Name: **Register Number:** **Class:**

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NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION 2024
SECONDARY FOUR EXPRESS

MATHEMATICS **4052/01**

Paper 1 **15 August 2024, Thursday**

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 90.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer all the questions.

1 State whether each of the following statements is **True** or **False**.

(a) All natural numbers are either composite or prime number.

Answer ..... [1]

(b) For all real values of x, x^3 is always greater than x^2.

Answer ..... [1]

---

2 $a = 2 \times 3^2 \times 5$

$b = 2^2 \times 5 \times 7$

Using only prime factorisation, explain why $\frac{105b}{a}$ is not a whole number.

Answer

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.....

..... [2]

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3 A rope is 44 m long.
Jay cuts the rope into two pieces so that the ratio of their lengths is 4:7.
He subsequently cuts away the same length of rope from each piece. The new ratio of the remaining lengths of the ropes is 2:5.
Calculate the length that Jay had cut away from each piece of rope.

Answer .....m [3]

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4 Solve $7^{9-x^2} - 1 = 0$. Find the values of x.

Answer ..... [2]

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5 It is given that y^3 is inversely proportional to the square of x.
The difference between the value of y^3 when $x = 3$ and $x = 6$ is 5.
Find the values of x when $y = 2$.

Answer ..... [3]

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6 There are 12.044×10^{23} atoms in 24 g of carbon.
Find the number of atoms in 5 kg of carbon. Leave your answer in standard form.

Answer ..... [2]

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7 The diagrams show the graphs of six different functions.

Diagram A

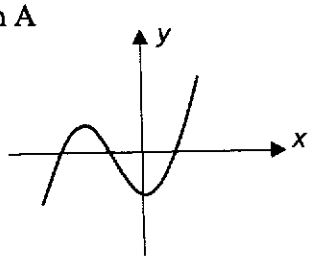


Diagram B

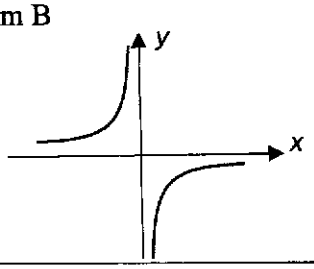


Diagram C

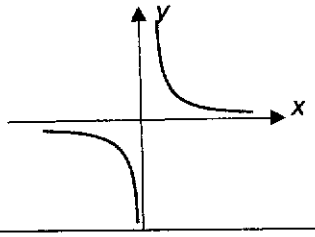


Diagram D

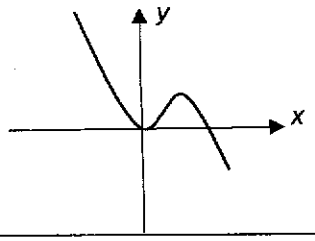


Diagram E

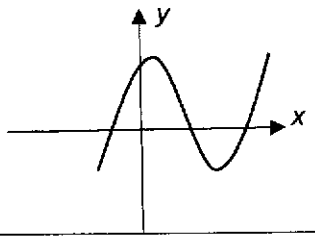
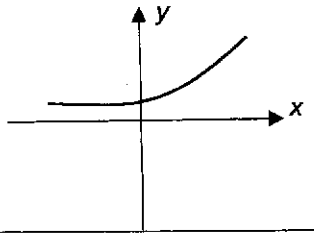


Diagram F



In each case, select one of the graphs that best represent the equation.

(a) $y = (x^2 - 4)(x + 4)$ *Answer* Diagram ..... [1]

(b) $y = -\frac{2}{x}$ *Answer* Diagram ..... [1]

(c) $y = x^2 - x^3$ *Answer* Diagram ..... [1]

8 Explain whether $x = 22$ and $y = 1$ is a possible solution of the equation $9x + 2.5y = 3y - 2x$.

Answer

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..... [2]

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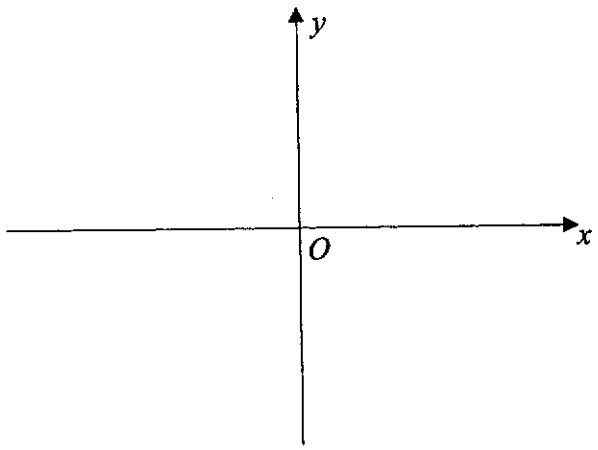
9 On the axes provided, indicating the y-intercept clearly, sketch the graph of

(a) $y = -x$, [1]

(b) $3y - kx - 9 = 0$, where $k < -3$. [2]

Answer



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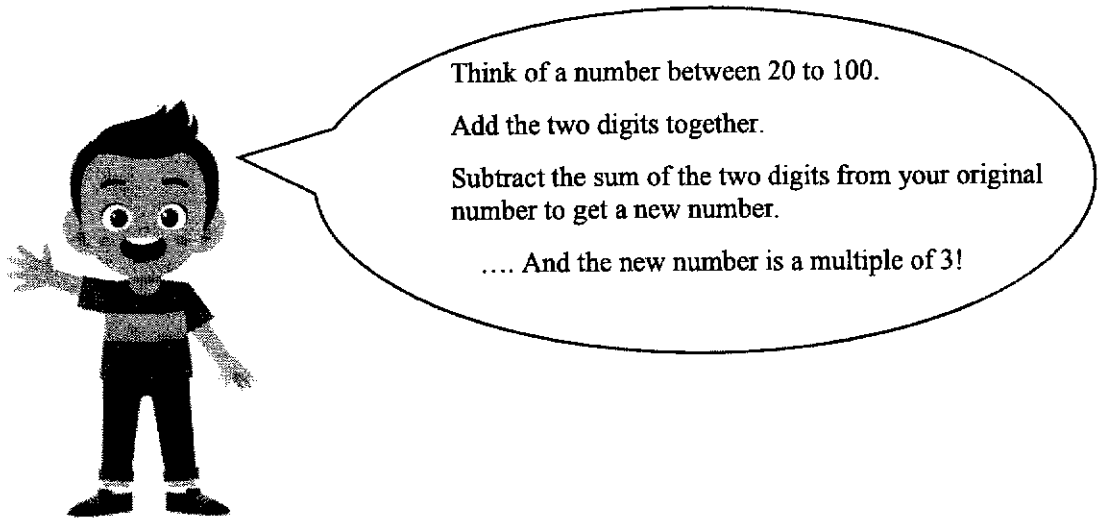
10 Jake plays a mind-reading game with his friends.



Using algebraic expression, explain clearly why Jake is able to predict that the new number is a multiple of 3.

Answer

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..... [3]

11 It is given that $2(mn)^2 - 2mn = 3mn + 3$.
Find n in terms of m.

Answer ..... [2]

---

12 Simplify $\frac{4}{3} + \frac{2}{9m} \div \sqrt[4]{81m^{16}}$, giving your answer as a single fraction.

Answer ..... [3]

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13 Simplify $\frac{-9x^2+12x-4}{3-\frac{1}{x-\frac{x}{2}}}$.

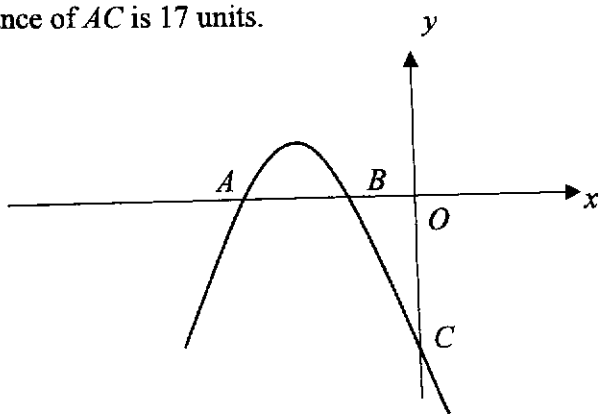
Answer ..... [3]

14 The curve $y = \frac{1}{5}x^2 - 2x + 7$ intersects a line which is equidistant from the positive x and y-axes. Find the coordinates of the points of intersection.

Answer ..... [3]

15 In the diagram, the quadratic curve cuts the x-intercept at point A $(-8, 0)$ and point B. The curve also cuts the y-intercept at point C.

The distance of point B from the origin is one-eighth of the distance of point C from the orign. The distance of AC is 17 units.



(a) Find the coordinates of point C.

Answer ..... [2]

(b) Find the coordinates of the maximum point.

Answer ..... [4]

16 A bag contains 8 blue marbles, 5 green marbles and x yellow marbles.

The marbles are drawn one after the other from the bag, at random, without replacement.

(a) The probability that two blue marbles are drawn is $\frac{4}{33}$. Calculate the value of x.

Answer ..... [3]

(b) The two marbles drawn in **part (a)** are returned into the bag.

Hence, calculate the probability that only the second marble is green when three marbles are drawn from the bag.

Answer ..... [2]

17 Given that

$\varepsilon = \{\text{positive integers less than } 17\}$,

$X = \{2, 3, 4\}$

$Y = \{\text{prime numbers}\}$

$Z = \{2, 3, 4, 5, \ldots, n-1, n\}$

(a) Draw a Venn Diagram to represent the set ε, X and Y, indicating the elements clearly. [3]

Answer

(b) **(i)** List all subsets of X.

Answer

..... [2]

(ii) Hence, deduce the number of subsets of Z in term of n.

Answer ..... [2]

18 The table represents the weekly salary of workers in a company.

Weekly salary	\$150	\$200	\$300	\$400	\$500	\$2400
Number of workers	12	10	8	x	5	1

(a) Given that the weekly mean salary of the workers is \$325, calculate the total number of workers in the company.

Answer ..... [3]

(b) Explain which measures of central tendency is a better gauge of workers' salary.

Answer

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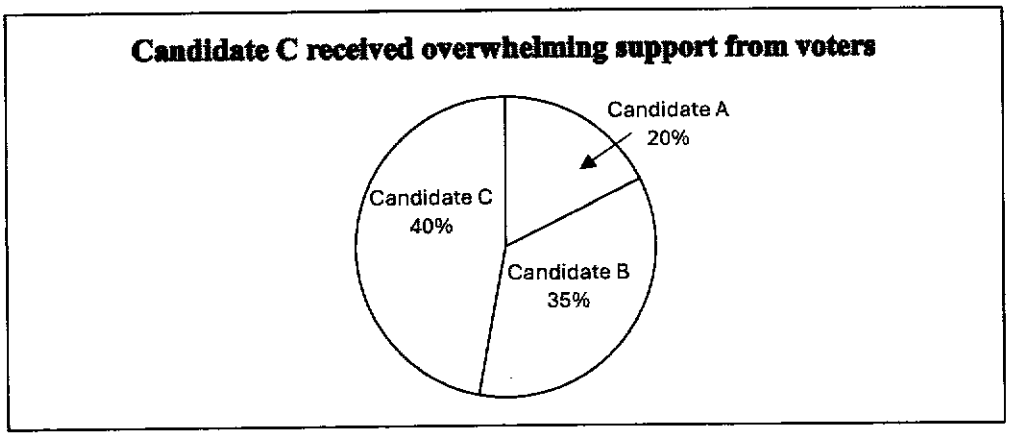
..... [2]

19 The pie chart shows the results of votes for Student Council President election.



(a) Give one reason to explain why the pie chart shown is considered misleading.

