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JURONG SECONDARY SCHOOL
2022 GRADUATION EXAMINATION 2
SECONDARY 4 EXPRESS

CANDIDATE NAME	

CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/01**

PAPER 1
Candidates answer on the Question Paper.
Additional Materials : Writing Paper (1 sheet)

29 August 2022
2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

This document consists of **15** printed pages including this page.

[Turn Over]

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer, and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 It is given that $\sin A = 0.3$, where A is obtuse.
Find the following trigonometric ratios.

(a) $\sec A$, [2]

Solution	Mark
$-\frac{10\sqrt{91}}{91}$	B1 correct value of $\cos A$ B1 change $\sec A$ to $1/\cos A$

(b) $\cos 2A$. [2]

Solution	Mark
$\cos 2A = 2\cos^2 A - 1$ $= 2(1 - 0.3^2) - 1$ $= 0.82$	M1 double angle formula A1

(c) $\tan(A + 45°)$. [4]

Solution	Mark
$\tan(A + 45°) = \frac{\tan A + \tan 45°}{1 - \tan A \tan 45°}$ $= \frac{\frac{3}{-\sqrt{91}}+1}{1+\frac{7}{\sqrt{51}}}$ $= \frac{\sqrt{91}-3}{\sqrt{91}+3} \times \frac{\sqrt{91}-3}{\sqrt{91}-3}$ $= \frac{50-3\sqrt{91}}{41}$	M1 addition formula B1 correct value for $\tan 45°$ M1 rationalisation A1

2 **(a)** Find the range of values of k such that $3x^2 - 5x + k$ is always positive. [2]

Solution	Mark
$(-5)^2 - 4(3)(k) < 0$ $k > \frac{25}{12}$	M1 A1

(b) Hence, solve the inequality $\frac{x^2-x-2}{3x^2-5x+4} < 0$. [3]

Solution	Mark
$k = 4 > \frac{25}{12}$, hence $3x^2 - 5x + 4 > 0$ $(x+1)(x-2) < 0$ $-1 < x < 2$	M1 using part (a) M1 correct quadratic inequality in factorised form A1

3 It is given that $f(x) = Ax(e^{kx})$, where A and k are constants.
Find the value of A and of k such that $f'(x) + 2ke^{kx} + 6f(x) = 0$ [6]

Solution	Mark
$f'(x) = Akxe^{kx} + Ae^{kx}$	M1 product rule
$Akxe^{kx} + Ae^{kx} + 2ke^{kx} + 6Axe^{kx} = 0$	M1 substitution
$A + 2k = 0$ --------------- (1)	M1
$Ak + 6A = 0$ ------------ (2)	M1
From (2) $A(k + 6) = 0$, $A = 0$ (rej) or $k = -6$	A1
sub $k = -6$ into (1), $A = 12$	A1

4 A curve has equation $y = \frac{4}{\sqrt{x+3}}$. A point (x, y) is moving along the curve.

Find the coordinates of the point at the instant where the y-coordinate is decreasing at a rate twice of the rate of increase of the x-coordinate. [5]

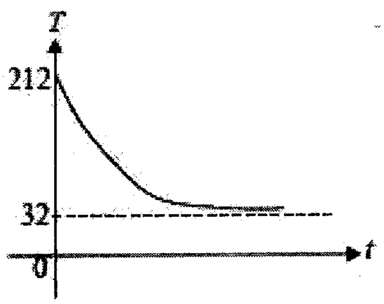
Solution	Mark
$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $\frac{dy}{dx} = -2$ $4 \times \left(-\frac{1}{2}\right)(x+3)^{-\frac{3}{2}} = -2$ $x + 3 = 1$ $(-2, 4)$	M1 correct formula for connected rate of change M1 correct value of $\frac{dy}{dx}$ M1 correct derivative A1 correct values of x and y A1 coordinate form

5 A metal cube is heated to a temperature of 212°C before being dropped into a liquid. As the cube cools, its temperature T°C, t minutes after it enters the liquid is given by $T = P + 180e^{-kt}$, where P and k are constants.
It is recorded that when $t = 5$, $T = 185$.

(a) Find the value of k and of P. [4]

Solution	Mark
$P = 212 - 180 = 32$	B1
$32 + 180e^{-5k} = 185$	M1 substitution
$e^{-5k} = 0.85$	M1 isolating the exp term
$-5k = \ln 0.85$	
$k = 0.0325$ (3 s.f.)	A1

(b) By sketching the graph of T against t, explain why T cannot be 30. [3]

Solution	Mark
 From the graph, the graph is completely above $T = 32$, hence, T cannot be 30.	B1 correct shape B1 correct y-intercept and horizontal asymptote -1 if $t < 0$ is included -1 if no labelling of axis B1 accept any reasonable answer

6 It is given that the first three terms, in ascending powers of x, of the binomial expansion of $(2 + ax)^6$ are $64 - 960x + bx^2$.

