Secondary 4E Prelims 2022 Add Math Paper 1 (Solution)

Q.	Solution	Remarks
1	$x+2y=3 \rightarrow x=3-2y$ — (1) $x^2+12x+5y^2=18$ — (2) Subst (1) into (2), $(3-2y)^2+12(3-2y)+5y^2=18$ M1 $9-12y+4y^2+36-24y+5y^2-18=0$ $9y^2-36y+27=0$ M1 $y^2-4y+3=0$ $(y-3)(y-1)=0$ $y=3$ or $y=1$ M1 $x=3-2(3)=-3$ $x=3-2(1)=1$ Coordinates are (−3 , 3) and (1 , 1) A1	
2	$2x^2-3x-11$ $=2\left(x^2-\frac{3}{2}x-\frac{11}{2}\right)$ or $2\left(x^2-\frac{3}{2}x\right)-11$ M1 $=2\left[\left(x-\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2-\frac{11}{2}\right]$ or $2\left[\left(x-\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2\right]-11$ $=2\left[\left(x-\frac{3}{4}\right)^2-\frac{97}{16}\right]$ or $2\left[\left(x-\frac{3}{4}\right)^2-\frac{9}{16}\right]-11$ M1 $=2\left(x-\frac{3}{4}\right)^2-\frac{97}{8}$ or $2(x-0.75)^2-12.125$ A1 Turning point $=\left(\frac{3}{4}, -\frac{97}{8}\right)$ or $(0.75, -12.125)$ B1	(allow ecf)
3(a)	Long division: $10x+0$ divided by $-5x+3$ gives quotient -2; $10x-6$; remainder 6 M1 $\frac{10x}{3-5x}=-2+\frac{6}{3-5x}$ (using long division) A1 OR $\frac{10x}{3-5x}=\frac{6-(6-10x)}{3-5x}$ M1 $=\frac{6}{3-5x}-\frac{2(3-5x)}{3-5x}=-2+\frac{6}{3-5x}$ A1	

3(b)	OR $\frac{10x}{3-5x} = a+\frac{b}{3-5x} = \frac{a(3-5x)+b}{3-5x}$ $\frac{10x}{3-5x} = \frac{-5ax+3a+b}{3-5x}$ Comparing coeff of x, $10=-5a \rightarrow a=-2$ $3a+b=0 \rightarrow b=6$ $\frac{10x}{3-5x} = -2+\frac{6}{3-5x}$ $\int \frac{10x}{3-5x}\,dx = \int\left(-2+\frac{6}{3-5x}\right)dx$ $= -2x-\frac{6\ln(3-5x)}{5}+c$ (minus 1m if $+c$ is missing)	 M1 A1 B1 for each term	(allow ecf)
4	Max: $c+A=5$ -- (1) , where A is the amplitude Min: $c-A=1$ -- (2) (1) + (2), $2c=6 \rightarrow c=3$ $A=5-3=2$ Since it is a negative sine graph, $a=-2$ Period = 8π $\frac{2\pi}{\left(\frac{1}{b}\right)}=8\pi$ $b=\frac{8\pi}{2\pi}=4$	 B1 M1 A1 M1 A1	
5(a)	$V=\frac{4}{3}\pi r^3$ $\frac{dV}{dr}=4\pi r^2$ $\frac{dr}{dt}=\frac{dr}{dV}\times\frac{dV}{dt}$ $=\frac{1}{4\pi(4)^2}\times 12$ $=0.0597$ cm per second	 M1 M1 A1	

