NORTH VISTA SECONDARY SCHOOL

END-OF-YEAR EXAMINATION 2018

40

NAME: ______________________ (   ) **CLASS:** ________

SUBJECT: ADDITIONAL MATHEMATICS **DATE: 5 OCTOBER 2018**

LEVEL/STREAM: SECONDARY 3 EXPRESS **TIME: 1 HOUR**

CODE : 4047/1

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give your non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **40**.

This question paper consists of 4 printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

Answer **all** the questions.

1 The line $4x = 2 + y$ cuts the curve $x^2 + 2y^2 - 1 = 0$ at points A and B. Find the coordinates of A and B. [4]

2 Express $\dfrac{5x^3 - x - 6}{x^2 - 1}$ in the form $Ax + \dfrac{B}{x+1} + \dfrac{C}{x-1}$. [4]

3 Find in ascending powers of x, the first four terms in the expansion of

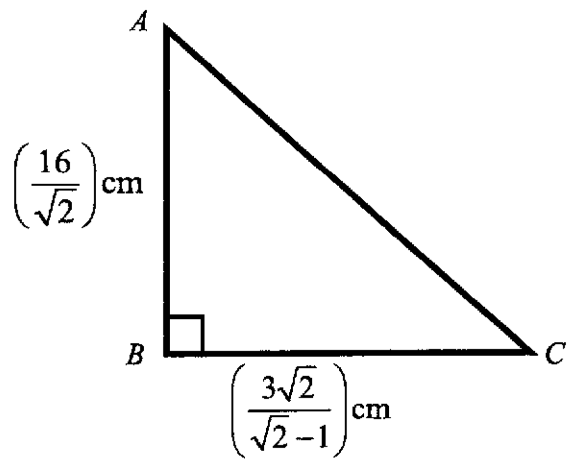
(a) $(3+x)^7$, [2]

(b) $(1-kx)^5$. [2]

Hence find the value of k for which the coefficient of x^3 in the expansion of $(1-kx)^5 + (3+x)^7$ is 2565. [1]

4 **(i)** Simplify $\dfrac{3\sqrt{2}}{\sqrt{2}-1}$. [2]

(ii) Hence, in the right-angled triangle ABC, $BC = \dfrac{3\sqrt{2}}{\sqrt{2}-1}$ cm and $AB = \dfrac{16}{\sqrt{2}}$ cm, find the area of triangle ABC in the form $a\left(\sqrt{2}+1\right)$ where a is a constant. [3]



5 **(i)** Find the range of the values of x for which $x(x+3) - x \geq 0$. [2]

(ii) Find the range of values of m such that the expression $(2x-1)^2 + 4m^2 - 9$ is always positive for all real values of x. [4]

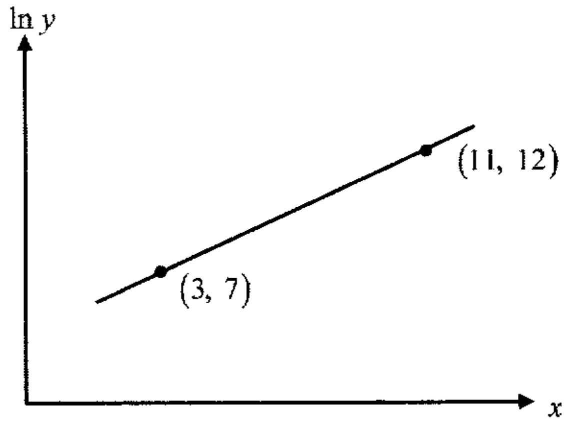
[Turn over

6 Solve

(i) $\log_2(3x-5)+3=\log_2(4x+5)$, [3]

(ii) $2\log_3 y-\log_y 27=1$. [5]

7



The variables x and y are connected by the equation $\frac{y}{p}=q^x$, where p and q are constants. Experimental values of x and y are obtained. The diagram above shows the straight line graph, passing through the points $(3,\ 7)$ and $(11,\ 12)$, obtained by plotting $\ln y$ against x. Estimate

(i) the value of p and of q, to 2 significant figures, [6]

(ii) the value of x when $y=15$. [2]

END OF PAPER

NORTH VISTA SECONDARY SCHOOL

END-OF-YEAR EXAMINATION 2018

60

NAME: ______________________( ) **CLASS:** ________

SUBJECT: ADDITIONAL MATHEMATICS **DATE: 10 OCTOBER 2018**

LEVEL/STREAM: SECONDARY 3 EXPRESS **TIME: 1 HOUR 30 MINUTES**

CODE: 4047/2

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
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The total of the marks for this paper is **60**.

