Class	Index Number	Name

ANG MO KIO SECONDARY SCHOOL
MID-YEAR EXAMINATION 2018
SECONDARY THREE EXPRESS

ADDITIONAL MATHEMATICS **4047**

Setter: Mdm Karen Teng

Thursday **03 May 2018** **2 hours**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **80**.

This document consists of **5** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

[Turn Over

1 Find the coordinates of the points of intersection of the line $2y = x + 3$ and the curve $-3y^2 + x^2 + 3y + 5x - 6 = 0$. [5]

2 Express $\dfrac{20 - 3x - 5x^2}{x^3 - 4x}$ in partial fractions. [5]

3 The roots of the quadratic equation $x^2 - 6x + 8 = 0$ are α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [3]

(ii) Find the equation whose roots are $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$. [4]

4 The function f is defined by $f(x) = 4x^3 + px^2 + 5x + 2$, where p is a constant.

(i) Given that $x - 1$ is a factor of f(x), find the value of p. [2]

(ii) Using the value of p found in **(i)**,

(a) find the remainder when f(x) is divided by $2x - 3$, [2]

(b) factorise f(x) completely, [3]

(c) hence solve the equation $4(y-1)^3 + p(y-1)^2 + 5y - 3 = 0$. [2]

5 **(a)** Solve the inequality $x(3x + 5) > 2$. [2]

(b) Find the smallest value of the integer a for which $3x^2 - 9x + a$ is always positive for all real values of x. [3]

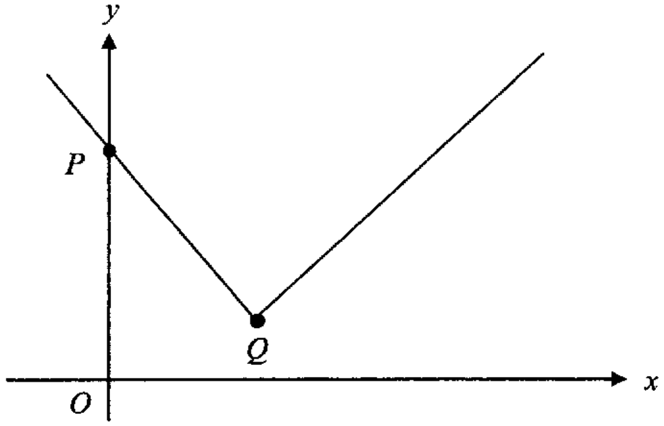
(c) Find the range of values of m for which the line $y + m = x$ cuts the curve $y = mx^2 + 9x$ at two real and distinct points. [4]

6 **(i)** Express $y = 7x - 6 - x^2$ in the form of $y = -(x+a)^2 + b$ where a and b are constants. Hence find the coordinates of the maximum point. [3]

(ii) Sketch the graph of $y = \left|7x - 6 - x^2\right|$ for $0 \le x \le 7$. Label clearly the intercepts and turning point. [3]

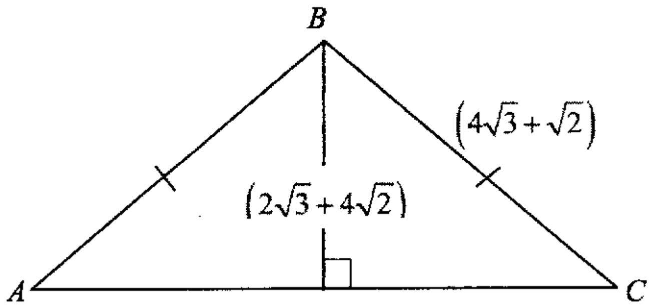
7 **(a)** Solve $|x-4| = 3 - 6x$. [3]

(b) The graph of $y = |2x-3| + 4$ is shown in the diagram. Find the coordinates of the points P and Q.