5(b)	$\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dt}$ $=4\pi(7)^2\times0.015$ $=9.236\text{ cm}^3$ per second Hence, air is leaking at a rate of $12-9.236=2.76$ cm^3 per second. OR $12-x=4\pi(7)^2\times0.015$ $x=12-9.236=2.76$ cm^3 per second.	 M1 A1 M1 A1	
6	$\frac{14x^2-15x-29}{(x-1)^2(2x+3)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(2x+3)}$ $14x^2-15x-29=A(2x+3)(x-1)+B(2x+3)+C(x-1)^2$ Let $x=1$, $14-15-29=B(2+3)\rightarrow B=-6$ Let $x=-1.5$, $14(-1.5)^2-15(-1.5)-29=C(-1.5-1)^2$ $25=6.25C\quad\rightarrow C=4$ Let $x=0$, $-29=A(3)(-1)+B(3)+C(-1)^2$ $-29-(-6)(3)-(4)(-1)^2=-3A\rightarrow A=5$ $\frac{14x^2-15x-29}{(x-1)^2(2x+3)}=\frac{5}{(x-1)}-\frac{6}{(x-1)^2}+\frac{4}{(2x+3)}$ OR $\frac{14x^2-15x-29}{(x-1)^2(2x+3)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(2x+3)}$ $14x^2-15x-29=A(2x+3)(x-1)+B(2x+3)+C(x-1)^2$ $=2Ax^2+Ax-3A+2Bx+3B+Cx^2-2Cx+C$ Comparing coeff of x^2: $2A+C=14\rightarrow C=14-2A$ -- (1) x: $A+2B-2C=-15$ -- (2) const: $-3A+3B+C=-29$ -- (3)	M1 M1 A1 M1 A1 A1 M1 M1	

	Subst (1) into (2) and (3), $A+2B-2(14-2A)=-15 \rightarrow 5A+2B=13$ -- (4) $-3A+3B+(14-2A)=-29 \rightarrow -5A+3B=-43$ -- (5) (4) + (5) $5B=-30 \quad \rightarrow B=-6$ $5A+2(-6)=13 \rightarrow A=5$ $C=14-2(5) \rightarrow C=4$ $\frac{14x^2-15x-29}{(x-1)^2(2x+3)}=\frac{5}{(x-1)}-\frac{6}{(x-1)^2}+\frac{4}{(2x+3)}$	M1 A1 A1 A1	
7(a)	Let $x=0.5$, Remainder = $4(0.5)^3-6(0.5)^2+k(0.5)+3$ $14=0.5k+2$ $k=24$	 M1 A1	
7(b)	$4x^3-6x^2+2x+3=(2x^2+px+3)(ax+b)$ Comparing coeff of x^3, $4=2a \rightarrow a=2$ Comparing constant, $3=3b \rightarrow b=1$ OR $4x^3-6x^2+2x+3=(2x^2+px+3)(2x+1)$ Comparing coeff of x OR coeff of x^2 $2=p+6$ $-6=2+2p$ $p=-4$ $p=-4$ OR (using long division) $$\begin{array}{r} 2x+(-3-p) \\ (2x^2+px+3)\overline{)4x^3-6x^2+2x+3} \\ \underline{4x^3+2px^2+6x} \\ (-6-2p)x^2-4x+3 \\ \underline{(-6-2p)x^2+(-3p-p^2)x+(-9-3p)} \\ (p^2+3p-4)x+(12+3p) \end{array}$$ Since $(2x^2+px+3)$ is a factor, remainder = $0x+0$ Hence, $p^2+3p-4=0$ AND $12+3p=0$ $p=-4,\ p=1$ $p=-4$ $\rightarrow p=-4$	M1 A1 M2 M1 A1 M1 (quotient) A1 (remainder) M1 A1	

8(a)	$3x=\sqrt{1-x}-1$ $3x+1=\sqrt{1-x}$ $(3x+1)^2=1-x$ M1 $9x^2+6x+1-1+x=0$ $9x^2+7x=0$ $x(9x+7)=0$ $x=0$ or $x=-\frac{7}{9}$ (reject) A1, A1	
8(b)	Area of triangle = $\frac{1}{2}\times b\times h$ $14+8\sqrt{3}=\frac{(1+\sqrt{3})h}{2}$ $h=\frac{2(14+8\sqrt{3})}{1+\sqrt{3}}$ M1 $=\frac{28+16\sqrt{3}}{1+\sqrt{3}}\ \frac{\times(1-\sqrt{3})}{\times(1-\sqrt{3})}$ M1 $=\frac{28-28\sqrt{3}+16\sqrt{3}-16(3)}{1^2-3}$ M1 $=\frac{-20-12\sqrt{3}}{-2}$ $=(10+6\sqrt{3})$ cm A1	
9(a)	Gradient of OP = $=\frac{6-0}{-2-0}=-3$ Gradient of perp bisector = $\frac{1}{3}$ M1 Midpoint $M=\left(\frac{-2+0}{2},\frac{6+0}{2}\right)=(-1,3)$ M1 $y=\frac{1}{3}x+c$ Subst $(-1,3)$ into equation, $3=\frac{1}{3}(-1)+c\ \rightarrow\ c=\frac{10}{3}$ Equation of perp bisector is $y=\frac{1}{3}x+\frac{10}{3}$ A1	