This question paper consists of 5 printed pages.

[Turn over

Mathematical Formulae

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1 Given that the roots of the quadratic equation $x^2 = 4x + 2$ are α and β,

(a) find the values of $\alpha + \beta$ and $\alpha\beta$, [2]

(b) form a quadratic equation in x whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [4]

2 The quantity, N, of a particle decaying is given by $N = 3500 + 2000e^{-0.04t}$, where t is the time in years after the particle starts decaying.

(i) Find the quantity of the particle at which the particle has not started decaying. [1]

(ii) Find the quantity of the particle when $t = 14$. [1]

(iii) Express t in terms of N. [3]

(iv) Explain why the quantity of the particle can never reach 3500. [1]

3 The function $f(x) = x^3 + ax^2 + bx + 3$, where a and b are constants, is exactly divisible by $(x+1)$ and leaves a remainder of 48 when divided by $(x-3)$.

(i) Find the value of a and of b. [4]

(ii) Determine, showing all necessary working, the number of real roots of the equation $f(x) = 0$. [4]

4 The curve $y = e^{3x} - 7$ intersects the coordinate axes at the points A and B.

(i) Given that the line AB passes through the point with coordinates $(\ln 7,\ k)$, find the value of k. [5]

(ii) In order to solve the equation $x = \ln \sqrt[3]{x+8}$, a graph of a suitable straight line is drawn on the same set of axes as the graph of $y = e^{3x} - 7$. Find the equation of this straight line. [3]

[Turn Over

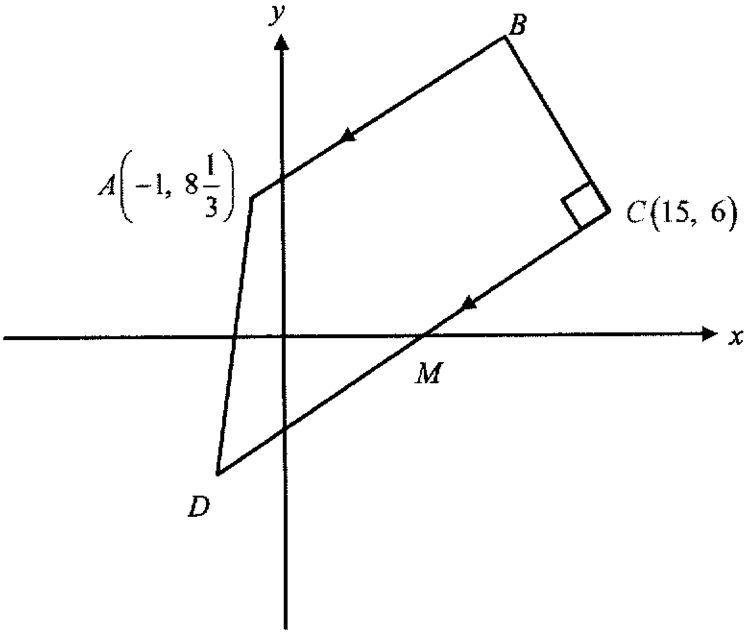
5 **(a)** Use the substituition $u = 8^x$ to solve $8^{x+1} = 64^x + 16$. [3]

(b) Given that $a = \log_2 x$ and $b = \log_4 y$, express in terms of a and/or b,

(i) $\log_2 64x^3$, [2]

(ii) $\log_y x$. [2]

6



The diagram shows a trapezium $ABCD$ in which AB is parallel to DC and angle $BCD = 90°$. The vertices of the trapezium are at the points $A\left(-1,\ 8\frac{1}{3}\right)$, B, $C(15,\ 6)$ and D. DC cuts the x-axis at M, the midpoint of DC. Given that the equation of AB is $3y = 2x + 27$, find

