\end{pmatrix}$ where $\lambda\in\mathbb{R}$, and the rooftop is modelled by the plane $p: 2x+y-z=7$.

(a) On Day 1, the drone flies towards the rooftop using the flight path. Upon reaching the rooftop, the drone flies in a new flight path, m. The acute angle between the original flight path and the normal to the rooftop is the same as the acute angle between the new flight path and the same normal to the rooftop. Both flight paths and the normal lie on the same plane. Find the cosine of the angle between the original flight path and the new flight path in an exact non-trigonometrical form. [3]

(b) On Day 2, the drone starts to descend from point A with coordinates $(2,1,0)$ towards the rooftop using the shortest possible path. Find the exact coordinates of the point where the drone lands on the rooftop. [4]

(c) After landing, the drone glides on the rooftop in another new flight path, s, that is perpendicular to the flight path l. Find a cartesian equation of the flight path s. [3]

A second rooftop is built parallel to the first rooftop p such that the perpendicular distance between these two rooftops is d units. Find two possible cartesian equations of the second rooftop in terms of d. [3]

[Turn Over

11 An architect designs a flower bed for a garden. The flower bed is enclosed by the curve C with equation $\left(\frac{x+y}{2}\right)^2+\left(\frac{y-x}{4}\right)^2=1$.

The region in the first quadrant bounded by C, the x-axis and the lines $x=0$ and $x=0.5$, is to be planted with tulips.

(a) By finding the Maclaurin series expansion of y up to and including the term in x^2, where $y>0$, find an approximation for the area to be planted with tulips. [7]

The curve C is obtained by rotating an ellipse S about the origin in the x-y plane. It is known that the curve C can be expressed by the following parametric equations

$$x=\cos t-2\sin t,\ y=\cos t+2\sin t,$$

where $-\pi<t\leq\pi$.

(b) Let L be the distance between the origin and a point on C. By using differentiation, determine the exact maximum and minimum values of L. You do not need to show that these values are maximum and minimum. [5]

(c) Write down a possible cartesian equation of S. Hence, determine the total area of the flower bed. [3]

End of Paper

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TAMPINES MERIDIAN JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

CANDIDATE NAME: ____________________

CIVICS GROUP: ____________________

H2 MATHEMATICS

9758/02

Paper 2

22 SEPTEMBER 2025

3 hours

Additional materials: Printed Answer Booklet

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[Turn Over

Section A: Pure Mathematics [40 marks]

1 **(a)** Without using a calculator, solve the inequality

$$\frac{x+5}{-2x^2+5x+3}<1.$$ [4]

(b) Hence, solve exactly the inequality

$$\frac{\cos x+5}{-2\cos^2 x+5\cos x+3}<1$$

for $0 \le x \le \pi$. [2]

2 It is given that $\mathrm{f}(\theta)=a\theta^2+b\theta+c$, where a, b and c are real constants. The curve with equation $y=\mathrm{f}(\theta)$ passes through the point with coordinates $(0,1)$ and has a turning point at $\left(-\frac{\sqrt{3}}{3},\frac{1}{2}\right)$.

(a) Find the exact values of a, b and c. [3]

(b) In a triangle ABC, $AB=1$, $AC=\sqrt{3}$ and angle $BAC=\theta+\frac{\pi}{6}$ radians. Given that θ is sufficiently small for θ^3 and higher powers of θ to be neglected, show that $BC^2 \approx \mathrm{f}(\theta)$. [3]

(c) Hence, show that $BC \approx 1+\frac{\sqrt{3}}{2}\theta+\frac{3}{8}\theta^2$. [3]

3 The function f is defined by

$$\mathrm{f}(x)=\begin{cases} x+4 & \text{for} \quad -4 \le x < -1, \\ x^2+2 & \text{for} \quad -1 \le x < 2. \end{cases}$$

It is given that $\mathrm{f}(x)=\mathrm{f}(x+6)$ for all real values of x.