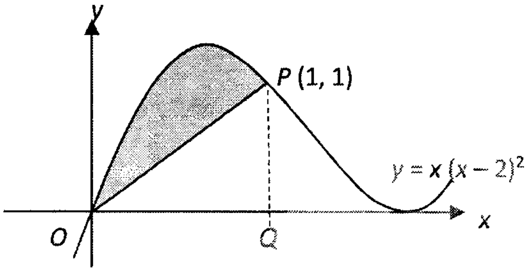
9(b)	Let $y=0$, $0=\frac{1}{3}x+\frac{10}{3}$ $x=-10$ $\quad R(-10,0)$ Since R lies on the perp bisector, $MPQR$ is a rectangle. $M(-1,3) \rightarrow P(-2,6)$: $x-1, y+3$ $R(-10,0) \rightarrow Q(-10-1, 0+3)$ $Q(-11,3)$ OR Using point P (–2, 6) and R (–10, 0) Midpoint of diagonals $=\left(\frac{-2-10}{2},\frac{6+0}{2}\right) = (-6,3)$ Using point Q (x, y) and M (–1, 3) $(-6,3) = \left(\frac{x-1}{2},\frac{y+3}{2}\right)$ $x=-11, y=3 \rightarrow Q(-11,3)$ OR Equation of PQ: $y=\frac{1}{3}x+c$ Subst $P(-2,6)$ into equation, $6=\frac{1}{3}(-2)+c \rightarrow c=\frac{20}{3}$ Equation is $y=\frac{1}{3}x+\frac{20}{3}$ -- (1) Equation of RQ: $y=-3x+c$ Subst $R(-10,0)$ into equation, $0=-3(-10)+c \rightarrow c=-30$ Equation is $y=-3x-30$ -- (2) Solving simultaneous equation, $x=-11, y=3 \rightarrow Q(-11,3)$	B1 M1 A1 M1 A1 M1 A1	

9(c)	Area = $\frac{1}{2}\begin{vmatrix} 0 & -2 & -11 & -10 & 0 \\ 0 & 6 & 3 & 0 & 0 \end{vmatrix}$ $= \frac{1}{2}[0+(-6)+0+0-0-(-66)-(-30)-0]$ M1 $= 45$ units^2 A1 OR Length of $OP = \sqrt{(-2-0)^2+(6-0)^2} = \sqrt{40}$ Length of $QR = \sqrt{(-11+10)^2+(3-0)^2} = \sqrt{10}$ Length of $PQ = \sqrt{(-11+2)^2+(3-6)^2} = \sqrt{90}$ M1 Area of trapezium $= \frac{1}{2}(\sqrt{40}+\sqrt{10})(\sqrt{90})$ $= 45$ units^2 A1 OR Area = Area of parallelogram $OMQR$ + Area of triangle MPQ $= (OR\times$perp ht$) + (\frac{1}{2}\times MQ\times$perp ht$)$ $= (10\times 3) + (\frac{1}{2}\times 10\times 3)$ M1 $= 45$ units^2 A1	(allow ecf) (allow ecf)
10(a)	$y = 2x^3+6x^2-18x+5$ $\frac{dy}{dx} = 6x^2+12x-18$ M1 Let $\frac{dy}{dx}=0$, $6x^2+12x-18=0$ M1 $x^2+2x-3=0$ $x=1$ or $x=-3$ $y=2(1)^3+6(1)^2-18(1)+5$ $y=2(-3)^3+6(-3)^2-18(-3)+5$ $y=-5$ $y=59$ A1 $x=1$: x: $(1)^-$, (1), $(1)^+$; $\frac{dy}{dx}$: -ve, 0, +ve (1, –5) Min point (A1) $x=-3$: x: $(-3)^-$, (-3), $(-3)^+$; $\frac{dy}{dx}$: +ve, 0, -ve (–3, 59) Max point (A1)	