(i) the coordinates of B, [4]

(ii) the coordinates of D, [3]

(iii) the area of trapezium $ABCD$. [2]

7 (i) Solve the equation $5-|2x+3|=0$. [2]

(ii) Sketch the graph of $y=5-|2x+3|$, indicating the axes intercepts and vertex. [2]

8 A circle, centre C, has a diameter AB where A is the point $(-13,\ -4)$ and B is the point $(3,\ 8)$.

(i) Find the coordinates of C and the radius of the circle. [2]

(ii) Find the equation of the circle. [2]

(iii) Show that the equation of the tangent to the circle at A is $3y+4x=-64$. [3]

9 The table below shows experimental values of two variables x and y.

x	2	4	6	8	10
y	0.41	0.49	0.57	0.66	0.77

It is known that x and y are related by the equation of the form $\frac{\sqrt{x}}{y}=ax+b\sqrt{x}$, where a and b are constants.

(i) Using a scale of 4 cm to 1 unit on each axis, plot $\frac{1}{y}$ against $\sqrt{x}$ and draw a straight line graph. [2]

(ii) Use your graph to estimate the value of a and of b. [3]

END OF PAPER

Marking Scheme paper 1

1	$x^2+2y^2-1=0$ $y=4x-2$ $x^2+2(4x-2)^2-1=0$ $33x^2-32x+7=0$ $(3x-1)(11x-7)=0$ $x=\frac{1}{3}$ *or* $\frac{7}{11}$ $y=-\frac{2}{3}$ *or* $\frac{6}{11}$ $A\left(\frac{1}{3},-\frac{2}{3}\right)$ *and* $B\left(\frac{7}{11},\frac{6}{11}\right)$	M1 M1 M1 A1
2	Using Long Division $\frac{5x^3-x-6}{x^2-1}=5x+\frac{4x-6}{(x+1)(x-1)}$ Let $x=-1,\ B=5----B1$ Let $x=1,\ C=-1---B1$ $A=5---B1$ $\frac{5x^3-x-6}{x^2-1}=5x+\frac{5}{x+1}-\frac{1}{x-1}$	M1 B1 B1 B1
3(ai)	$(3+x)^7=2187+5103x+5103x^2+2835x^3+...$	B2
3(ai)	$(1-kx)^5=1-5kx+10k^2x^2-10k^3x^3+...$	B2
3(ai)	$(3+x)^7=2187+5103x+5103x^2+2835x^3+...$ $(1-kx)^5=1-5kx+10k^2x^2-10k^3x^3+...$ $2835x^3-10k^3x^3=2565x^3$ $2835-10k^3=2565$ $k^3=27$ $k=3$	B1
4(i)	$\frac{3\sqrt{2}}{\sqrt{2}-1}=\frac{3\sqrt{2}}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$ $=\frac{3(2)+3\sqrt{2}}{1}$ $=6+3\sqrt{2}$	M1 A1

4(ii)	$\text{Area}=\frac{1}{2}\left(\frac{16}{\sqrt{2}}\right)\left(\frac{3\sqrt{2}}{\sqrt{2}-1}\right)$ $=\left(\frac{8}{\sqrt{2}}\right)\left(6+3\sqrt{2}\right)$ $=\left(\frac{4\sqrt{2}}{1}\right)\left(6+3\sqrt{2}\right)$ $=24\sqrt{2}+24$ $=24\left(\sqrt{2}+1\right)\text{ cm}^2$	M1 M1 A1
5(i)	$x(x+3)-x\geq 0$ $x^2+2x\geq 0$ $x(x+2)\geq 0$ $x\leq -2 \quad and \quad x\geq 0$	M1 A1
5(ii)	$(2x-1)^2+4m^2-9$ $=4x^2-4x+1+4m^2-9$ $b^2-4ac<0$ $(-4)^2-4(4)\left(4m^2-8\right)<0$ $16-64m^2+128<0$ $144-64m^2<0$ $9-4m^2<0$ $(3-2m)(3+2m)<0$ $m<-\frac{3}{2} \quad or \quad m>\frac{3}{2}$	M1 M1 M1 A1
6(a)	$\log_2(3x-5)+3=\log_2(4x+5)$ $\log_2(3x-5)+\log_2 2^3=\log_2(4x+5)$ $\log_2 8(3x-5)=\log_2(4x+5)$ $24x-40=4x+5$ $x=2\frac{1}{4}$	[M1] [M1] [A1]