Answer

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..... [1]

(b) Propose one way that you could improve the statistical diagram.

Answer

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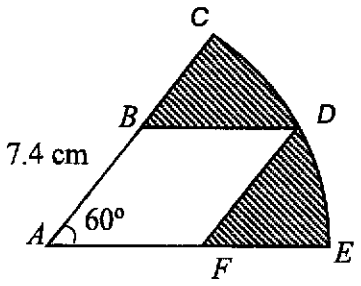
..... [1]

20 The diagram shows a figure consisting of a rhombus, $ABDF$ inscribed in a sector, ACE with centre A. $AB = 7.4$ cm and $\angle CAE = 60°$.



(a) Calculate the shaded area.

Answer ..... cm^2 [3]

(b) Calculate the perimeter shaded area.

Answer .....cm [2]

21 Consider the following pattern.

Line 1: $2^2 + 0^2 = (2-0)^2 + 2(2)(0)$

Line 2: $4^2 + 1^2 = (4-1)^2 + 2(4)(1)$

Line 3: $6^2 + 2^2 = (6-2)^2 + 2(6)(2)$

Line 4: $8^2 + 3^2 = (8-3)^2 + 2(8)(3)$

⋮ ⋮

⋮ ⋮

(a) Write down Line 5 of the sequence of pattern.

Answer ..... [1]

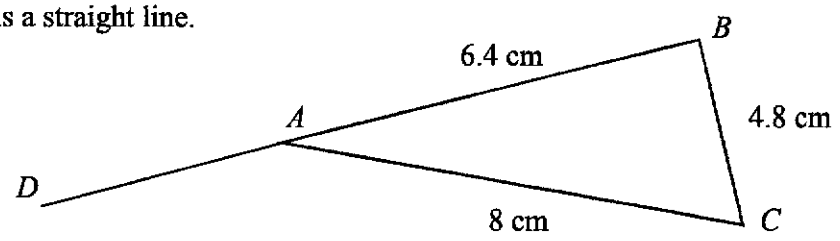
(b) Write down Line n of the sequence of pattern.

Answer ..... [1]

(c) Hence find the value of p and of q, given that $p^2 + q^2 = 1249$ is part of the pattern where $q < p$.

Answer $p =$ ..... and $q =$ ..... [4]

22 The diagram shows a triangle ABC with $AC = 8$ cm, $AB = 6.4$ cm and $BC = 4.8$ cm. DAB is a straight line.



(a) Without calculating the value of angles in the triangle, find the value of sin $\angle ACB$.

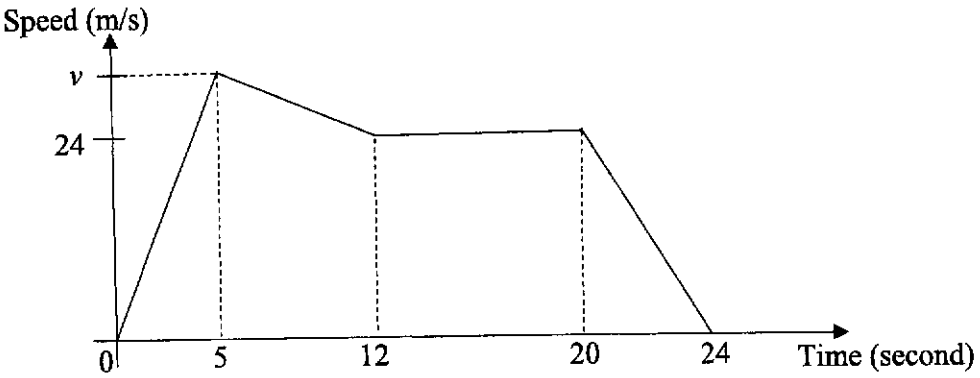
Answer ..... [3]

(b) State the value of cos $\angle DAC$.

Answer ..... [1]

23 The diagram shows the speed-time graph for a car journey.

The total distance travelled by the car is 504 m.



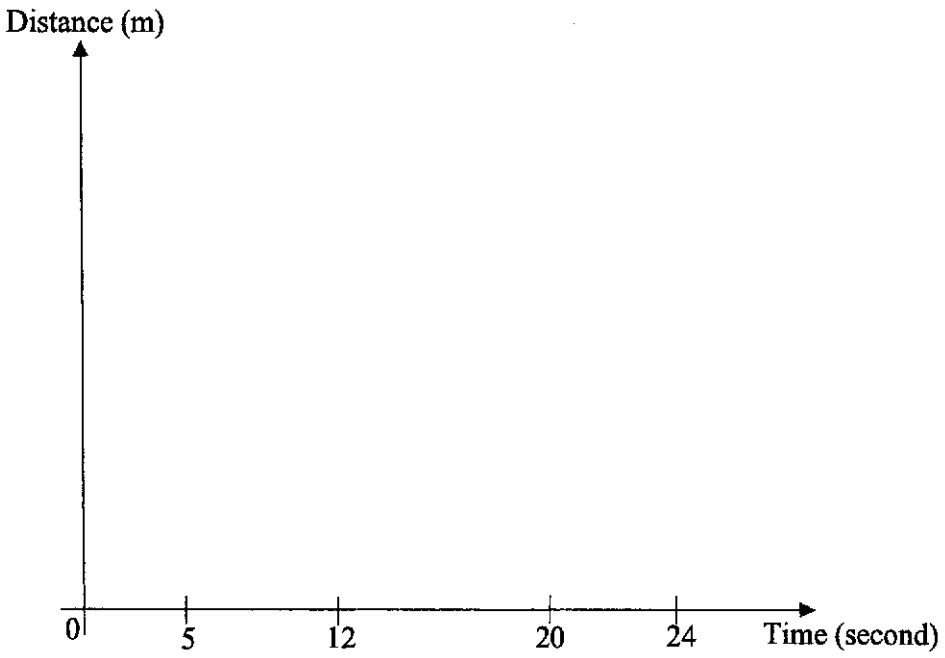
(a) Convert the constant speed of the car during the journey into km/h.

Answer .....km/h [1]

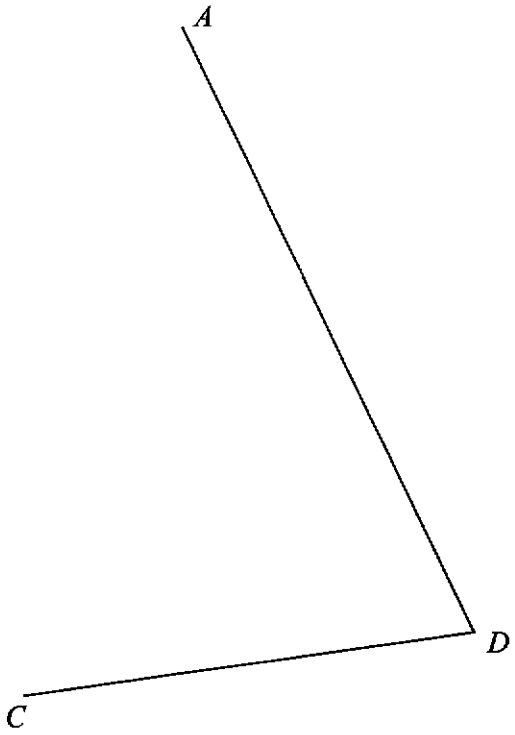
(b) Calculate the greatest speed, v m/s of the car.

Answer .....m/s [2]

(c) Sketch the distance-time graph for the entire journey. [2]



24 *ABCD* represents a plot of flat land, drawn on a scale of 1 cm : 15 km.



(a) Construct and mark clearly point *B* such that its actual distance is 145.5 km from point *A* and 109.5 km from point *C*. [1]

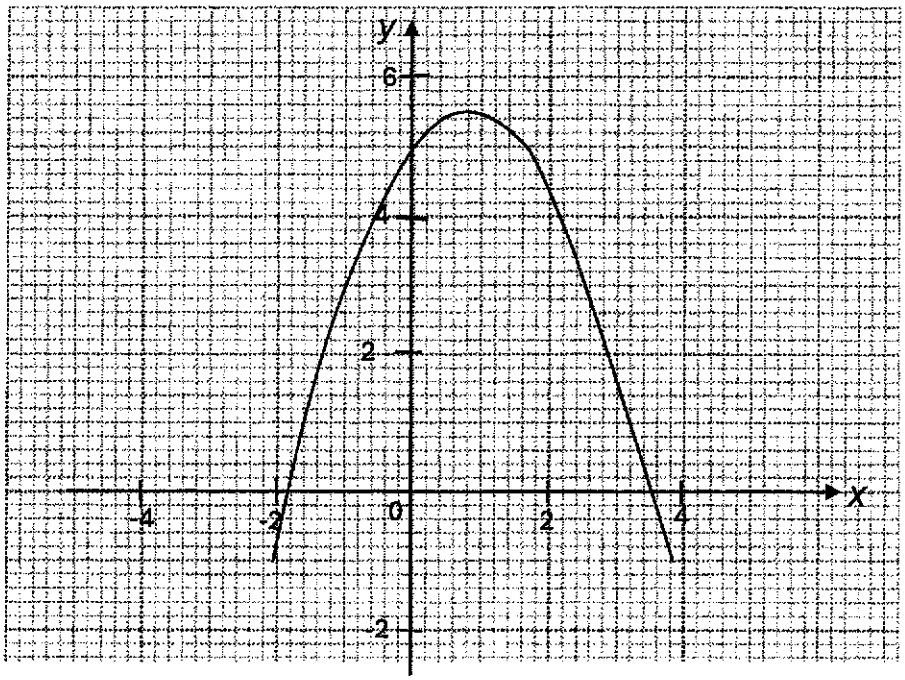
(b) A vertical Tower, *T*, is equidistant from *A* and *C* and equidistant from *AD* and *CD*.

With clear constructions, mark the letter, *T* on the diagram to indicate the position of the Tower. [2]

(c) Hence, measure and calculate the actual distance from *T* to *C*.

Answer .....km [2]

25 A portion of the curve $y = -\frac{1}{4}(3x^2 - 5x + m)$ is drawn on the grid.



Using the graph,

(a) state the value of m.

Answer ..... [1]

(b) state the coordinates where the gradient of the curve is 0.

Answer ..... [1]

(c) state the value of k for which the line $y = 2x + k$ is a tangent to the curve.

Answer ..... [2]

- End of Paper -

Name: **Register Number:** **Class:**

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90

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION 2024
SECONDARY FOUR EXPRESS

MATHEMATICS **4052/02**

Paper 2 **19 August 2024, Monday**

Candidates answer on the Question Paper. **2 hours 15 minutes**

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Mathematical Formulae

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$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

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$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

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$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

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$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer all questions.