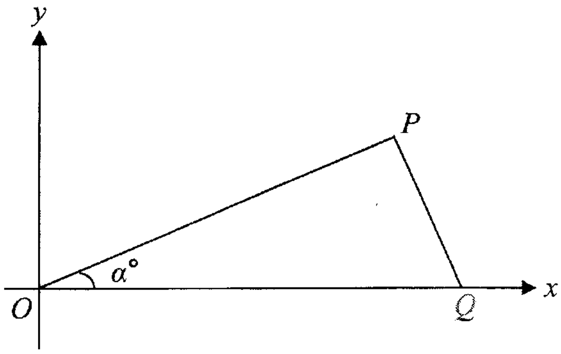
(a) Find the value of a and of b. [3]

Solution	Mark
$2^6 + \binom{6}{1}2^5(ax) + \binom{6}{2}2^4(ax)^2 = 32 - 960x + bx^2$	M1 binomial expansion
$192a = -960$, $b = 240a^2$	M1 compare coefficient
$a = -5$, $b = 6000$	A1

(b) Using the values found in part **(a)**, find the coefficient of x^3 in the expansion of $(1 + 3x^2)^5(2 + ax)^6$. [4]

Solution	Mark
$(1 + 15x^2 + \ldots)(32 - 960x + 6000x^2 - 20000x^3 + \ldots)$	M1 binomial expansion of $(1 + 3x^2)^5$
	M1 correct x^3 in $(2 + ax)^6$
coefficient of $x^3 = 1(-20000) + (15)(-960)$	M1
$= -34400$	A1

7 In the diagram below, the line OP makes an angle of $\alpha°$ with the positive x-axis such that $\tan \alpha = 0.2$ and Q lies on the x-axis.



(a) Given that $OP = \frac{\sqrt{26}}{2}$ units, show that $P = (2.5, 0.5)$. [3]

Solution	Mark
Let $OP = (x, y)$ $\sqrt{x^2 + y^2} = \frac{\sqrt{26}}{2}$ ---------------- (1) $\frac{y}{x} = 0.2$ ------------------------ (2) $x^2 + (0.2x)^2 = 6.5$ $x = 2.5, y = 0.5$	M1 using length of OP M1 using gradient A1 with working

(b) Given that the area of ΔOPQ is 0.65 units^2, find the coordinates of Q. [2]

Solution	Mark
$\frac{1}{2} \times OQ \times 0.5 = 0.65$ $OQ = 2.6$ $Q = (2.6, 0)$	M1 A1

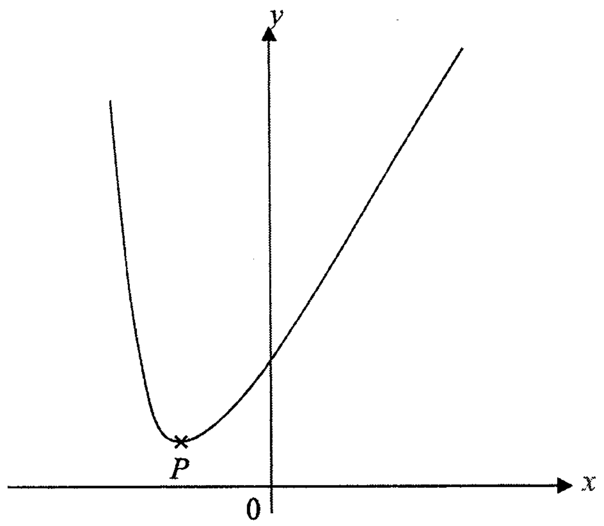
(c) Explain, with calculations, why ΔOPQ is a right-angle triangle. [3]

Solution	Mark
gradient of $PQ = \frac{0-0.5}{2.6-2.5} = -5$ gradient of PQ × gradient of $OP = -5 \times 0.2 = -1$ Hence, OP is perpendicular to PQ, ΔOPQ is a right-angle triangle	M1 M1 A1 must identify the perpendicular lines/right angle

(d) Find the coordinates of R such that $OPQR$ is a rectangle. [2]

Solution	Mark
Let $R = (x, y)$ $\left(\frac{0+2.6}{2}, 0\right) = \left(\frac{x+2.5}{2}, \frac{y+0.5}{2}\right)$ $R = (0.1, -0.5)$	M1 A1

8 The diagram below shows part of the graph of $y = \frac{x^2+2x+5}{x+3}$.



Find the coordinates of the minimum point P. [7]