[2]

8 An isosceles triangle ABC has an area of 20 cm^2 with sides $AB = BC = \left(4\sqrt{3} + \sqrt{2}\right)$ and height $\left(2\sqrt{3} + 4\sqrt{2}\right)$ cm. Without using a calculator, find the perimeter of the triangle in the form $\left(a\sqrt{3} + b\sqrt{2}\right)$ cm, where a and b are integers. [5]



9 **(a)** **(i)** Expand $\left(1-\frac{x}{2}\right)^6$ in ascending powers of x, up to and including the term in x^3. [2]

(ii) Hence find the coefficient of x^2 in the expansion of $(3+2x)\left(1-\frac{x}{2}\right)^6$. [3]

(b) **(i)** Write down the general term in the binomial expansion of $(a+2x)^8$. [1]

(ii) If the coefficient of the x^5 term is equal to 28, find the value of a. [3]

10 **(a)** **(i)** Express 81^{10-x} as a power of 3. [1]

(ii) Hence find the value of x for which $\frac{3^{2x}}{81^{10-x}}=9^{-2}$. [3]

(b) By using an appropriate substitution, solve the equation $2^{2+x}+2^{-x}=5$. [4]

11 Given that $\log_2 a=b$, express

(i) a in terms of b, [1]

(ii) $\log_2\left(\frac{a^4}{32}\right)$ in terms of b, [2]

(iii) $\left(\frac{1}{8}\right)^b$ in terms of a. [2]

12 Solve the following equations

(a) $\lg(1-x)-\lg(x+3)=2\lg 3$, [3]

(b) $2\log_x 2=3+\log_2 x^2$. [4]

END OF PAPER

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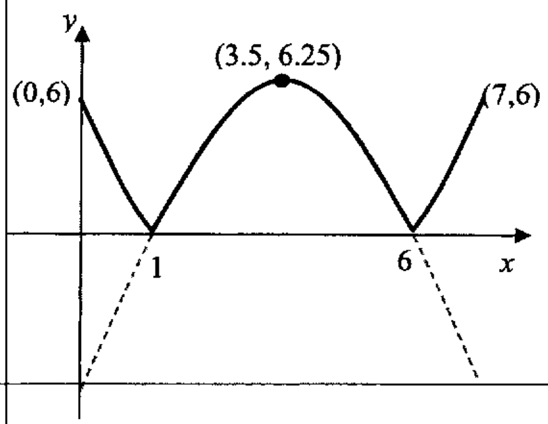
AMKSS 2018 MYE 3E AM Answers

Qn	Answer	Qn	Answer
1	Coordinates are $(3,3)$ and $(-11,-4)$	**7a**	$x=1\text{(Rej)}\ or\ x=-\frac{1}{5}$
2	$\frac{20-3x-5x^2}{x(x^2-4)}=\frac{-5}{x}-\frac{3}{4(x-2)}+\frac{3}{4(x+2)}$	**7b**	$P(0,7)$ $Q(1.5,4)$
3i	$\alpha^2+\beta^2=20$	**8**	$4\sqrt{3}+10\sqrt{2}$
3ii	$x^2-9x+8=0$	**9ai**	$1-3x+\frac{15}{4}x^2-\frac{5}{2}x^3+...$
4i	$p=-11$	**9aii**	$5\frac{1}{4}$
4ii	Remainder $=f(1.5)=-1.75$	**9bi**	${}^8C_r a^{8-r}(2x)^r$
4b	$(x-1)(4x+1)(x-2)$	**9bii**	$a=\frac{1}{4}$
4c	Let $y-1=x$ in (b) $y=2,\ 3$ or $\frac{3}{4}$	**10ai**	$81^{10-x}=3^{40-4x}$
5a	$x<-2$ or $x>\frac{1}{3}$	**10aii**	$x=6$
5b	$a>6\frac{3}{4}$, Smallest integer $a=7$	**10b**	$x=0\ or\ x=-2$
5c	$-4<m<4$	**11i**	$a=2^b$
6i	$y=-\left(x-\frac{7}{2}\right)^2+\frac{25}{4}$ Max pt is (3.5, 6.25)	**11ii**	$4b-5$
6ii		**11iii**	$\frac{1}{a^3}$
		12a	$x=-2\frac{3}{5}$
		12b	$x=2^{\frac{1}{2}}=\sqrt{2}$ or $x=2^{-2}=\frac{1}{4}$

EX	JURONG SECONDARY SCHOOL 2018 MID-YEAR EXAMINATION SECONDARY 3 EXPRESS

CANDIDATE NAME	

CLASS			INDEX NUMBER	

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For Examiner's Use
80

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1 Solve the simultaneous equations.

$$2y+x=3$$
$$\frac{1}{y}-\frac{4}{x}=3$$
[4]

2 Given that (1, *k*) is a point of intersection between the curve $4x^2-4xy+y^2=1$ and the line $y=7-4x$, find the

(a) value of *k*, [1]