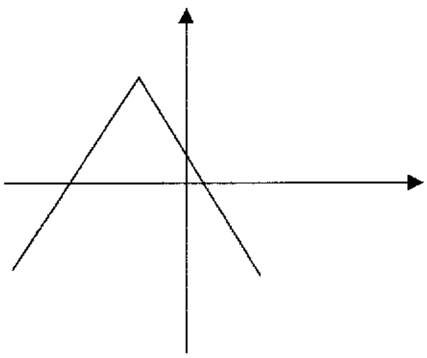
6b	$2\log_3 y - \log_y 27 = 1$ $2\log_3 y - \frac{3\log_3 3}{\log_3 y} = 1$ Let $x = \log_3 y$ $2x - \frac{3}{x} = 1$ $2x^2 - x - 3 = 0$ $(2x-3)(x+1) = 0$ $x = \frac{3}{2}$ or $x = -1$ $\log_3 y = \frac{3}{2}$ or $\log_3 y = -1$ $y = \sqrt{27} = 3\sqrt{3}$ $y = \frac{1}{3}$	[M1 – change of base] [M1] [M1] [A2]
7	$\frac{y}{p} = q^x$ $y = pq^x$ $\ln y = \ln p + x\ln q$ $\ln q =$ gradient $= \frac{12-7}{11-3}$ $= \frac{5}{8}$ $q = e^{\frac{5}{8}}$ $= 1.868$ $= 1.9$ $7 = \ln p + \frac{5}{8}(3)$ $p = 168.17 \approx 170$	[M1] [M1] [M1] [A1] [M1] [A1]
	$\ln 15 = 5\frac{1}{8} + \frac{5}{8}x$ $x \approx -3.87$	M1 A1