Solution	Mark
$\frac{dy}{dx} = \frac{(x+3)(2x+2)-(x^2+2x+5)(1)}{(x+3)^2}$	M1 quotient rule
$\frac{2x^2+8x+6-x^2-2x-5}{(x+3)^2} = 0$	M1 $\frac{dy}{dx} = 0$
$x^2 + 6x + 1 = 0$	
$x = \frac{-6\pm\sqrt{6^2-4(1)(1)}}{2}$	M1 quadratic formula
$= -3 \pm 2\sqrt{2}$	A1 accept -0.17157 and -5.8284
When $x = -3 - 2\sqrt{2}$, y is a maximum	M1 1st or 2nd derivative test
When $x = -3 + 2\sqrt{2}$, y is a minimum	A1 nature of s.p.
$P = (-0.172, 1.66)$	A1

9 **(a)** It is given that $\frac{2^x \times 32(2^x)}{8^{x+1}} = \frac{9(5^{2x}) - 5^{2x+1}}{5^x - 5^{x-1}}$.

Evaluate 10^x without using a calculator. [5]

Solution	Mark
$\frac{2^x \times 2^5 \times 2^x}{2^{3x+3}} = \frac{9 \times 5^{2x} - 5^{2x} \times 5}{5^x - 5^x \times \frac{1}{5}}$	M1 change to common base
$\frac{2^{2x+5}}{2^{3x+3}} = \frac{4 \times 5^{2x}}{\frac{4}{5} \times 5^x}$	M1 factorisation of RHS
	M1 simplification to one term on each side
$\frac{4}{2^x} = 5^x \times 5$	
$\frac{4}{5} = 5^x \times 2^x$	M1 isolate 2^x and 5^x
$10^x = 0.8$	A1

(b) Solve $\sqrt{x+7} - x - 1 = 0$. [3]

Solution	Mark
$\sqrt{x+7} = x + 1$	
$x + 7 = x^2 + 2x + 1$	M1 getting rid of square root
$x^2 + x - 6 = 0$	
$(x - 2)(x + 3) = 0$	M1 method to solve eqn
$x = 2$ or -3 (reject as $x + 1 > 0$)	A1 with rejection

10 A particle moves in a straight line so that its velocity, v m/s is given by $v = t^2 - 4t + 3$, where t is the time in seconds after passing a fixed point O.

(a) Find the acceleration of the particle when $t = 1$. [2]

Solution	Mark
$a = 2t - 4$ when $t = 1$, $a = -2$ m/s^2	M1 A1

(b) Find the value(s) of t when the particle comes to instantaneous rest. [2]

Solution	Mark
$(t-1)(t-3) = 0$ $t = 1$ or 3	M1 $v = 0$ A1

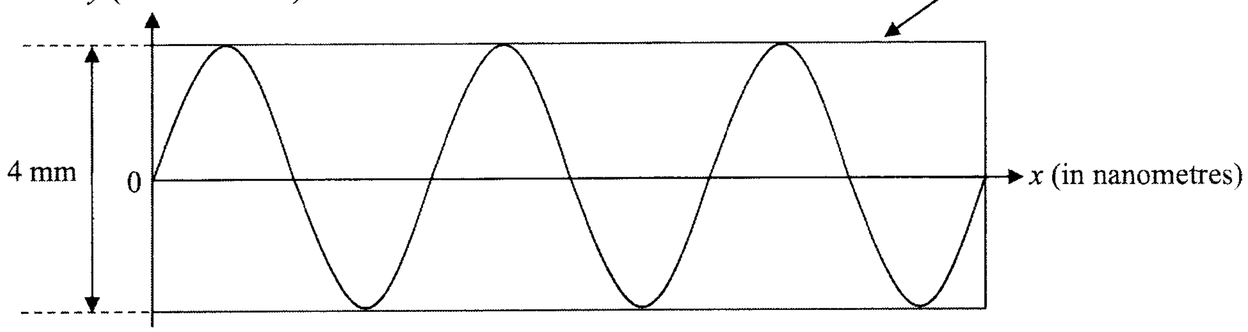
(c) Find the displacement(s) of the particle at the instant when it comes to rest. [3]

Solution	Mark
$s = \frac{1}{3}t^3 - 2t^2 + 3t + c$ $s = 0$ when $t = 0$, $c = 0$ $s = \frac{1}{3}t^3 - 2t^2 + 3t$ When $t = 1$, $s = \frac{4}{3}$ m When $t = 3$, $s = 0$ m	M1 indefinite integral of v B1 correct expression for s A1 for both values -1 if no unit

(d) Find the average speed of the particle for the first 4 seconds. [3]

Solution	Mark
When $t = 4$, $s = \frac{4}{3}$ m Total distance travelled = 4 m average speed = 1 m/s	M1 find s when $t = 4$ M1 total distance A1

11 The diagram below shows a portion a plastic fibre cable, which allows light waves to pass through. The path of the light wave can be modelled by a trigonometric function.