(a) Sketch the graph of $y=\mathrm{f}(x)$ for $-6 \le x < 6$. [3]

(b) Hence sketch the graph of $y=2\mathrm{f}(x-1)$ for $-6 \le x < 6$. [3]

4 A viral trend on social media gains popularity rapidly but eventually slows down due to oversaturation. Let P represent the popularity score of the trend at time t (measured in days), where $0 \le t \le 100$. The change in the trend's momentum is modelled by $\frac{d^2P}{dt^2} = -0.05\left(\frac{dP}{dt}\right)^2$.

(a) By substituting $v = \frac{dP}{dt}$, show that $v = \frac{20}{t+C}$ where C is a real constant. [3]

(b) Given that $P = 0$ and $v = 10$ when $t = 0$, find the particular solution of P in terms of t. [4]

(c) Find the popularity score of the trend when $t = 30$. [1]

5 Referred to the origin O, the points A, B and C have position vectors **a**, **b** and **c** respectively where **a** is a non-zero vector and $\mathbf{b} \neq \mathbf{c}$.

(a) Given that $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, show that $\mathbf{b} - \mathbf{c} = \lambda\mathbf{a}$, where λ is a non-zero constant. [2]

It is given that the area of the trapezium $OABC$ is 10 units^2, $|\mathbf{a}| = 3$, $|\mathbf{c}| = 5$ and the angle between **a** and **c** is 30°.

(b) Show that $BC = 5$ units. [3]

(c) State the value of k such that $\overrightarrow{OA} = k\overrightarrow{CB}$, where k is a positive constant. [1]

(d) The line OC meets the line AB at point D. Find $\overrightarrow{OD}$ in terms of **c**. [5]

Section B: Probability and Statistics [60 marks]

6 For events A, B and C, it is given that $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.42$, $P(A' \cap B \cap C) = x$ and $P(A \cap B \cap C) = 0.02$. It is also given that events A and B are independent, and that events A and C are independent.

(a) Find $P(A \cap B)$ and $P(A \cap C)$. [2]

(b) Draw a Venn diagram to represent this situation, showing the probability in each of the eight regions, in terms of x where necessary. [3]

(c) Find the greatest and least possible values of $P(A' \cap B' \cap C)$. [2]

7 A coffee shop owner claims that its signature coffee has a mean caffeine content of 120 mg per cup. To investigate this claim, a random sample of 60 cups of coffee is selected and the caffeine content, x mg per cup, is summarised as follows.

$$\sum x = 6960 \qquad \sum x^2 = 822465$$

(a) Calculate unbiased estimates of the population mean and variance for the caffeine content per cup. [2]

(b) Test, at the 5% level of significance, whether the coffee shop owner's claim is supported by the data. You should state your hypotheses and define any symbols that you use. [4]

The coffee shop also sells premium coffee, where its caffeine content is normally distributed with population variance 200 mg^2. The coffee shop owner now claims that its premium coffee has a mean caffeine content less than 120 mg per cup. Another large random sample of n cups of premium coffee is selected and the sample mean caffeine content per cup is found to be 116.6 mg. A test is carried out at the $2\frac{1}{2}$% level of significance and the result supports the owner's claim.

(c) Find the set of values that n can take. [4]

8 A manufacturer produces mystery boxes, each containing either a regular or a seasonal toy. On average, p% of the mystery boxes contain a seasonal toy.

Amber orders n mystery boxes from the manufacturer. The number of seasonal toys that Amber gets is the random variable S.

(a) State, in the context of the question, two assumptions needed for S to be well modelled by a binomial distribution. [2]

You are now given that S can be modelled by a binomial distribution.

(b) Given that $\mathrm{P}(S=2)=\mathrm{P}(S=3)$ and $\mathrm{E}(S)=2.96$, find the value of p. [4]

Assume now that $p=5$.

The manufacturer now packs the mystery boxes into cartons of 12 each for sale. Each carton is checked for quality control. If there is at most 1 mystery box containing a seasonal toy, the carton is accepted. Otherwise, the carton is rejected.