2018 EOY SEC 3 AM P2

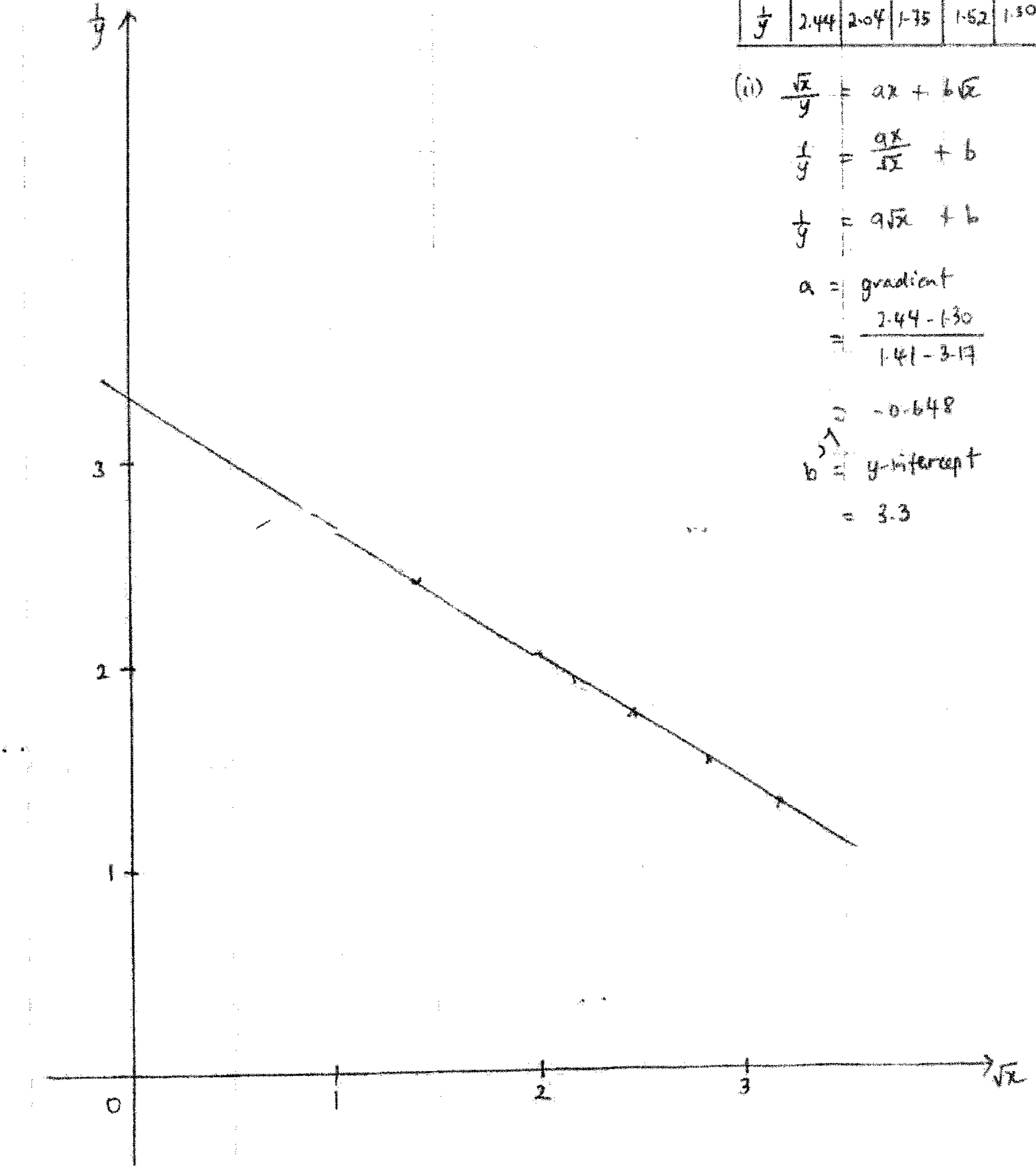
1	a	$\alpha+\beta=4$ $\alpha\beta=-2$	[B1] [B1]
	b	$\frac{1}{\alpha^3}\times\frac{1}{\beta^3}=-\frac{1}{8}$ $\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\alpha^3+\beta^3}{\alpha^3\beta^3}$ $=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha^3\beta^3}$ $=\frac{64+24}{-8}$ $=-11$ $x^2+11x-\frac{1}{8}=0$ $8x^2+88x-1=0$	[M1 [M1] [M1] [A1]
2	(i)	$N=3500+2000e^{-0.04(0)}$ $=5500$	[B1]
2	(ii)	$N=3500+2000e^{-0.04t(14)}$ $=4642$ $=4640$	[B1]
2	(iii)	$N=3500+2000e^{-0.04t}$ $N-3500=2000e^{-0.04t}$ $\frac{N-3500}{2000}=e^{-0.04t}$ $-0.04t=\ln\left(\frac{N-3500}{2000}\right)$ $t=-25\ln\left(\frac{N-3500}{2000}\right)$	 [M1] [M1] [A1]
2	(iv)	$0<2000e^{-0.04t}\leq 2000$ $3500<3500+2000e^{-0.04t}\leq 5500$ $3500<N\leq 5500$ *or* As t approaches infinity, $2000e^{-0.04t}$ approaches 0. As t approaches infinity, $3500+2000e^{-0.04t}$ approaches 3500.	[B1]