(a) It is given that the diameter of the cable is 4 millimetres.
Find the amplitude of the light wave. [1]

Solution	Mark
2 mm	B1

(b) It is given further that the period of the light wave is 500 nanometres.
Find the length of the portion of cable shown in the diagram. [1]

Solution	Mark
1500 nanometers	B1

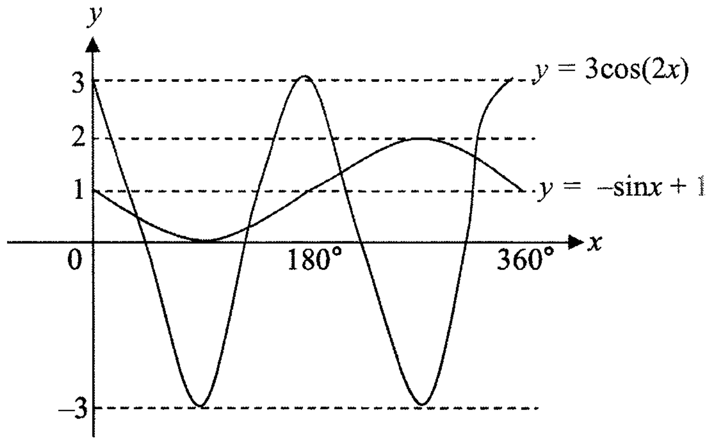
(c) Which of the following can be a suitable model for the light wave?

$y = 2\sin(\pi x)$ $\quad\quad$ $y = 2\cos(\pi x)$ $\quad\quad$ $y = 2\sin\left(\frac{\pi}{250}x\right)$

Explain your answer. [3]

Solution	Mark
$y = 2\sin\left(\frac{\pi}{250}x\right)$ The graph starts from the centre position 0, hence it is a sine graph. The coefficient of x, b is such that $\frac{2\pi}{b} = 500$, hence $b = \frac{\pi}{250}$	B1 B1 choice of trigo ratio B1 coefficient of x

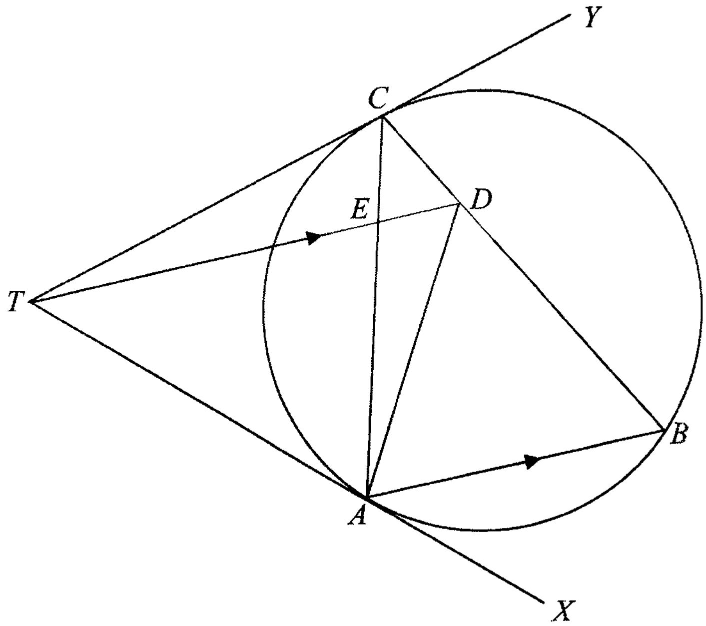
12 **(a)** Sketch the graph of $y = -\sin x + 1$ and $y = 3\cos(2x)$ for $0° \leq x \leq 360°$ on the same axes. [4]

Solution	Mark
	B1, B1 each amplitude and max/min value B1, B1 each correct period, correct shape

(b) Hence, state the number of solutions to the equation $-\sin x + 1 = 3\cos(2x)$ for $0° \leq x \leq 360°$. [1]

Solution	Mark
4	B1

13 In the diagram below, TAX and TCY are tangents to the circle at A and C respectively. AC meets TD at E and D is on BC such that TD is parallel to AB.



(a) Prove that angle ACB is equal to angle ATD. [2]

Solution	Mark
$\measuredangle ATD = \measuredangle XAB$ (corresponding angles, $AB \mathbin{//} TD$) $= \measuredangle ACB$ (angles in alternate segments)	B1 B1 -1 if no reason/wrong reason

(b) Explain why a circle can be drawn passing through the points T, A, D and C. [1]

Solution	Mark
Angles in the same segment	B1

(c) Hence, prove that $CE \times EA = DE \times TE$. [4]

Solution	Mark
$\measuredangle ATE = \measuredangle DCE$ (from part a) $\measuredangle TEA = \measuredangle CED$ (vertically opposite angles) ΔATE is similar to ΔDCE (AA similarity test) $\frac{TE}{CE} = \frac{EA}{ED}$ $CE \times EA = DE \times TE$	M1 two reasons A1 similar triangles with test M1 A1