	OR $\frac{d^2y}{dx^2}=12x+12$ when $x=1$ $\frac{d^2y}{dx^2}=24>0$ → (1, –5) Min point when $x=-3$ $\frac{d^2y}{dx^2}=-24<0$ → (–3, 59) Max point	 A1 A1	
10(b)	gradient, $m=\frac{dy}{dx}=6x^2+12x-18$ $\frac{dm}{dx}=\frac{d^2y}{dx^2}=12x+12$ Let $\frac{dm}{dx}=0$ $12x+12=0$ $x=-1$ $\frac{d^2m}{dx^2}=12>0$ (minimum gradient) Hence, minimum gradient = $6(-1)^2+12(-1)-18=-24$	 M1 A1 A1	
11(a)	angle *BAF* = angle *FAC* (common angle) angle *AFB* = angle *ACF* (alternate segment theorem) hence, triangles *ABF* and *AFC* are similar (AA property)	M1 M1 A1	
11(b)	Using similar triangles *ABF* and *AFC*, $\frac{AF}{AB}=\frac{AC}{AF}$ → $AF^2=AB\times AC$	 B1	
11(ci)	triangle *BEF* (must be in corresponding order)	B1	
(c)(ii)	Using similar triangles *DEC* and *BEF*, $\frac{ED}{EB}=\frac{EC}{EF}$ $ED\times EF=EC\times EB$ Given that $3EC=2EB$, $\frac{EC}{EB}=\frac{2}{3}$ → $\frac{EC}{BC}=\frac{2}{5}$ and $\frac{EB}{BC}=\frac{3}{5}$	 M1 M1	

	Hence, $\left(\frac{2}{5}BC\right)\times\left(\frac{3}{5}BC\right)=EF\times ED$ $\frac{6}{25}(BC)^2=EF\times ED \rightarrow k=\frac{6}{25}$	 A1	
12(a)	$3^{2x-1}=2^{3-x}$ $\frac{3^{2x}}{3^1}=\frac{2^3}{2^x}$ $9^x\times 2^x=8\times 3$ $18^x=24$ $x=\log_{18}24$ $x=\frac{\lg 24}{\lg 18}$	 M1 M1 A1	
12(b)	$\log_6 4y=\log_6 3x^2-1$ $\log_6 4y=\log_6 3x^2-\log_6 6$ $\log_6 4y=\log_6\left(\frac{3x^2}{6}\right)$ $4y=\frac{x^2}{2}$ $y=\frac{x^2}{8}$ OR $\log_6 4y=\log_6 3x^2-1$ $\log_6 4y-\log_6 3x^2=-1$ $\log_6\left(\frac{4y}{3x^2}\right)=-1$ $\frac{4y}{3x^2}=6^{-1}$ $y=\frac{3x^2}{4\times 6}$ $y=\frac{x^2}{8}$	 M1 A1 M1 A1	
12(c)	$\log_2 x^2=8\log_x 2$ $2\log_2 x=\frac{8\log_2 2}{\log_2 x}$		

	$2\log_2 x = \frac{8}{\log_2 x}$	M1	
	Let $u = \log_2 x$, $2u = \frac{8}{u}$ $u^2 = 4$		
	$u = 2$ or $u = -2$	M1	
	$\log_2 x = 2$ $\log_2 x = -2$ $x = 2^2 = 4$ $x = 2^{-2} = 0.25$	A1 (for both ans)	
13(a)	Gradient of $OP = \frac{1-0}{1-0} = 1$	M1	
	$y = x + c$ Subst $P(1,\ 1)$ into equation, $1 = 1 + c \rightarrow c = 0$		
	Equation of OP is $y = x$ (shown)	A1	
13(b)	$y = x(x-2)^2$ $= x(x^2 - 4x + 4)$ $= x^3 - 4x^2 + 4x$		
	$\frac{dy}{dx} = 3x^2 - 8x + 4$	M1	
	OR $u = x$ $v = (x-2)^2$ $\frac{du}{dx} = 1$ $\frac{dv}{dx} = 2(x-2)^1 \times 1 = 2x - 4$ $\frac{dy}{dx} = (x-2)^2(1) + (x)(2x-4)$ $= x^2 - 4x + 4 + 2x^2 - 4x$		
	$= 3x^2 - 8x + 4$	M1	
	when $x = 1$ (point P) $\frac{dy}{dx} = 3(1)^2 - 8(1) + 4 = -1$ Gradient of normal $= \frac{-1}{-1} = 1$ = gradient of OP		
	Hence, OP is the normal to the curve at P.	A1	