(b) coordinates of the other point of intersection. [3]

3 (a) Given that $2x^3+5x^2-x-2=(Ax+3)(x+B)(x-1)+C$, where *A*, *B* and *C* are constants, find the values of *A*, *B* and *C*. [3]

(b) Given that x^2+mx+n and x^2+ax+b have the same remainder when divided by $x+p$, express *p* in terms of *a*, *b*, *m* and *n*. [3]

4 The function $f(x)=54x^4-9x^3-6a^2x^2+7x+2$ has a factor of $3x+a$.

(i) Show that $a^3-7a+6=0$. [2]

(ii) Hence, find the possible values of *a*. [4]

5 Express $\frac{2x^3+5x^2-x+1}{(x-2)(x^2+1)}$ in partial fractions. [5]

6 The curve $y=(k+2)x^2-(2k+1)x+k$ has a minimum point and lies completely above the *x*-axis. Find the range of values of *k*. [4]

7 (i) Find the values of *m* for which the curve $y=x^2-5x+m+8$ touches the line $y=mx$ only once. [4]

(ii) Hence, state the range of values of *m* for which the curve $y=x^2-5x+m+8$ cuts the line $y=mx$ twice. [1]

(iii) Using answers in (i) and (ii), state what can be deduced about the curve $y=x^2-5x+11$ and the straight line $y=3x$, giving a reason for your answer. [2]

8 The roots of the equation $3x^2=2kx-k-4$ are α and β. If $\alpha^2+\beta^2=\frac{16}{9}$, find possible values of *k*. [5]

9 The equation $2x^2+4x+5=0$ has roots α and β.

(a) Find $\alpha+\beta$ and $\alpha\beta$. [2]

(b) Show that $\alpha^3+\beta^3=7$. [3]

(c) Find the quadratic equation, with integer coefficients, whose roots are $\frac{\alpha}{\beta^2}$ and $\frac{\beta}{\alpha^2}$. [3]

10 Find the range of values for which $\frac{-5}{3x^2+13x-10}>0$. [3]

11 Without using a calculator, find the value of 6^x, given that $3^{2x+2}=4^{-3-x}$. [4]

12 (a) Solve the equation $4^{x+1}=2-7(2^x)$. [3]

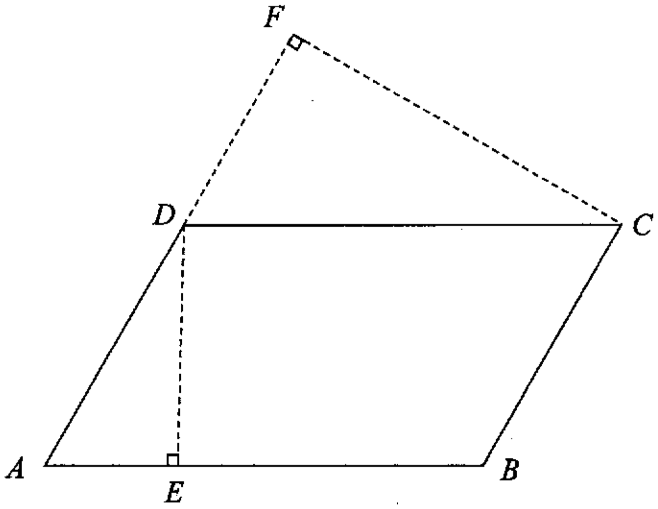
(b) Solve the simultaneous equations

$$4^{x-2}=\frac{64}{2^y}$$

$$\log_x(y+2)-1=\log_x 4$$ [5]

13 Solve the equation $\sqrt{x-5}=\sqrt{x}+2$. [3]

14 In the diagram, $ABCD$ is a parallelogram, with heights DE and CF. It is given that $CD = (7 + 4\sqrt{2})$ cm, $BC = (11 - 2\sqrt{2})$ cm and $DE = (3 + 3\sqrt{2})$ cm.



Find

(a) the perimeter of $ABCD$, [2]

(b) the area of $ABCD$, [2]

(c) the length of CF. [3]

15 **(a)** Solve $\log_3(\frac{1}{x}) = 2$. [2]

(b) Given that $\lg x = a$ and $\lg y = b$, express $\lg(\frac{1000x^2}{y})$ in terms of a and b. [2]

(c) Solve $\ln(x-1) = 3$, leaving your answer in exact terms. [2]

End of Paper