3	(i)	By factor theorem, $f(-1)=0$ $-1+a-b+3=0$ $a-b=-2---(1)$ By remainder theorem, $f(3)=48$ $27+9a+3b+3=48$ $9a+3b=18----(2)$ $a=1 \quad and \quad b=3$	[M1] [M1] [A2]
3	(ii)	$x^3+x^2+3x+3=0$ Long Division $(x+1)(x^2+3)=0$ $x+1=0$ or $(x^2+3)=0$ $x=-1$ or Discriminant $=(0)^2-4(1)(3)=-12<0$ $f(x)=0$, $\therefore$ only 1 real root.	[M1] [M1] [M1] [A1]
4	(i)	$y=e^{3x}-7$ when $y=0$, $x=\frac{1}{3}\ln 7$ when $x=0$, $y=-6$ gradient of $AB=\frac{-6}{-\frac{1}{3}\ln 7}$ $=\frac{18}{\ln 7}$ $\frac{k+6}{\ln 7-0}=\frac{18}{\ln 7}$ $k=12$	[M1] [M1] [M1] [M1] [A1]
	(ii)	$x=\ln\sqrt[3]{x+8}$ $=\frac{1}{3}\ln(x+8)$ $3x=\ln(x+8)$ $e^{3x}=x+8$ $e^{3x}-7=x+1$ $y=x+1$	[M1] [M1] [A1]

5	(a)	$8^{x+1}=64^x+16$ $8\left(8^x\right)=8^{2x}+16$ $sub\quad u=8^x$ $u^2-8u+16=0$ $u=4$ $8^x=4$ $2^{3x}=2^2$ $x=\frac{2}{3}$	 [M1] [M1] [A1]
5	(b)	(i) $\log_2 64x^3=\log_2 64+\log_2 x^3$ $=3(2+a)$	 [M1] [A1]
5	(b)	(ii) $\log_y x=\frac{\log_2 x}{\log_2 y}$ $=a\div\frac{\log_4 y}{\log_4 2}$ $=a\div\frac{b}{\frac{1}{2}}$ $=\frac{a}{2b}$	 [M1] [A1]
6	(i)	$3y=2x+27$ $y=\frac{2}{3}x+9\quad$ —(1) $m_{BC}=-\frac{3}{2}$ eqn of BC: $y-6=-\frac{3}{2}(x-15)$ $y=-\frac{3}{2}x+\frac{57}{2}\quad$ —(2) $\frac{2}{3}x+9=-\frac{3}{2}x+\frac{57}{2}$ $x=9$ $y=15$ $B(9,15)$	 [M1] [M1] [M1] [A1]

6	(ii)	Let $M(m,0)$ $\frac{0-6}{m-15}=\frac{2}{3}$ $m=6$ Let $D(x,y)$ $\left(\frac{x+15}{2},\frac{y+6}{2}\right)=(6,0)$ $x=-3$ $y=-6$ $D(-3,-6)$	[M1] [M1] [A1]
6	(iii)	Area of trapezium $ABCD$ $=\frac{1}{2}\begin{vmatrix}-1 & -3 & 15 & 9 & -1\\ 8\frac{1}{3} & -6 & 6 & 15 & 8\frac{1}{3}\end{vmatrix}$ $=\frac{1}{2}\left\|(6-18+225+75)-(-25-90+54-15)\right\|$ $=182\text{units}^2$	[M1] [A1]
7	(i)	$5-\|2x+3\|=0$ $\|2x+3\|=5$ $2x+3=5$ or $2x+3=-5$ $x=1$ $x=-4$	
7	(ii)		
8	(i)	$centre=\left(\frac{-13+3}{2},\frac{-4+8}{2}\right)$ $=(-5,2)$ radius $=\sqrt{(3+5)^2+(8-2)^2}$ $=10$ units	[B1] [B1]
8	(ii)	$(x+5)^2+(y-2)^2=100$	[B2]

8	(iii)	gradient of $AC = \frac{-4-2}{-13+5}$ $= \frac{3}{4}$ gradient of tangent at $A = -\frac{4}{3}$ $y+4 = -\frac{4}{3}(x+13)$ $y = -\frac{4}{3}x - \frac{64}{3}$ $3y+4x = -64$	[M1] [M1] [A1]



$\sqrt{x}$	1.41	2	2.45	2.83	3.17
$\frac{1}{y}$	2.44	2.04	1.75	1.52	1.30

(ii) $\frac{\sqrt{x}}{y} = ax + b\sqrt{x}$

$\frac{1}{y} = \frac{ax}{\sqrt{x}} + b$

$\frac{1}{y} = a\sqrt{x} + b$

a = gradient

$= \frac{2.44 - 1.30}{1.41 - 3.17}$

$= -0.648$

b = y-intercept

$= 3.3$