13(c)	 Area under curve $= \int_0^1 x(x-2)^2\,dx$ M1 $= \int_0^1 (x^3-4x^2+4x)\,dx$ $= \left[\frac{x^4}{4}-\frac{4x^3}{3}+\frac{4x^2}{2}\right]_0^1$ $= \left[\frac{(1)^4}{4}-\frac{4(1)^3}{3}+2(1)^2\right]-[0]$ $= \frac{11}{12}$ units^2 A1 Area of triangle $OPQ = \frac{1}{2}\times1\times1=\frac{1}{2}$ units^2 B1 Shaded area $= \frac{11}{12}-\frac{1}{2}=\frac{5}{12}$ units^2 (or 0.417 units^2) A1 OR Area under curve $= \int_0^1\left[x(x-2)^2-x\right]dx$ M1, M1 $= \int_0^1 (x^3-4x^2+4x-x)\,dx$ $= \int_0^1 (x^3-4x^2+3x)\,dx$ $= \left[\frac{x^4}{4}-\frac{4x^3}{3}+\frac{3x^2}{2}\right]_0^1$ M1 $= \left[\frac{(1)^4}{4}-\frac{4(1)^3}{3}+\frac{3(1)^2}{2}\right]-[0]$ $= \frac{5}{12}$ units^2 A1	
14(a)	$(\text{cosec}\,\theta+\cot\theta)^2=\frac{1+\cos\theta}{1-\cos\theta}$ LHS $=(\text{cosec}\,\theta+\cot\theta)^2$ $=\left(\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}\right)^2$ M1	

	$=\left(\dfrac{1+\cos\theta}{\sin\theta}\right)^2$ $=\dfrac{(1+\cos\theta)^2}{\sin^2\theta}$ $=\dfrac{(1+\cos\theta)^2}{1-\cos^2\theta}$ $=\dfrac{(1+\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$ $=\dfrac{1+\cos\theta}{1-\cos\theta}$ = RHS (proved)	 M1 M1 A1
	OR LHS $=(\text{cosec}\,\theta+\cot\theta)^2$ $=\text{cosec}^2\theta+2\text{cosec}\,\theta\cot\theta+\cot^2\theta$ $=\dfrac{1}{\sin^2\theta}+\left(\dfrac{2}{\sin\theta}\right)\left(\dfrac{\cos\theta}{\sin\theta}\right)+\dfrac{\cos^2\theta}{\sin^2\theta}$ $=\dfrac{1+2\cos\theta+\cos^2\theta}{\sin^2\theta}$ $=\dfrac{(1+\cos\theta)^2}{1-\cos^2\theta}$ $=\dfrac{(1+\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$ $=\dfrac{1+\cos\theta}{1-\cos\theta}$ = RHS (proved)	 M1 M1 M1 A1
14(b)	$(\text{cosec}2\theta+\cot 2\theta)^2(1-\cos 2\theta)=2\cos\theta$ $1+\cos 2\theta=2\cos\theta$ $1+2\cos^2\theta-1=2\cos\theta$ $2\cos^2\theta-2\cos\theta=0$ $\cos\theta(\cos\theta-1)=0$ $\cos\theta=0$ or $\cos\theta=1$ $\alpha=\cos^{-1}0=90°$; $\alpha=\cos^{-1}1=0°$ $\theta=90°$ or $\theta=360°-90°$; $\theta=0°$ or $\theta=360°-0°$ $\theta=90°, 270°$ (A1) ; $\theta=0°, 360°$ (A1)	 M1 M1 M1