--END OF PAPER--

4E AM GE2 2022 Paper 2 Mark Scheme

1 **(a)** Differentiate $2x\cos 3x$ with respect to x. [2]

$\frac{\mathrm{d}}{\mathrm{d}x}(2x\cos 3x)$
$= 2x(-3\sin 3x) + 2\cos 3x$
$= -6x\sin 3x + 2\cos 3x$ [B1,B1]: each term

(b) Hence, find $\int x\sin 3x \, \mathrm{d}x$. [3]

$\int x\sin 3x \, \mathrm{d}x$
$= -\frac{1}{6}(2x\cos 3x) + \frac{1}{3}\int \cos 3x \, \mathrm{d}x$ [M1]: reverse differentiation
$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + \mathrm{C}$ [A1,A1] each term

Deduct 1 mark for missing +C.

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2 **(a)** $a^3+b^3=(a+b)(a^2-ab+b^2)$ [1]

(b)
$\frac{(\sin x+\cos x)(\sin^2 x-\sin x\cos x+\cos^2 x)}{\sin x+\cos x}$ [M1]: Cubic Identity
$=1-\sin x\cos x$ [M1]: simplify
$=1-\frac{2\sin x\cos x}{2}$ ~~[A1]: show change of double angle, fraction~~
$=1-\frac{\sin 2x}{2}$

(c) Hence, solve the equation $\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}=1-\sin^2 2x$ for $0 \le x \le \pi$. [5]

$1-\frac{\sin 2x}{2}=1-\sin^2 2x$ [M1]: Hence
$2\sin^2 2x-\sin 2x=0$
$\sin 2x(2\sin 2x-1)=0$ [M1]: Factorisation mtd
$\sin 2x=0$ or $\sin 2x=\frac{1}{2}$ [M1]: Correct Trigo Equations
$x=0,\frac{\pi}{2},\pi$ [A1] or $\frac{\pi}{12},\frac{5\pi}{12}$ [A1]

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3 **(a)** Express $\dfrac{3x^2-4}{x^2(3x-2)}$ in partial fractions. [4]

Let $\dfrac{3x^2-4}{x^2(3x-2)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{3x-2}$ [B1] Correct form

$3x^2 - 4 = Ax(3x-2) + B(3x-2) + Cx^2$

Compare constant term, $B = 2$ [M1]

By substitution, $A = 3, C = -6$ [M1]

$\dfrac{3x^2-4}{x^2(3x-2)} = \dfrac{3}{x} + \dfrac{2}{x^2} - \dfrac{6}{3x-2}$ [A1]

(b) Hence, evaluate $\int \dfrac{3x^2-4}{x^2(3x-2)}\, dx$. [3]

$$\int \frac{3}{x} + \frac{2}{x^2} - \frac{6}{3x-2}\, dx$$
$$= 3\ln x - \frac{2}{x} - 2\ln(3x-2) + C$$

[B1,B1,B1] each term, penalise 1 mark if missing +C

4 It is given that $\log_2(4-x^2)-\log_{\sqrt{2}}(x-1)=1$.

(a) Explain clearly why $1<x<2$. [4]

$4-x^2>0$
$(x-2)(x+2)<0$ [M1] and $x-1>0$, $x>1$ [M1]
$-2<x<2$ [A1]

Reasonable conclusion using words or number line. [A1]

$\therefore\ 1<x<2$

(b) Hence, solve the equation and show that it has only one solution. [5]

$\log_2(4-x^2)-\dfrac{\log_2(x-1)}{\log_2\sqrt{2}}=1$ [M1]: Change base

$\log_2(4-x^2)-\log_2(x-1)^2=1$ [M1]: Power Law

$\log_2\dfrac{4-x^2}{(x-1)^2}=1$ [M1]: Quotient Law

$\dfrac{4-x^2}{(x-1)^2}=2$ [M1]: Convert form/Equivalence

$3x^2-4x-2=0$

$x=\dfrac{4\pm\sqrt{16-4(-6)}}{6}$ [A1]
$=1.72$ or -0.387 (rejected)

5 f(x) is such that $f''(x) = 18x - 4$. Given that f(0) = 1 and f(2) = 9,

(a) find f(x). [4]

$f'(x) = 9x^2 - 4x + c$ [M1]

$f(x) = 3x^3 - 2x^2 + cx + d$ [M1]

show correct usage of notation/substitution [M1]

$d = 1, c = -4$

$f(x) = 3x^3 - 2x^2 - 4x + 1$ [A1]

(b) Show that $x + 1$ is a factor. [1]

$f(-1)$

$= 3(1)^3 - 2(-1)^2 - 4(-1) + 1$

$=0$

By factor/remainder theorem, $x+1$ is a factor.

Must state appropriate conclusion to show how the substitution 'show'.