(c) Given that a randomly chosen carton is rejected, find the probability that no more than 30% of the boxes in the carton each contains a seasonal toy. [3]

9 A car dealer is investigating how the value of a car depreciates over time. A random sample of eight cars of the same model is selected and the current resale value, y thousand dollars of each car is recorded along with its age in x years. The results are shown in the table.

Age of car (x years)	0	1	2	3	4	5	6	7
Resale Value (y thousand dollars)	30.0	25.8	22.3	19.2	16.5	14.4	12.5	10.9

(a) Draw the scatter diagram for these values, labelling the axes clearly. [1]

It is thought that the resale value of a car, y, can be modelled by one of the formulae

$$y=a+bx \quad \text{or} \quad \ln y=c+dx,$$

where a, b, c and d are real constants.

(b) Find, correct to 5 decimal places, the value of the product moment correlation coefficient between

(i) y and x,

(ii) $\ln y$ and x. [2]

(c) Use your answers to parts **(a)** and **(b)** to explain which of $y=a+bx$ and $\ln y=c+dx$ is the better model. [2]

It is required to estimate the age of a car with a resale value of $18000.

(d) Find the equation of a suitable regression line and use it to find the required estimate. [2]

(e) Without the use of a graphing calculator, re-write your equation from part **(d)** so that it can be used to estimate the resale value when the age is given in months. [2]

10 Wardrobe A contains 4 distinct blouses, labelled T_1, T_2, T_3 and T_4. Wardrobe B contains 3 distinct pairs of pants, labelled P_1, P_2 and P_3. The following table shows the formality (formal or casual) of all the clothing.

Wardrobe	Clothing	Formal	Casual
A	Blouse	T_1, T_2	T_3, T_4
B	Pants	P_1, P_2	P_3

Nancy selects an outfit at random, by first choosing one blouse from Wardrobe A, followed by choosing one pair of pants from Wardrobe B.

X denotes the score of a randomly chosen outfit.

- If the blouse and pants have the same formality, then the score of the outfit is the sum of their respective indices. For example, the score of an outfit comprising T_4 and P_3 is 7.
- If the blouse and pants have different formality, then the score of the outfit is the product of their respective indices. For example, the score of an outfit comprising T_2 and P_3 is 6.

(a) Show that $\mathrm{P}(X=4)=\frac{1}{6}$. [1]

(b) Find the probability distribution of X. [3]

(c) Show that $\mathrm{E}(X)=\frac{55}{12}$ and find $\mathrm{Var}(X)$. [3]

(d) Nancy selects three outfits in succession **without replacement**. Find the probability that there are exactly two outfits that comprise both a formal blouse and a pair of formal pants. [3]

11 Anastasia leaves her house to travel to school every school day. On each school day, she will first take C minutes to cycle from her house to the train station, followed by taking a train ride of T minutes, and finally taking a bus ride of B minutes, before she reaches her school. You should assume that all waiting and walking times are negligible. It is given that C, T and B follow independent normal distributions, with means and standard deviations as shown in the table.

	Mean (minutes)	**Standard Deviation (minutes)**
C	8	0.2
T	17	σ
B	15	2.1

Anastasia is supposed to reach school by 7.30 am so that she is not late for school.

(a) Given that 10% of the train rides will take at most 15 minutes, find the value of σ. [2]

For the rest of the question, use $\sigma = 1.4$.

(b) Find the probability that Anastasia spends between 35 and 50 minutes travelling from her house to school on a randomly chosen school day. [3]

(c) Show that the probability that Anastasia is late for school if she leaves house at 6:48 am is 0.215, correct to 3 significant figures. [1]

(d) Find the latest time that Anastasia needs to leave her house if she wants to be at least 90% confident that she will not be late for school. [2]

(e) Find the probability that the total time Anastasia spends cycling from her house to the train station on five randomly chosen school days exceeds twice the time she spends on the train ride on a randomly chosen school day. [3]

(f) Anastasia attends school five days a week and she leaves house at 6:48 am every day for school. Find the probability that she is late for school twice in the first two weeks and her 3rd late coming occurs in the 6th week. [4]

End of Paper