1 **(a)** Simplify $\frac{9a^2-4b^2}{3ap+6aq-4bq-2bp}$.

Answer ..... [3]

(b) Write as a single fraction in its simplest form $\frac{x}{6-7x-5x^2}+\frac{2}{5x-3}$.

Answer ..... [3]

(c) Solve the inequality $-\frac{1}{2} < \frac{2y}{5} - \frac{1+y}{3} \leq \frac{5}{6}$.

Answer ..... [3]

2 A closed container, in the shape of a cone, is partially filled with water to a height of 25 cm as shown in **Diagram I**.
The diameter of the container is 38 cm and the height of the container is 40 cm.

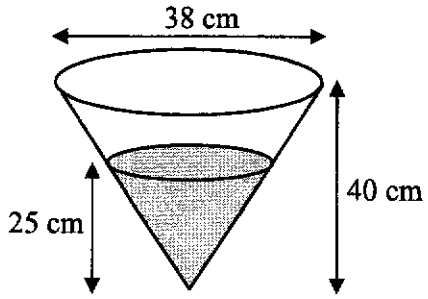


Diagram I

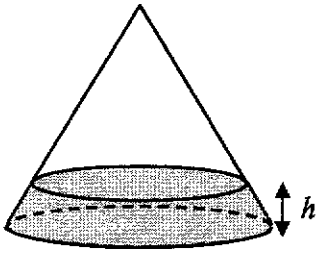


Diagram II

(a) Calculate the amount of water in the container, leaving your answer in litres.

Answer .....litres [3]

The container is subsquently inverted so that the water flows to the base of the cone as shown in **Diagram II**.

(b) Calculate the height of the water, h cm, in the cone as shown in **Diagram II**.

Answer ..... cm [4]

3 Sam, Jane and Peter each decided to purchase a new car priced at $108 888.

(a) Sam paid for his new car in cash and was given a discount by the salesman. Given that Sam paid $100 176.96, find the percentage discount he received.

Answer ..... % [2]

(b) Jane paid 15% of the cash price as deposit and take a loan for the remaining amount from the bank which charges simple interest at the rate of 2.98% per annum over a period of five years.

Calculate the monthly instalment Jane had to pay over the five years period.

Answer $..... [4]

(c) Peter paid \$18 888 as deposit and take a loan for the remaining amount from the bank which charges compound interest at the rate of R% per annum over a period of ten years. He paid a monthly instalment of \$960.

Calculate the value of R.

Answer ..... [3]

4 Three bakery outlets A, B and C tracked the number of caramel, strawberry and mint cupcakes sold in a week in the table shown below.

	Caramel Cupcakes	Strawberry Cupcakes	Mint Cupcakes
Outlet A	60	68	55
Outlet B	49	56	71
Outlet C	53	70	80

This information can be represented by the matrix $\mathbf{P} = \begin{pmatrix} 60 & 68 & 55 \\ 49 & 56 & 71 \\ 53 & 70 & 80 \end{pmatrix}$.

(a) Caramel cupcakes cost \$1.20 each to bake.
Strawberry cupcakes cost \$1.50 each to bake.
Mint cupcakes cost \$0.95 each to bake.

Represent these costs in a 3×1 column matrix **Q**.

Answer **Q** = [1]

(b) Evaluate the matrix $\mathbf{R} = \mathbf{PQ}$.

Answer **R** = [2]

(c) State what each element of matrix **R** means.

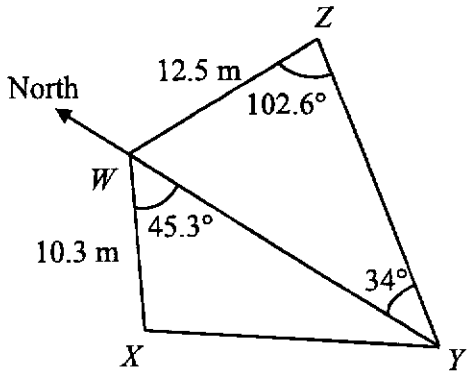
.....

..... [1]

(d) The selling price of each caramel, strawberry and mint cupcake are marked up by 200%, 150% and 300% of the respective cost price.
Calculate the total amount of profit the three bakery outlets collected in **a month**.

Answer $ ..... [4]

5



The diagram shows four points W, X, Y and Z in an open field where W is due north of Y.
$WZ = 12.5$ m and $WX = 10.3$ m.
Angle $WZY = 102.6°$, angle $WYZ = 34°$ and angle $XWY = 45.3°$.

(a) Calculate XY.

Answer ..... m [4]

(b) Calculate the bearing of W from Z.

Answer ..... [3]

A vertical tree of height 8 m is situated at point Z.
A man whose height is 1.9 m, walks from point Y to point W.

(c) Find the greatest angle of elevation of the top of the tree from the top of the man's head.

Answer ..... [3]

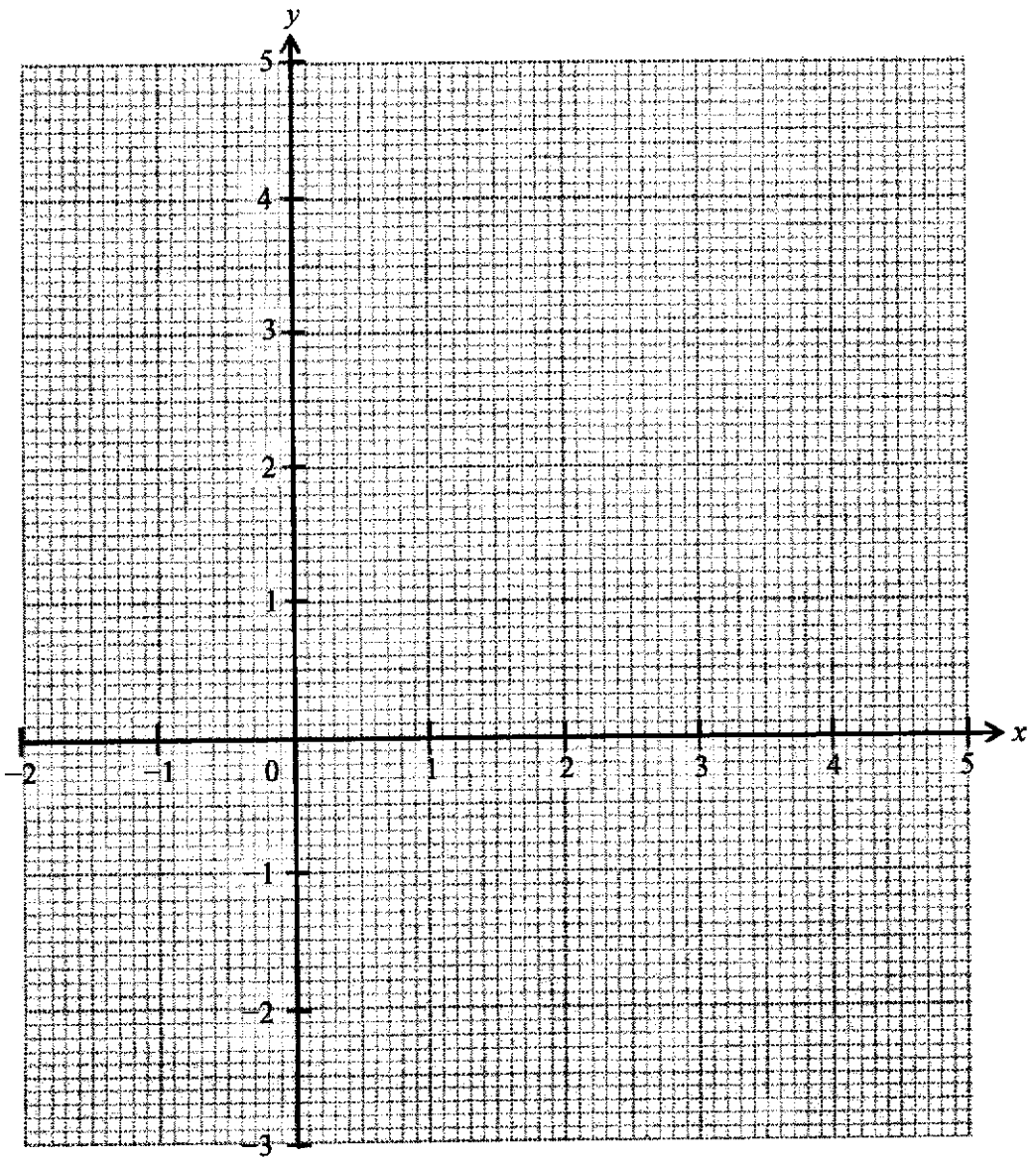
6 The variables x and y are connected by the equation $y = \frac{x^2}{5} + \frac{1}{x} - 1$.
Some corresponding values of x and y are given in the table.

x	-2	-1	-0.5	0.5	1	2	3	4	5
y	-0.7	-1.8	-3	1.1	0.2	0.3	1.1	2.5	m

(a) Find the value of m.

Answer $m =$ ..... [1]

(b) On the grid, draw the graph of $y = \frac{x^2}{5} + \frac{1}{x} - 1$ for $-2 \leq x \leq 5$. [3]



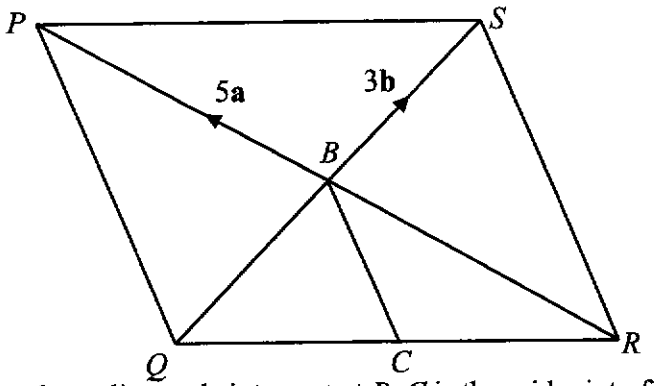
(c) By drawing a tangent, find the gradient of the curve at (1, 0.2).

Answer ..... [2]

(d) Use your graph to find the solutions of the equation $x^2 + \frac{5}{x} - 4x - 4 = 0$ in the range $-2 \leq x \leq 5$.

Answer $x =$ ..... or ..... or ..... [3]

7



$PQRS$ is a parallelogram whose diagonals intersect at B. C is the midpoint of QR.
$\overrightarrow{BP} = 5\boldsymbol{a}$ and $\overrightarrow{BS} = 3\boldsymbol{b}$.

(a) Show that triangles RBC and RPQ are similar.

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.....

.....

..... [2]

(b) Show that $\overrightarrow{PC} = -\frac{3}{2}(5\boldsymbol{a} + \boldsymbol{b})$.
Answer

[3]

(c) Given that A is a point on QB such that $QA : AB = 2 : 1$.
Express $\overrightarrow{AC}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

Answer ..... [2]

(d) State one fact about P, A and C.

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..... [1]

(e) Explain why the areas of triangle PAB and triangle QAC are equal.