Accept long division+conclusion

(c) Solve f(x) = 0. [4]

Long Division / Selective Expansion [M1]

$(x+1)(3x^2 - 5x + 1) = 0$ [M1]

$x = -1$ or $x = \dfrac{5 \pm \sqrt{13}}{6}$ [M1]

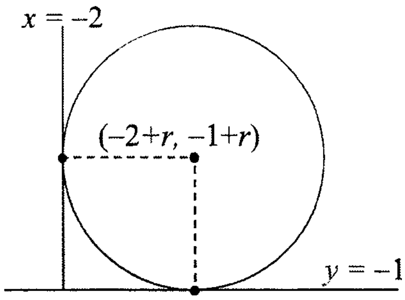
$x = -1$ or 0.232 or 1.43 [A1] : Accept exact surd form.

Penalise for missing method to solve quadratic equation.

6 It is given that $x = -2$ and $y = -1$ are tangents to a circle.
The x-coordinate and y-coordinate of the centre of the circle are positive.
The line $3y = 2x + 5$ is a normal to the circle.

(a) Show that the centre of the circle is (2, 3). [4]

Let radius be r.



Centre = (–2+r, –1+r) [M1]

$3(-1+r) = 2(-2+r) + 5$ ----------------------② tangent ⊥ radius [M1]

Solve r [M1]

Substitute back to find centre = (2, 3) [A1]

Alternative method:

Let centre be (a,b). Simultaneous equations centre/radius relationship equation with normal equation.

(b) Find the equation of the circle. [2]

$(x-2)^2 + (y-3)^2 = 16$ [B1, B1 for 16]

7 The height of a coin from the ground, h meters, after it has been flipped in the air for t seconds, can be represented by $h = -6t^2 + 24t + 12$.

(a) By completing the square, find the greatest height which the coin reaches. [3]

$h = -6(t^2 - 4t - 2)$

$= -6\left(t^2 - 4t + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - 2\right)$ [M1]

$= 36 - 6(t-2)^2$ [A1]

Greatest $h = 36$ m [A1]

(b) Find the exact duration of the coin from the time it is flipped in the air till it lands on the ground. [3]

$36 - 6(t-2)^2 = 0$ [M1]

$t = \sqrt{6} + 2$ or $-\sqrt{6} + 2$(rejected) [M1]

Duration $= \left(\sqrt{6} + 2\right)$s [A1]

8 A pot of melted chocolate is cooled from its initial temperature to a temperature of T °C in x minutes and follows the equation of the form $T = A(B^x)$, where A and B are constants. The freezing point of the chocolate is 17°C. The table below shows the corresponding values of T and x recorded.

x	5	10	15	20	25
T	35.429	20.921	12.353	7.295	4.307

(a) Draw the graph of lg T plotted against x, using a scale of 2 cm for 5 unit on the x-axis and a scale of 1 cm for 0.1 unit on the lg T axis. [3]

x	5	10	15	20	25
lg T	1.55	1.32	1.09	0.86	0.63

B1: table
G1: axes
G1: plot and straight line

(b) Using your graph, state whether the chocolate is frozen at 13 minutes. Justify your answer. [2]

Possible solution 1

At $x = 13$, lg $T = 1.18$, $T = 15.1$°C< 17°C. [M1]

Therefore, it has frozen. [A1]

Possible solution 2

If $T = 17$°C, lg $T = 1.23$. $x = 11.75$ minutes < 13 minutes. [M1]

Therefore, it has frozen. [A1]

(c) estimate the value of each of the constants A and B, [5]

$\lg T = x \lg B + \lg A$ [M1]

$\lg A = 1.78$ (± 0.01) [M1]

$A = 60.3$ (± 2) [A1]