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..... [2]

(f) Find

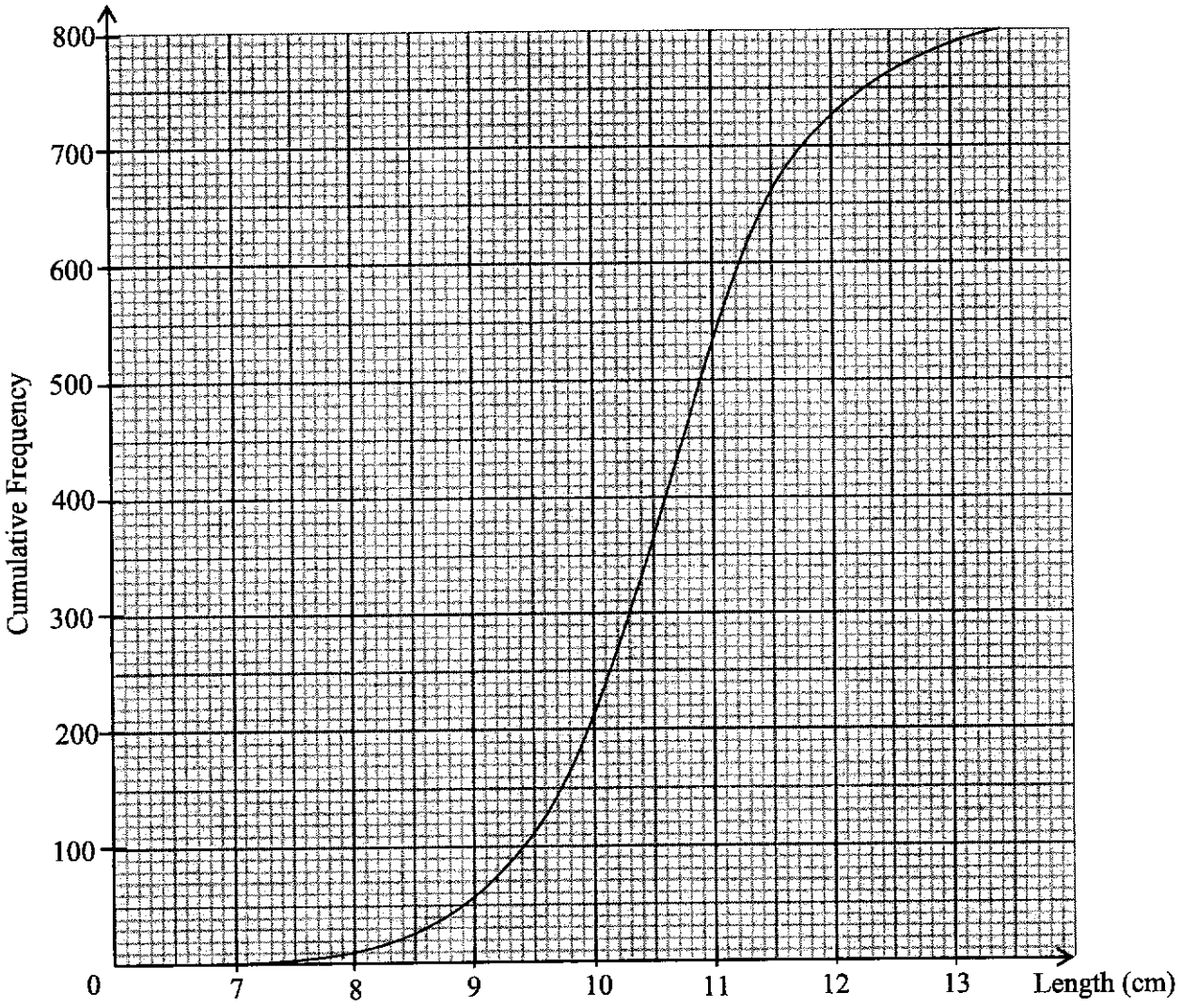
(i) $\frac{area\ of\ \Delta ABC}{area\ of\ \Delta QBC}$,

Answer ..... [1]

(ii) $\frac{area\ of\ \Delta ABC}{area\ of\ \Delta RPQ}$.

Answer ..... [2]

8 The cumulative frequency curve shows the distribution of the lengths (cm) of 800 lobsters caught in May during a lobster fishing trip.



(a) Use the graph to estimate

(i) the median length,

Answer ..... cm [1]

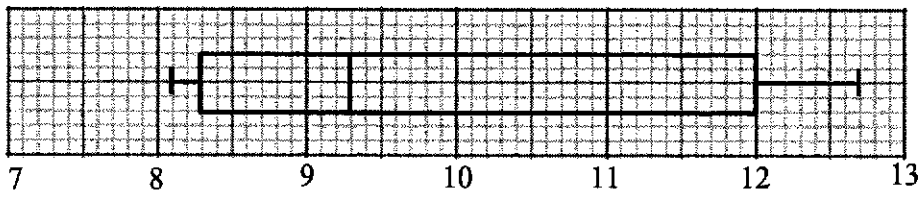
(ii) the interquartile range.

Answer ..... cm [2]

(b) To ensure sustainability of the lobster population, it is illegal to catch lobsters shorter than 8.3 cm or longer than 12.7 cm.
Estimate the percentage of lobsters that needs to be released.

Answer ..... % [2]

(c) The lengths of another 800 lobsters caught in June are recorded.
The box-and-whisker plot shows the distribution of the lengths.



Lengths of lobsters (cm)

Make two comments comparing the lengths of the lobsters caught in May and June.
Use figures to support your answers.

1. .....

.....

2. .....

..... [2]

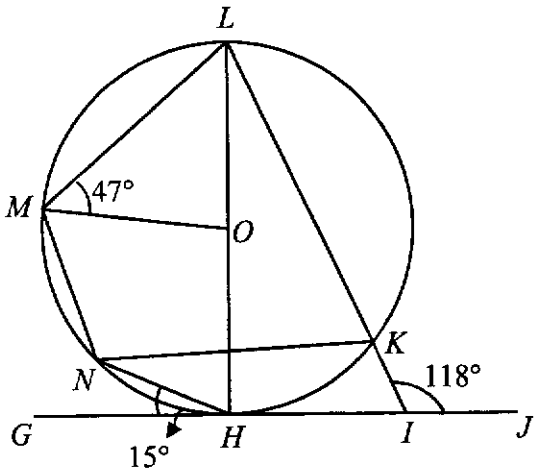
(d) Jill makes a claim that "fishermen are able to keep a larger percentage of their lobsters in June as compared to May". Do you agree?

.....

..... [1]

18

9



K, *L*, *M*, *N* and *H* are points on the circle, centre *O*.
GJ is a tangent to the circle at *H* and *LOH* and *LKI* are straight lines.
Angle *NHG* = 15°, angle *OML* = 47° and angle *LIJ* = 118°

(a) Find, giving reasons for each answer,

(i) angle *MNH*,

Answer ..... [2]

(ii) angle *MOH*,

Answer ..... [1]

(iii) angle *KNH*.

Answer ..... [2]

(b) Explain whether MN and LI are parallel lines.

.....

.....

.....

.....

.....

.....

.....

..... [2]

10 Jack visited the Super Nintendo World in Japan and bought a night light in the shape of a star as shown in **Diagram I**. The star-shaped night light can be modelled as a prism with a cross sectional area as shown in **Diagram II**.

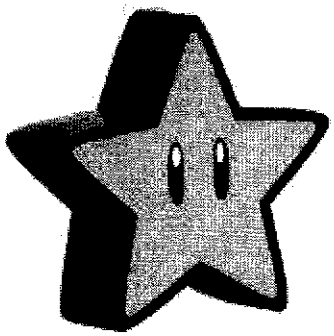


Diagram I

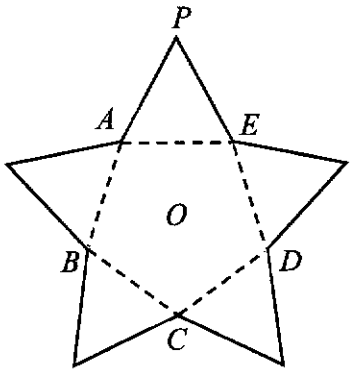


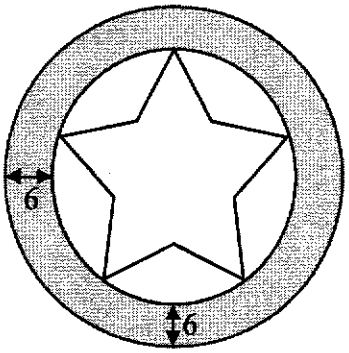
Diagram II

The star-shaped cross sectional area is formed by a regular pentagon, $ABCDE$ and five identical equilateral triangles. O is the centre of the star.
The thickness of the prism is 12 cm and $AP = 10$ cm.

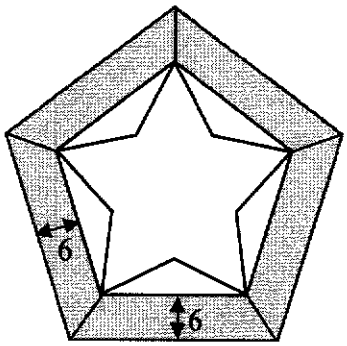
(a) Calculate the star-shaped cross sectional area.

Answer ..... cm^2 [3]

Jack wants to build a wooden frame for the night light and mount it onto the wall using an adhesive tape. The night light fits exactly inside the wooden frames as shown in **Design I** and **Design II**. The adhesive tape is applied to one face of the cross surface area of the wooden frame before mounting onto the wall.
In **Design I**, the wooden frame has an uniform width of 6 cm.
In **Design II**, the wooden frame is made up of five identical isosceles trapezium prisms with a height of 6 cm.
The thickness of the wooden frame is 12 cm.



Design I



Design II

Density of Wood = 900 kg/m^3
Mass of the night light = 600 grams
Maximum mass that the adhesive tape can hold = 8 kg

(b) Which design should Jack choose to satisfy the condition of the maximum mass that the adhesive tape can hold? Justify your decision with calculations.

Continuation of working space for question **10(b)**.

Answer ..... [7]

End of paper

Nan Chiau High School

2024 Secondary 4 Mathematics

Mathematics Prelim Examination Paper 1 Solutions

Qn	Solution
1a	False
1b	False
2	$\frac{105b}{a} = \frac{(3\times5\times7)(2^2\times5\times7)}{2\times3^2\times5}$ $= \frac{2^2\times3\times5^2\times7^2}{2\times3^2\times5}$ $= \frac{2\times5\times7^2}{3}$ Numerator $2 \times 5 \times 7^2$ does not include the factor 3 to reduce $\frac{105b}{3}$ to a whole number.
3	Length of shorter portion of rope $= \frac{4}{11} \times 44$ $= 16$ cm Length of longer portion of rope = 28 cm Let x be the length rope cut. $\frac{16-x}{28-x} = \frac{2}{5}$ $5(16-x) = 2(28-x)$ $80 - 5x = 56 - 2x$ $24 = 3x$ $x = 8$
4	$7^{9-x^2} - 1 = 0$ $7^{9-x^2} = 1$ $7^{(3-x)(3+x)} = 7^0$ $x = 3 \quad or \quad x = -3$
5	$y^3 = \frac{k}{x^2}$ $\frac{k}{3^2} - \frac{k}{6^2} = 5$ $\frac{k}{9} - \frac{k}{36} = 5$ $\frac{k}{12} = 5$ $k = 60$ $y^3 = \frac{60}{x^2}$ $2^3 = \frac{60}{x^2}$ $x^2 = 7.5$ $x = \pm 2.74$

6

24 g → 12.044×10^{23} atoms

1 g → $0.5.01833 \times 10^{23}$ atoms

5000g → $0.5.01833 \times 10^{23} \times 5000$

$= 2509.167 \times 10^{23}$

$= 2.51 \times 10^{26}$

7a Diagram A

7b Diagram B

7c Diagram D

8

$9x + 2.5y = 3y - 2x$

$11x = 0.5y$

$22x = y$

or $\frac{x}{y} = \frac{1}{22}$

Hence, $x = 22$ and $y = 1$ is not a solution, as $22(22) \neq (1)$

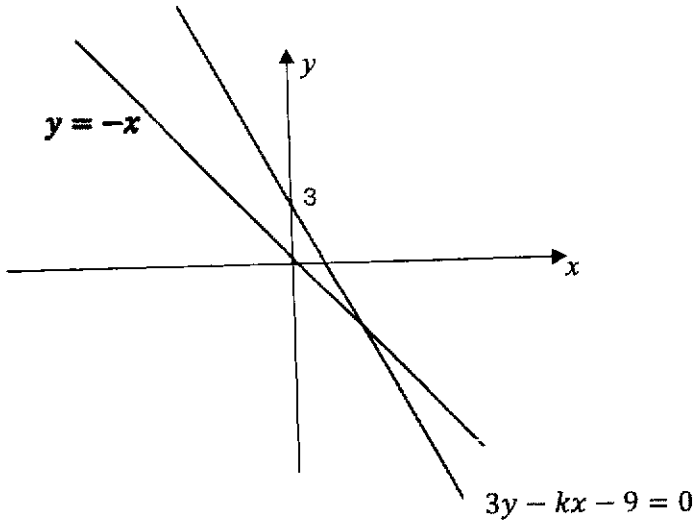
Or

LHS = $9(22) + 2.5(1) = 200.5$

RHS = $3(1) - 2(22) = -41$

Since LHS ≠RHS, $x = 22$ and $y = 1$ is not a solution.

9



$3y - kx - 9 = 0$

$3y = kx + 9$

$y = \frac{k}{3}x + 3$, where $k < -3$

10

Let the first digit be x and second digit be y

A number between 20 to 100	Let the number be $10x + y$
Add the two digits together	$x + y$
Subtract the sum of the two digits from your original number to get a new number	$(10x + y) - (x + y)$ $= 9x$
The new number is a multiple of 3	Since the new number is a multiple of 9 **and 3 is a factor** of 9, hence the new number is a multiple of 3.

11

$$2(mn)^2 - 2mn = 3mn + 3$$
$$2m^2n^2 - 5mn - 3 = 0$$
$$(mn - 3)(2mn + 1) = 0$$
$$mn = 3 \quad \text{or} \quad mn = -\frac{1}{2}$$
$$n = \frac{3}{m} \quad \text{or} \quad n = -\frac{1}{2m}$$

12

$$\frac{4}{3} + \frac{2}{9m} \div \sqrt[4]{81m^{16}} = \frac{4}{3} + \frac{2}{9m} \div 3m^4$$
$$= \frac{4}{3} + \frac{2}{9m} \times \frac{1}{3m^4}$$
$$= \frac{4}{3} + \frac{2}{27m^5}$$
$$= \frac{36m^5 + 2}{27m^5}$$

13

$$\frac{-9x^2 + 12x - 4}{3 - \frac{1}{x - \frac{x}{2}}} = \frac{-(3x-2)^2}{3 - \frac{1}{\frac{2x-x}{2}}}$$
$$= \frac{-(3x-2)^2}{3 - \frac{2}{x}}$$
$$= \frac{-(3x-2)^2}{\frac{3x-2}{x}}$$
$$= -x(3x - 2)$$

14

$y = \frac{1}{5}x^2 - 2x + 7$ ---------- [1]

$y = x$ ---------- [2]

Sub eqn[1] into eqn[2]

$$x = \frac{1}{5}x^2 - 2x + 7$$

$$\frac{1}{5}x^2 - 3x + 7 = 0$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(\frac{1}{5})(7)}}{2(\frac{1}{5})}$$

$x = 2.89$ or $x = 12.1$

$y = 2.89$ or $y = 12.1$

(2.89, 2.89) and (12.1, 12.1)

15 a

Let the y-intercept (vertical distance) be d

By Pythagoras' Thm

$d = \sqrt{17^2 - 8^2}$

$d = 15$

$\therefore C(0, -15)$

15 b

Let the x-coordinate of point B be b

$b = \frac{1}{8}(-15)$

$b = -\frac{15}{8}$

Equation of line of symmetry, $x = \frac{-\frac{15}{8}+(-8)}{2}$

$x = -\frac{79}{16}$

Since $y = -15$ when $x = 0$

Coefficient of $x^2 = -1$

Since curve is $y = -(x+8)(x+\frac{15}{8})$ or $y = -\frac{1}{8}(x+8)(8x+15)$

$\therefore y = -(-\frac{79}{16}+8)(-\frac{79}{16}+\frac{15}{8})$

$y = \frac{2401}{256}$

the coordinate of the maximum point is $(-\frac{79}{16}, \frac{2401}{256})$ or (- 4.94, 9.38)

16 a

$(\frac{8}{8+5+x})(\frac{7}{7+5+x}) = \frac{4}{33}$

$(\frac{8}{13+x})(\frac{7}{12+x}) = \frac{4}{33}$

$33(8)(7) = 4(13+x)(12+x)$

$462 = x^2 + 25x + 156$

$x^2 + 25x - 306 = 0$

$(x-9)(x+34) = 0$

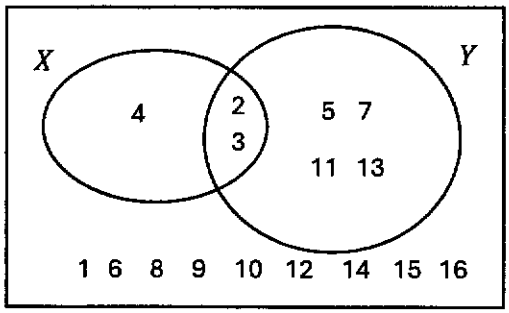
$x = 9$ or $x = -34$ (rej)

16 b

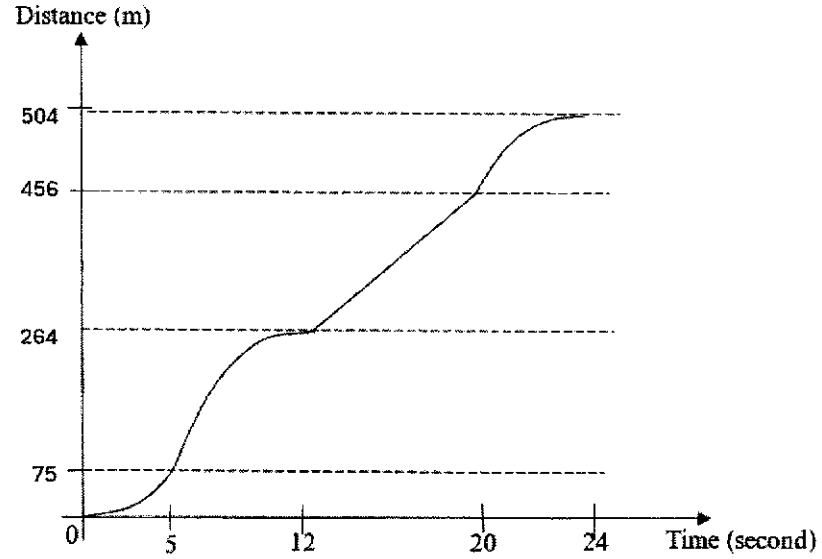
P(second marble is green when three marbles are drawn)

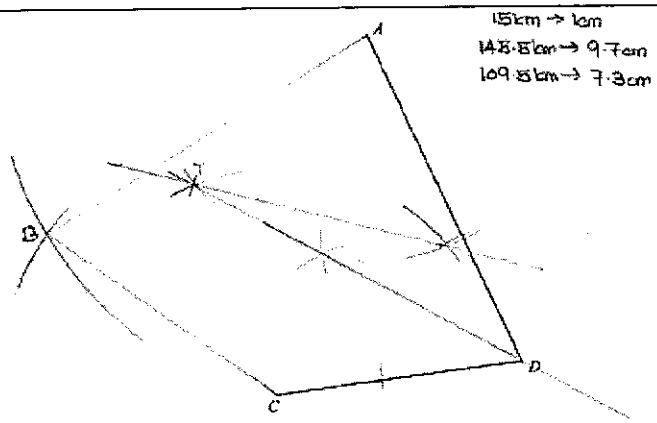
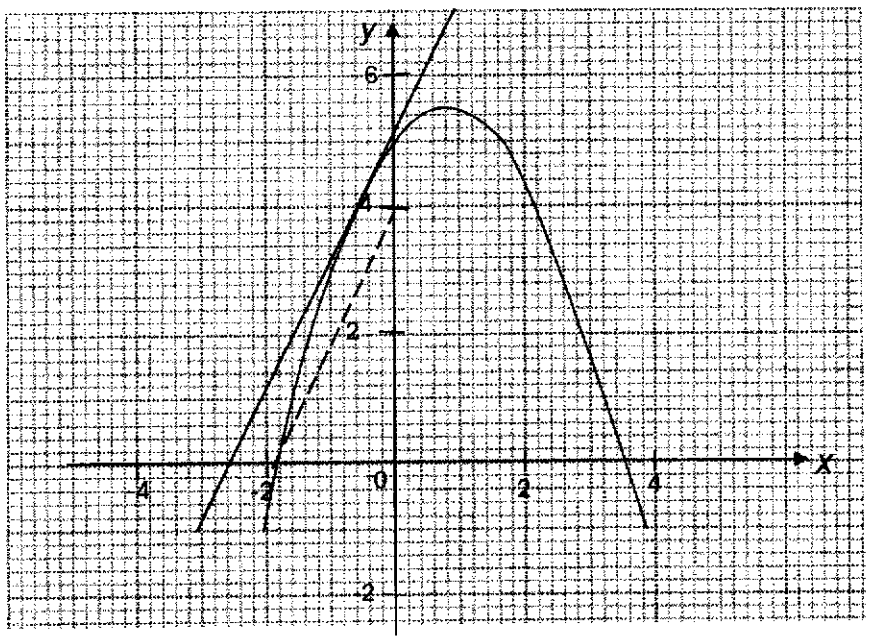
$= (\frac{17}{22})(\frac{5}{21})(\frac{16}{20})$

$= \frac{34}{231}$

17 a	
17 bi	$\emptyset$ *or* $\{\ \}$, $\{2\}, \{3\}, \{4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{2,3,4\}$
17 bii	Number of subsets $= 2^{n-1}$
18 a	$\text{Mean} = \frac{12(150)+10(200)+8(300)+x(400)+5(500)+2400}{12+10+8+x+5+1}$ $325 = \frac{11100+400x}{36+x}$ $11700 + 325x = 11100 + +400x$ $600 = 75x$ $x = 8$ $\therefore$ Total number of workers $= 12 + 10 + 8 + 8 + 5 + 1$ $= 44$
18 b	**Median** is a better gauge of workers' salary as there is salary that is **an extreme value of S2400** compared to the rest.
19 a	$40\% + 35\% + 20\% = 95\%$ As pie charts are used to visualize parts of a whole, all the parts should always add up to 100%. [or] The title exaggerated the responses of the support for Candidate C.
19 b	To include in uncounted / spoiled votes (5%) to show the full result of votes [or] To re-calculate the votes according to the three candidates according to base of 95%. [or] To recraft the title to be less bias and allow readers to analysis themselves.