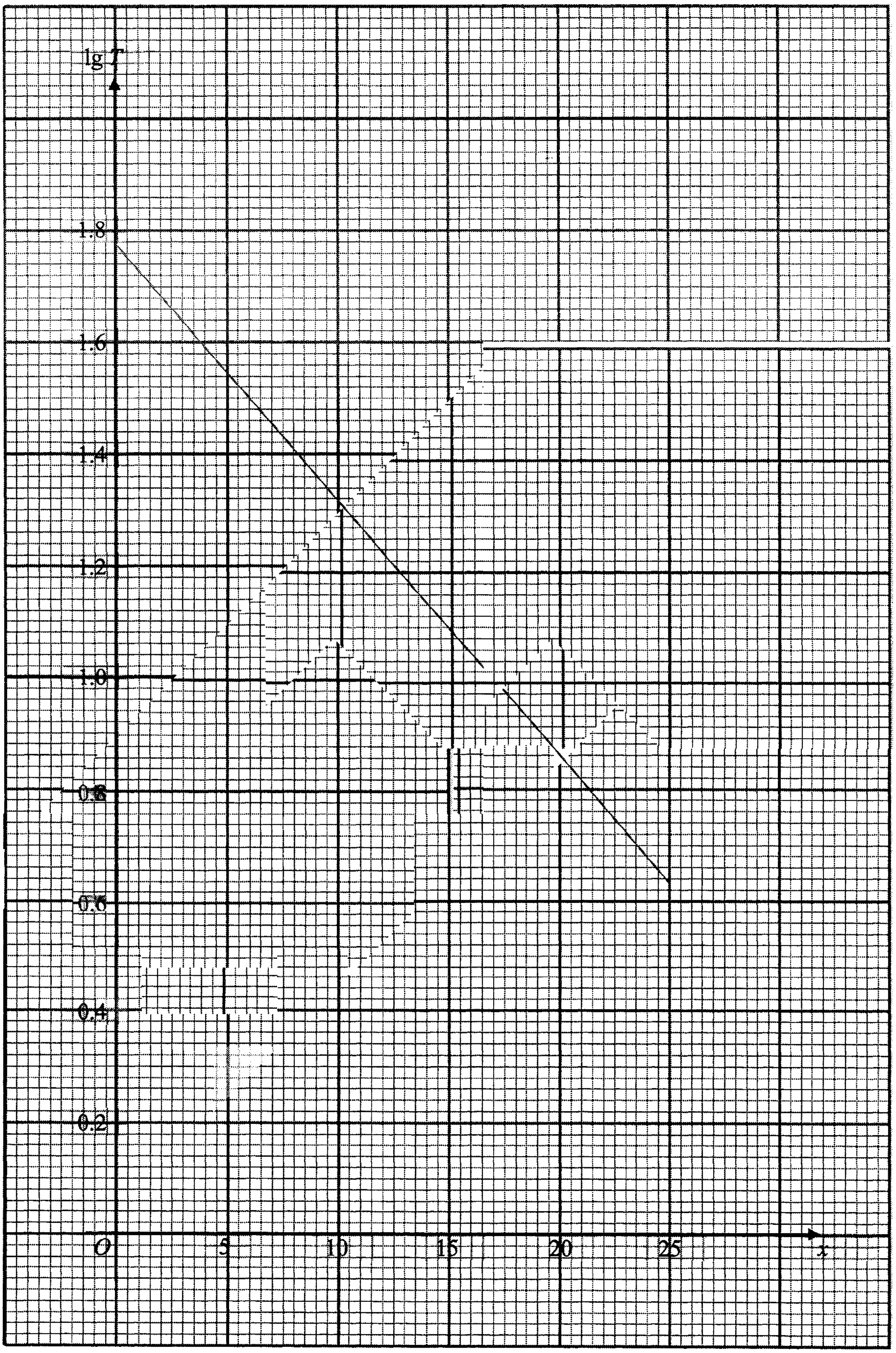
$\lg B$ = gradient = -0.046 [M1]

$B = 0.9$ (± 0.01) [A1]

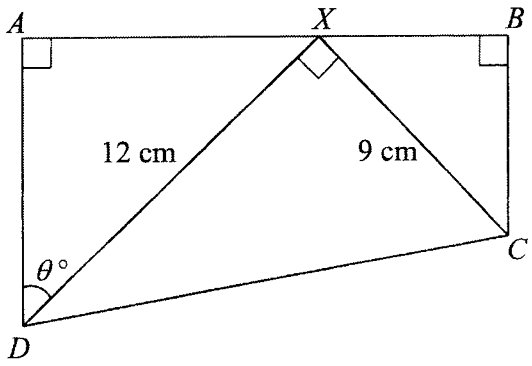
(d) find the amount of time taken for the chocolate to cool to half its original temperature. [2]

$\frac{A}{2} = 30.12798$ [M1]

Read at $\lg T = 1.48(\pm 0.01)$, time = 6.5 minutes (± 0.25) [A1]


lg T
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
O
5
10
15
20
25
x

9 The diagram shows a trapezium $ABCD$. The point X lies on line AB such that $DX = 12$ cm and $CX = 9$ cm. $\angle ADX = \theta°$ and $\angle DAX = \angle DXC = \angle XBC = 90°$.