20 a	Area of rhombus $= 2 \times \frac{1}{2}(7.4)(7.4)\sin 60$ $= 47.424\text{cm}^2$ **[Or]** height of rhombus $= (7.4)(\sin 60) = 6.4086$ cm Area of rhombus $= (6.4086)(7.4) = 47.424\ \text{cm}^2$ $AD^2 = 7.4^2 + 7.4^2 - 2(7.4)(7.4)\cos 120$ $AD = 12.817$ cm Area of sector $= \frac{60}{360} \times \pi(12.817)^2$ $= 86.014\ \text{cm}^2$ Shaded area $= 86.014 - 47.424$ $= 38.6\ \text{cm}^2$
20 b	Arc length $= \frac{60}{360} \times 2\pi(12.817)$ $= 13.422$ Perimeter $= 13.422 + 2(12.817 - 7.4) + 2(7.4)$ $= 39.1$ cm
21 a	$10^2 + 4^2 = (10-4)^2 + 2(10)(4)$
21 b	$(2n)^2 + (n-1)^2 = (2n-n+1)^2 + 2(2n)(n-1)$ *or* $= 4n^2 + n^2 - 2n + 1$ or $= 5n^2 - 2n + 1$
21 c	$p^2 + q^2 = 1249$ $5n^2 - 2n + 1 = 1249$ $5n^2 - 2n - 1248 = 0$ $(n-16)(5n+78) = 0$ $n = 16$ or $n = -\frac{78}{5}$ (rej) Hence, $p = 32$ and $q = 15$
22 a	$AB^2 + BC^2 = 6.4^2 + 4.8^2 = 64$ $AC^2 = 8^2 = 64$ Since $AB^2 + BC^2 = AC^2$, hence by **converse of Pythagoras' Theorem** , $\angle ABC = 90^o$ $\sin \angle ACB = \frac{6.4}{8}$ $= \frac{4}{5}$

22 b	$\cos\angle DAC = -\frac{4}{5}$
23 a	$24\text{ m/s} = \frac{24 \times \frac{1}{1000} km}{1 \times \frac{1}{3600} h}$ $= 86.4\text{km/h}$
23 b	$504 = \left(\frac{1}{2} \times 5 \times v\right) + \left(\frac{1}{2}(v+24)(7)\right) + \left(\frac{1}{2}(8+12)(24)\right)$ $504 = \frac{5}{2}v + \frac{7}{2}v + 84 + 240$ $180 = 6v$ $v = 30$ km/h
23 c	Distance from 0s to 5s = $\frac{1}{2} \times 5 \times 30 = 75\ m$ Distance from 0s to 12s = $75 + \frac{1}{2}(30+24)(7) = 264m$ Distance from 0s to 20s= $264 + (24)(8) = 456\ m$ 

24 a 24 b	
24 c	5.9 x 15 = 88.5 km
25 a	−20
25 b	(0.8, 5.5)
25 c	 $k = 5.2$

NCHS 2024 EM Paper 2 Solutions

1

(a) $\frac{9a^2-4b^2}{3ap+6aq-4bq-2bp}$

$= \frac{(3a+2b)(3a-2b)}{(3ap+6aq)-(2bp+4bq)}$

$= \frac{(3a+2b)(3a-2b)}{3a(p+2q)-2b(p+2q)}$

$= \frac{(3a+2b)(3a-2b)}{(p+2q)(3a-2b)}$

$= \frac{3a+2b}{p+2q}$

(b) $\frac{x}{6-7x-5x^2} + \frac{2}{5x-3}$

$= \frac{x}{-(5x-3)(x+2)} + \frac{2}{5x-3}$

$= \frac{-x+2(2+x)}{(5x-3)(x+2)}$

$= \frac{-x+4+2x}{(5x-3)(x+2)}$

$= \frac{x+4}{(5x-3)(x+2)}$

1(c)

$-\frac{1}{2} < \frac{2y}{5} - \frac{1+y}{3} \leq \frac{5}{6}$

$-\frac{1}{2} < \frac{3(2y)-5(1+y)}{15}$ *and* $\frac{3(2y)-5(1+y)}{15} \leq \frac{5}{6}$

$-\frac{1}{2} < \frac{6y-5-5y}{15}$ $\frac{6y-5-5y}{15} \leq \frac{5}{6}$

$-\frac{1}{2} < \frac{y-5}{15}$ $\frac{y-5}{15} \leq \frac{5}{6}$

$-15 < 2(y-5)$ $y-5 \leq \frac{25}{2}$

$-15 < 2y-10$ $y \leq 17\frac{1}{2}$

$-5 < 2y$

$-2\frac{1}{2} < y \leq 17\frac{1}{2}$

2(a)

$\frac{r}{19} = \frac{25}{40}$

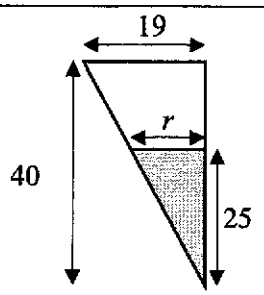
$r = \frac{25}{40} \times 19$

$= 11.875$

$V = \frac{1}{3}\pi(11.875)^2(25)$

$= 3691.78\ cm^3$

$= 3.69\ litres$



(b)

Method 1

Volume of container $= \frac{1}{3}\pi(19)^2(40)$

$\left(\frac{h_1}{h_2}\right)^3 = \frac{V_1}{V_2}$

$\left(\frac{h_1}{40}\right)^3 = \frac{\frac{1}{3}\pi(19)^2(40) - \frac{1}{3}\pi(11.875)^2(25)}{\frac{1}{3}\pi(19)^2(40)}$

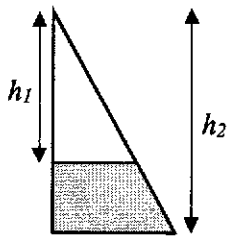
$\frac{h_1}{40} = \sqrt[3]{\frac{(19)^2(40)-(11.875)^2(25)}{(19)^2(40)}}$

$h_1 = 40 \times \sqrt[3]{\frac{(19)^2(40)-(11.875)^2(25)}{(19)^2(40)}}$

$= 36.437$

Depth of water $= 40 - 36.437$

$= 3.56\ cm\ (3sf)$



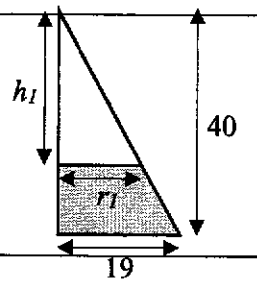
Method 2

$\frac{r_1}{19} = \frac{h_1}{40}$

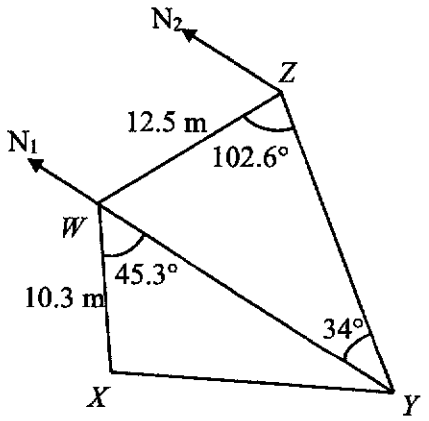
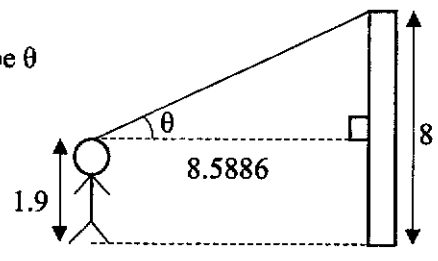
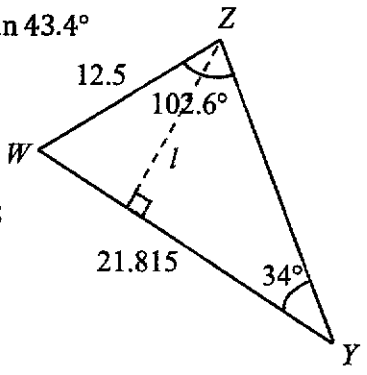
$r_1 = \frac{19}{40}h_1$

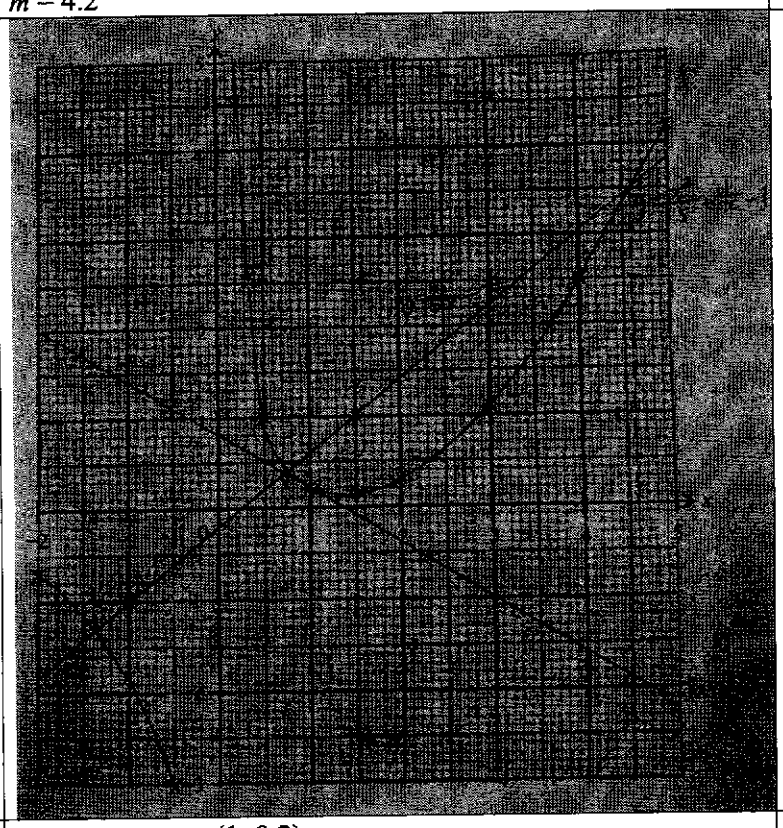
$\frac{1}{3}\pi(19)^2(40) - \frac{1}{3}\pi(r_1)^2(h_1) = 3691.78$

$\frac{1}{3}\pi(r_1)^2(h_1) = 11429.75$



	$\left(\frac{19}{40}h_1\right)^2(h_1) = 10914.61$ $h_1{}^3 = 48375.00277$ $h_1 = 36.437$ $Depth\ of\ water = 40 - 36.437$ $= 3.56\ cm\ (3sf)$	
3(a)	$\frac{108888-100176.96}{108888} \times 100\%$ $= 8\%$	
(b)	Balance owed $= \frac{85}{100} \times 108888$ $= \$92554.80$ Interest $= \frac{92554.80 \times 2.98 \times 5}{100}$ $= \$13790.6652$ Monthly instalment $= \frac{92554.80+13790.6652}{5 \times 12}$ $= \$1772.42\ (2dp)$	
(c)	$P = 108888 - 18888$ $= \$90000$ $A = 960 \times 10 \times 12$ $= \$115200$ $A = P\left(1 + \frac{R}{100}\right)^n$ $115200 = 90000\left(1 + \frac{R}{100}\right)^{10}$ $\left(1 + \frac{R}{100}\right)^{10} = \frac{115200}{90000}$ $1 + \frac{R}{100} = \sqrt[10]{\frac{115200}{90000}}$ $R = 100\left(\sqrt[10]{\frac{115200}{90000}} - 1\right)$ $= 2.499$ $= 2.50\ (3sf)$	
4(a)	$Q = \begin{pmatrix} 1.20 \\ 1.50 \\ 0.95 \end{pmatrix}$	
(b)	$R = \begin{pmatrix} 60 & 68 & 55 \\ 49 & 56 & 71 \\ 53 & 70 & 80 \end{pmatrix}\begin{pmatrix} 1.20 \\ 1.50 \\ 0.95 \end{pmatrix}$ $= \begin{pmatrix} 226.25 \\ 210.25 \\ 244.60 \end{pmatrix}$	
(c)	The elements in R represent the total cost of baking caramel, strawberry and mint cupcakes in outlet A, B and C respectively	
(d)	**Method 1** Selling Price $= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 4 \end{pmatrix}\begin{pmatrix} 1.20 \\ 1.50 \\ 0.95 \end{pmatrix}$ $= \begin{pmatrix} 3.60 \\ 3.75 \\ 3.80 \end{pmatrix}$	**Method 2** Profit for each type of cupcake $= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1.20 \\ 1.50 \\ 0.95 \end{pmatrix}$ $= \begin{pmatrix} 2.40 \\ 2.25 \\ 2.85 \end{pmatrix}$

	Profit earned in a week $= \begin{pmatrix} 60 & 68 & 55 \\ 49 & 56 & 71 \\ 53 & 70 & 80 \end{pmatrix} \begin{pmatrix} 3.60 \\ 3.75 \\ 3.80 \end{pmatrix} - \begin{pmatrix} 226.25 \\ 210.25 \\ 244.60 \end{pmatrix}$ $= \begin{pmatrix} 680 \\ 656.20 \\ 757.30 \end{pmatrix} - \begin{pmatrix} 226.25 \\ 210.25 \\ 244.60 \end{pmatrix}$ $= \begin{pmatrix} 453.75 \\ 445.95 \\ 512.70 \end{pmatrix}$ Total Profit earned in a month $= 4\left[(1 \quad 1 \quad 1) \begin{pmatrix} 453.75 \\ 445.95 \\ 512.70 \end{pmatrix} \right]$ $= 4(1412.40)$ $= (5649.60)$ Total Profit = \$5649.60	Profit earned in a week $= \begin{pmatrix} 60 & 68 & 55 \\ 49 & 56 & 71 \\ 53 & 70 & 80 \end{pmatrix} \begin{pmatrix} 2.40 \\ 2.25 \\ 2.85 \end{pmatrix}$ $= \begin{pmatrix} 453.75 \\ 445.95 \\ 512.70 \end{pmatrix}$ Total Profit earned in a month $= 4\left[(1 \quad 1 \quad 1) \begin{pmatrix} 453.75 \\ 445.95 \\ 512.70 \end{pmatrix} \right]$ $= 4(1412.40)$ $= (5649.60)$ Total Profit = \$5649.60
5(a)	$\frac{WY}{\sin 102.6^\circ} = \frac{12.5}{\sin 34^\circ}$ $WY = \frac{12.5}{\sin 34^\circ} \times \sin 102.6^\circ$ $= 21.815$ $XY^2 = 10.3^2 + 21.815^2 - 2(10.3)(21.815)(\cos 45.3^\circ)$ $XY = 16.306 \quad or \quad -16.306(NA)$ $= 16.3\ (3sf)$	
(b)	**Method 1** $\angle N_1WZ = 102.6^\circ + 34^\circ\ (ext\ \angle\ of\ \Delta)$ $= 136.6^\circ$ $\angle WZN_2 = 180^\circ - 136.6^\circ\ (int\ \angle, N_1W//N_2Z)$ $= 43.4^\circ$ $Bearing\ of\ W\ from\ Z = 360^\circ - 43.4^\circ\ (\angle\ at\ a\ pt)$ $= 316.6^\circ$	
	Method 2 $\angle YWZ = 180^\circ - (34^\circ + 102.6^\circ)\ (\angle\ sum\ of\ \Delta)$ $= 43.4^\circ$ $\angle WZN_2 = 43.4^\circ\ (alt\ \angle, N_1W//N_2Z)$ $Bearing\ of\ W\ from\ Z = 360^\circ - 43.4^\circ\ (\angle\ at\ a\ pt)$ $= 316.6^\circ$	
(c)	$\angle YWZ = 43.4^\circ\ (alt\ \angle, N_1Y//N_2Z)$ $l = 12.5 \sin 43.4^\circ$ OR $\frac{1}{2} \times 21.815 \times l = \frac{1}{2} \times 12.5 \times 21.815 \times \sin 43.4^\circ$ $= 8.5886$ $l = 8.5886$ Let the greatest angle of elevation be θ $\tan\theta = \frac{8-1.9}{12.5 \sin 43.4^\circ}$ $\theta = 35.38^\circ$ $= 35.4^\circ$	 

6(a)	$m = 4.2$	
(b)		
(c)	Draw tangent at (1, 0.2) Gradient = −0.5 to −0.7	
(d)	$x^2 + \frac{5}{x} - 4x - 4 = 0$ $\frac{x^2}{5} + \frac{1}{x} - \frac{4}{5}x - \frac{4}{5} = 0$ $\frac{x^2}{5} + \frac{1}{x} - 1 = \frac{4}{5}x - \frac{1}{5}$ $y = \frac{4}{5}x - \frac{1}{5}$ Plot graph of $y = \frac{4}{5}x - \frac{1}{5}$ $x = -1.35 \pm 0.05$ *or* 0.75 ± 0.05 *or* 4.6 ± 0.05	
7(a)	**Method 1** $\angle BRC = \angle PRQ$ *(common ∠)* $\frac{CR}{QR} = \frac{1}{2}$ *(C is midpt of QR)* $\frac{BR}{PR} = \frac{1}{2}$ *(diagonals of parallelogram bisect each other)* ΔRBC *and* ΔRPQ *are similar (SAS Similarity)* **Method 2** Since diagonals of a parallelogram bisect each other, B is the midpoint of PR. In addition C is given as the midpoint of QR, using midpoint theorem, BC is parallel to PQ. $\angle BRC = \angle PRQ$ *(common ∠)* $\angle BCR = \angle PQR$ *(corresponding ∠, BC//PQ)* ΔRBC *and* ΔRPQ *are similar (AA Similarity)*	

(b)

Method 1

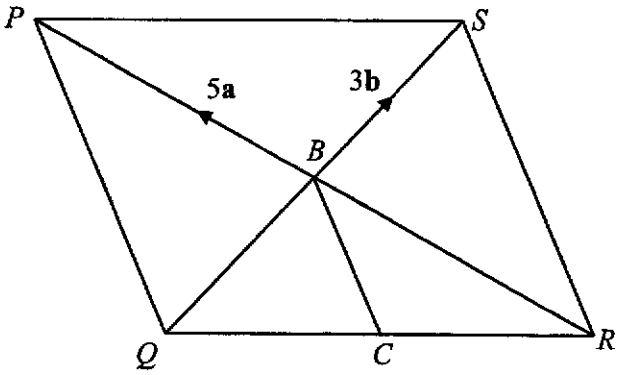
$\overrightarrow{PC} = \overrightarrow{PQ} + \overrightarrow{QC}$

$= (\overrightarrow{PB} + \overrightarrow{BQ}) + \frac{1}{2}\overrightarrow{QR}$

$= -5\boldsymbol{a} - 3\boldsymbol{b} + \frac{1}{2}(-5\boldsymbol{a} + 3\boldsymbol{b})$

$= -\frac{15}{2}\boldsymbol{a} - \frac{3}{2}\boldsymbol{b}$

$= -\frac{3}{2}(5\boldsymbol{a} + \boldsymbol{b})$ *(Shown)*



Method 2

$\overrightarrow{PC} = \overrightarrow{PB} + \overrightarrow{BC}$

$= \overrightarrow{PB} + \frac{1}{2}\overrightarrow{PQ}$

$= -5\boldsymbol{a} + \frac{1}{2}(-5\boldsymbol{a} - 3\boldsymbol{b})$

$= -\frac{15}{2}\boldsymbol{a} - \frac{3}{2}\boldsymbol{b}$

$= -\frac{3}{2}(5\boldsymbol{a} + \boldsymbol{b})$ *(Shown)*

(c)

Method 1

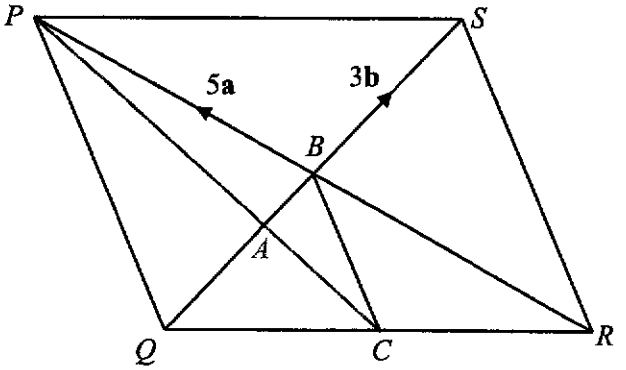
$\overrightarrow{AQ} = \frac{2}{3}(-3\boldsymbol{b})$

$= -2\boldsymbol{b}$

$\overrightarrow{AC} = \overrightarrow{AQ} + \overrightarrow{QC}$

$= -2\boldsymbol{b} + \frac{1}{2}(-5\boldsymbol{a} + 3\boldsymbol{b})$

$= -\frac{5}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$ ***or*** $-\frac{1}{2}(5\boldsymbol{a} + \boldsymbol{b})$



Method 2

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= \boldsymbol{b} + \frac{1}{2}(-5\boldsymbol{a} - 3\boldsymbol{b})$

$= -\frac{5}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$ ***or*** $-\frac{1}{2}(5\boldsymbol{a} + \boldsymbol{b})$

(d)

$\overrightarrow{PC} = -\frac{3}{2}(5\boldsymbol{a} + \boldsymbol{b})$ & $\overrightarrow{AC} = -\frac{1}{2}(5\boldsymbol{a} + \boldsymbol{b})$

$\overrightarrow{PC} = 3\overrightarrow{AC}$

$\therefore$ C is a common point, P, A and C are collinear.