(a) Show that $AB = 9\cos\theta + 12\sin\theta$. [2]

$\frac{AX}{12} = \sin\theta$

$\frac{BX}{9} = \cos\theta$ [M1] CAH, SOH

$\therefore AB = AX + BX$

$= 12\sin\theta + 9\cos\theta$ [A1]

(b) Express AB in the form $R\cos(\theta - \alpha)$, where R and α are constants, and hence state the maximum length of AB and its corresponding value of θ. [5]

$\therefore AB$

$= \sqrt{9^2 + 12^2}\cos(\theta - \tan^{-1}\frac{12}{9})$ [M1],[M1]

$= 15\cos(\theta - 53.1°)$ [A1]

Max $AB = 15$ m [B1]

$\theta = 53.1°$ [B1]

9(c) Find the value of θ for which $AB = 11$ cm. [3]

$15\cos(\theta - \tan^{-1}\frac{12}{9}) = 11$ [M1]

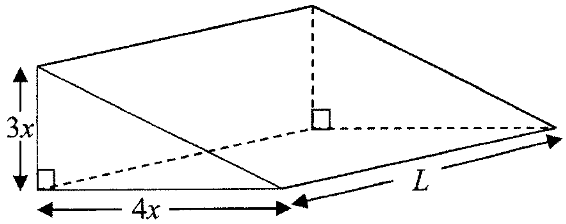
$\cos(\theta - \tan^{-1}\frac{12}{9}) = \frac{11}{15}$

$\alpha = 42.83343°$ [M1]

$\theta - \tan^{-1}\frac{12}{9} = \alpha$ or $360 - \alpha$

$\theta = 96.0°$ (rejected) or $10.3°$ (1dp) [A1]

10 The figure below shows a right-angled triangular prism. The height and base of the triangular faces of the prism are $3x$ meters and $4x$ meters respectively. The length of the prism is L meters.



(a) Given that the volume of the slope is 240 m^3, show that the surface area of the prism, A m^2, is given by

$$A = 12x^2 + \frac{480}{x}.$$ [4]

$\frac{1}{2}(3x)(4x)L = 240$ [M1] form SA equation

$L = \frac{40}{x^2}$ [M1] L the subject

Area $= (3x)(4x) + 5xL + 3xL + 4xL$ [M1] Surface Area

$= 12x^2 + 12x(\frac{40}{x^2})$ [A1] Sub,Simplify

$= 12x^2 + \frac{480}{x}$

(b) Given that x and L can vary, find the value of x for which A has a stationary value and determine whether this value of A is maximum or minimum. [5]

$\frac{dA}{dx} = 24x - \frac{480}{x^2}$ [M1]

Solve $\frac{dA}{dx} = 0$ [M1]

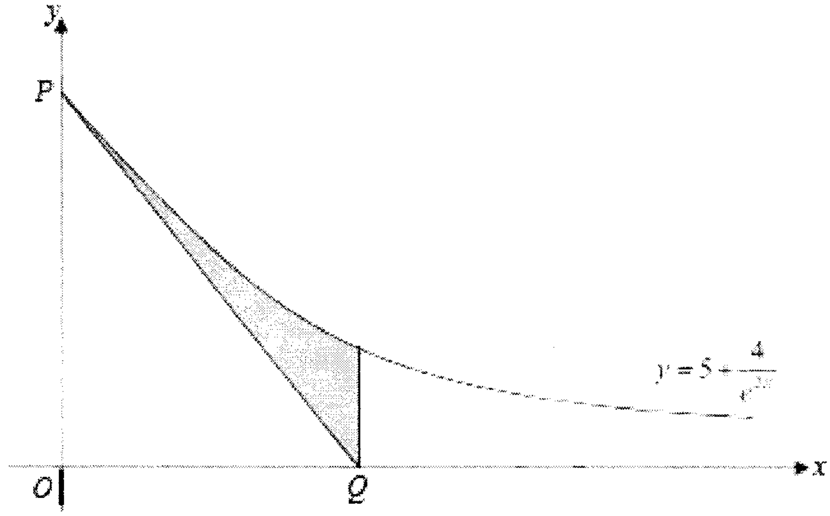
$x = 2.71$ (3sf) [A1]

$\frac{d^2A}{dx^2} = 24 + \frac{960}{x^3}$ [M1]

Since $x > 0$, $\frac{d^2A}{dx^2} > 0$ and surface area is minimum. [A1]

**can substitute x = 2.71 to show too*

11



The diagram shows part of the curve $y = 5 + \frac{4}{e^{2x}}$ intersecting the y-axis at point P.

The tangent to the curve at point P intersects the x-axis at Q.

Find the area of the shaded region. [9]

$\frac{dy}{dx} = -8e^{-2x}$ [M1]

At $P(0, 9)$, gradient $= -8$ [M1]

$y = -8x + 9$ [M1] Tangent PQ

$Q\left(\frac{9}{8}, 0\right)$ [A1]

Area under graph

$= \int_0^{9/8} 5 - 4e^{-2x}\, dx - \frac{1}{2}(9)\left(\frac{9}{8}\right)$ [M1, M1] Integrate, use triangle/ subtract functions

$= \left[5x - 2e^{-2x}\right]_0^{9/8} - \frac{81}{16}$ [M1]

$= \left(\frac{45}{8} - 2e^{-2(9/8)}\right) - (-2e^0) - \frac{81}{16}$ [M1]

$= 2.35$ units^2 [A1]