(e)

Method 1

$BC//PQ$ (ΔRBC and ΔRPQ are similar)

Area $\Delta BPQ = \frac{1}{2} \times PQ \times h$ or ***Area*** $\Delta\boldsymbol{PBC} = \frac{1}{2} \times \boldsymbol{BC} \times \boldsymbol{h}$

$= Area\ \Delta CPQ$ $= \boldsymbol{Area\ \Delta QBC}$

Since ΔAPQ *is common area* or $\boldsymbol{\Delta ABC}$ ***is common area***

$\therefore$ *Area* ΔPAB *and Area* ΔQAC *are the same*

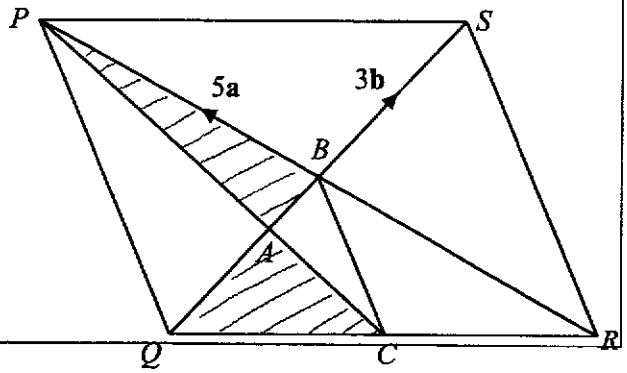
Method 2

$\angle PAB = \angle QAC$ *(vert opp* $\angle$*)*

$\frac{AB}{QA} = \frac{1}{2}$ *(from (c))* & $\frac{PA}{AC} = \frac{2}{1}$ *(from (d))*

$\frac{Area\ \Delta PAB}{Area\ \Delta QAC} = \frac{\frac{1}{2} \times PA \times AB \times \sin \angle PAB}{\frac{1}{2} \times QA \times AC \times \sin \angle QAC}$

$= \frac{PA}{AC} \times \frac{AB}{QA}$



	$= \frac{2}{1} \times \frac{1}{2}$ $= \frac{1}{1}$ $\therefore$ *Area* ΔPAB *and Area* ΔQAC *are the same* **Method 3** $\frac{Area\ of\ \Delta PAB}{Area\ of\ \Delta QAC} = \frac{Area\ of\ \Delta PAB}{Area\ of\ \Delta ABC} \times \frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta QAC}$ $= \frac{\frac{1}{2} \times PA \times h_1}{\frac{1}{2} \times AC \times h_1} \times \frac{\frac{1}{2} \times AB \times h_2}{\frac{1}{2} \times QA \times h_2}$ $= \frac{PA}{AC} \times \frac{AB}{QA}$ $= \frac{2}{1} \times \frac{1}{2}$ $= \frac{1}{1}$ $\therefore$ *Area* ΔPAB *and Area* ΔQAC *are the same*	
(f) (i)	$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta QBC} = \frac{1}{3}$	
(ii)	$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta RPQ} = \frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta QBC} \times \frac{Area\ of\ \Delta QBC}{Area\ of\ \Delta RBC} \times \frac{Area\ of\ \Delta RBC}{Area\ of\ \Delta RPQ}$ $= \frac{1}{3} \times \frac{1}{1} \times \left(\frac{1}{2}\right)^2$ $= \frac{1}{12}$	
8(a) (i)	Middle position = 400th Median Length = 10.6 cm	
(ii)	Q1 position = 200th Q3 position = 600th Q1 = 9.95 Q3 = 11.25 or 11.2 Interquartile Range = 11.25 – 9.95 or 11.2 – 9.95 = 1.3 cm or 1.25 cm	
(b)	$Percentage = \frac{15 + 25}{800} \times 100\%$ *or* $\frac{20 + 25}{800} \times 100\%$ $= 5\%$ *or* 5.625%	
(c)	Lobsters caught in June are **shorter** in length as the median length of **9.3 cm** is **lower** as compared to **10.6 cm** in May. The lengths of the lobsters caught in June has a **wider spread** as its ITR of **3.7 cm** is **higher** as compared to **1.3 cm** in May.	
(d)	I disagree with Jill as **25%** of lobsters needed to be released in June because they are shorter than 8.3 cm which is **higher** as compared to the total percentage released of **5.625%** in May. Or I disagree with Jill as **75%** of the lobsters caught in June are within the legal length which is **lower** as compared to **94.375%** of the lobster caught in May are within the legal length.	

9(a)

(i) $\angle MLO = 47°$ $(base\ \angle\ in\ isos\ \Delta)$

$\angle MNH = 180° - 47°$ $(\angle\ in\ opp\ segment)$

$= 133°$

(ii) $\angle MOH = 47° \times 2$ $(\angle\ at\ center = 2\ \angle\ at\ circumference)$

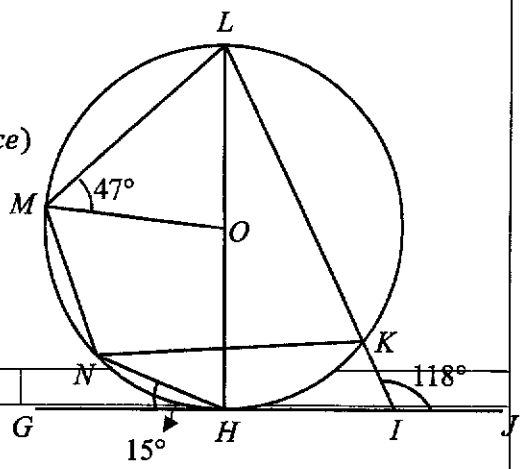
$= 94°$

(iii) $\angle LHJ = 90°$ $(tangent\ \perp\ radius)$

$\angle HLI = 118° - 90°$ $(ext\ \angle\ of\ \Delta)$

$= 28°$

$\angle KNH = 28°$ $(\angle\ in\ same\ segment)$



(b)

$\angle LHG = 90°$ $(tangent\ \perp\ radius)$

$\angle OHN = 90° - 15°$

$= 75°$

$\angle OMN = 360° - (133° + 75° + 94°)$ $(\angle\ sum\ of\ quadrilateral)$

$= 58°$

$\angle NML + \angle MLI = (58° + 47°) + (47° + 28°)$

$= 180°$

Using property of interior angles, $\angle NML$ and $\angle MLI$ add up to 180°, MN and LI are parallel lines.

10 (a)

Method 1

$\sin 60° = \frac{h_1}{10}$ $\qquad$ $\tan\left(\frac{\frac{3\times180}{5}}{2}\right)° = \frac{h_2}{5}$

$h_1 = 10 \sin 60°$ $\qquad$ $h_2 = 5 \tan 54°$

$= 8.6603$ $\qquad$ $= 6.8819$

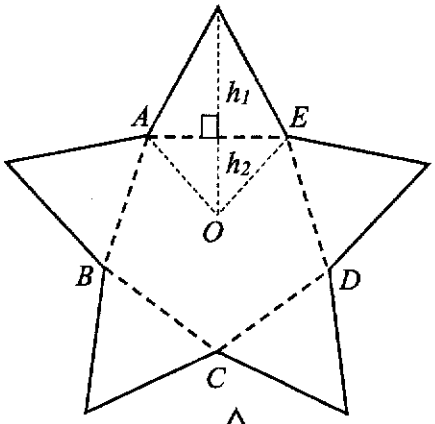
$h_1 + h_2 = 15.5422$

$Area\ PAO = \frac{1}{2} \times 15.5422 \times 5$

$Area\ of\ 5\ pointed\ star = 10 \times \frac{1}{2} \times 15.5422 \times 5$

$= 388.555$

$= 389\ cm^2\ (3sf)$



Method 2

Area of 1 equilateral Δ $\qquad$ Area of 1 isosceles Δ

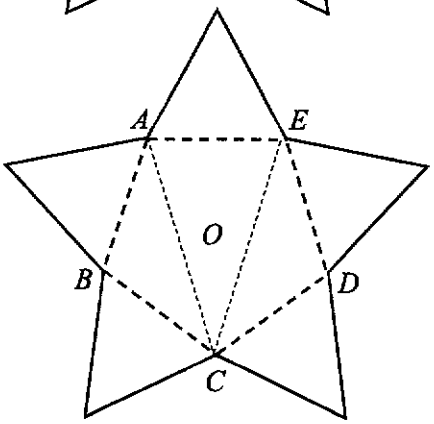
$= \frac{1}{2} \times 10 \times 10 \times \sin 60°$ $\qquad$ $= \frac{1}{2} \times 10 \times 5 \tan\left(\frac{\frac{3\times180}{5}}{2}\right)°$

$= 43.301$ $\qquad$ $= 34.410$

$Area\ of\ 5\ pointed\ star = 5 \times (43.301 + 34.410)$

$= 388.555$

$= 389\ cm^2\ (3sf)$



Method 3

$AC = 16.180\ cm$

$Area\ of\ 5\ pointed\ star$

$= 5 \times Area\ \Delta APE + 2 \times Area\ \Delta ABC + \times Area\ \Delta ACE$

$= 5 \times \left(\frac{1}{2} \times 10 \times 10 \times \sin 60°\right) + 2 \times \left(\frac{1}{2} \times 10 \times 10 \times \sin 108°\right) + \frac{1}{2} \times 16.180 \times 10 \times \sin 72°$

$= 216.506 + 95.106 + 76.940$

$= 388.552$

$= 389\ cm^2\ (3sf)$

(b)

$Density = 900kg/m^3$
$= 0.9g/cm^3$

Design I

$Outer\ Radius = 15.5422 + 6 = 21.5422$
$Cross\ SA = \pi(21.5422)^2 - \pi(15.5422)^2$
$= 699.024\ cm^2$
$Volume = 699.024 \times 12$
$= 8388.288\ cm^3$
$Mass\ (wood) = 8388.288 \times 0.9$
$= 7549.4592\ grams$
$Total\ Mass = 7549.4592 + 600$
$= 8149.4592\ grams$

Design II

$l = \dfrac{6}{\cos 36°}$
$= 7.4164$
$Cross\ SA$
$= 5\left[\frac{1}{2}(15.5422 + 7.4164)^2 \sin 72° - \frac{1}{2}(15.5422)^2 \sin 72°\right]$
$= 678.905\ cm^2$
$Volume = 678.905 \times 12$
$= 8146.86\ cm^3$
$Mass(wood) = 8146.86 \times 0.9$
$= 7332.174\ grams$
$Total\ Mass = 7332.174 + 600$
$= 7932.174\ grams$

Since $7932.174 < 8000 < 8149.4592\ grams$, Jack should choose **Design II**.