	2022 Prelim 4E AMath Paper 2 Solutions	
1	$3^{2y+1}-3^{y+2}+3=3^{y}$ Let $u=3^{y}$ $3u^{2}-9u+3=u$ $u=3, u=\frac{1}{3}$ $y=1, y=-1$	 M1 M1 A2[4]
2	$\mathrm{f}(x)=2x^{3}+5x^{2}+x-2$ $\mathrm{f}(-2)=0$ $x+2$ is a factor <u>Method 1</u> $2x^{3}+5x^{2}+x-2=(x+2)(ax^{2}+bx+c)$ Equating coeff of x^{3}, $a=2$ Equating constants, $c=-1$ Equating coeff of x, $c+2b=1$ $b=1$ OR Equating coeff of x^{2}, $4+b=5$ $b=1$ $2x^{3}+5x^{2}+x-2=(x+2)(2x^{2}+x-1)$ $2x^{3}+5x^{2}+x-2=(x+2)(2x-1)(x+1)$ $(x+2)(2x-1)(x+1)=0$ $x=-2,-1,\frac{1}{2}$ <u>Method 2</u> Show long division $2x^{3}+5x^{2}+x-2=(x+2)(2x^{2}+x-1)$ $2x^{3}+5x^{2}+x-2=(x+2)(2x-1)(x+1)$ $(x+2)(2x-1)(x+1)=0$ $x=-2,-1,\frac{1}{2}$	 B1 M1 M1 M1 A1[5] M1 M1 M1 A1[5]

	Method 3 Show synthetic division $2x^3+5x^2+x-2=(x+2)(2x^2+x-1)$ $2x^3+5x^2+x-2=(x+2)(2x-1)(x+1)$ $(x+2)(2x-1)(x+1)=0$ $x=-2,-1,\frac{1}{2}$	M1 M1 M1 A1[5]

3	(a)	Let $y = 4xe^{3x}$	
		$\frac{dy}{dx} = 4xe^{3x}(3) + e^{3x}(4)$	M2 (for each term)
		$= 12xe^{3x} + 4e^{3x}$	A1
	(b)	$\int 12xe^{3x} + 4e^{3x}dx = 4xe^{3x}$	M1
		$\int 12xe^{3x}dx = \int -4e^{3x}dx + 4xe^{3x}$	
		$\int 6xe^{3x}dx = \frac{\int -4e^{3x}dx + 4xe^{3x}}{2}$	
		$\int_0^3 6xe^{3x}dx = \frac{\int_0^3 -4e^{3x}dx + \left[4xe^{3x}\right]_0^3}{2}$	M1
		$= \frac{\left[\frac{-4}{3}e^{3x}\right]_0^3 + \left[4xe^{3x}\right]_0^3}{2}$	M1
		$= \frac{\frac{-4}{3}e^9 + \frac{4}{3} + 12e^9}{2}$	
		$= \frac{2}{3} + \frac{16}{3}e^9$	A1[7]

4	(a)	$T_{r+1}=\binom{12}{r}(x^2)^{12-r}\left(-\frac{m}{2x}\right)^r$	
		$=\binom{12}{r}\left(-\frac{m}{2}\right)^r x^{24-2r-r}$	M1
		$0=24-3r$	
		$r=8$	A1
		$\binom{12}{8}\left(-\frac{m}{2}\right)^8=126720$	M1
		$m^8=65536$	
		$m=4$	A1
	(b)	For $\left(x^2-\frac{m}{2x}\right)^{12}(8x^9+5)$,	
		term in $x^9=(8x^9)(126720)+5(?\,x^9)$	
		For $\left(x^2-\frac{m}{2x}\right)^{12}$, term in x^9	
		$24-3r=9$	
		$r=5$	B1
		$\binom{12}{5}\left(-\frac{4}{2}\right)^5 x^9=-25344$	
		Term in $x^9=(8x^9)(126720)+5(-25344x^9)$	M1
		coefficient of $x^9=887040$	A1[7]

5	**(a)**	$\frac{d}{dx}\left(x\sin x+\cos x\right)$ $=x\cos x+\sin x-\sin x$ $=x\cos x$	M2 A1
	(b)	$0=x\cos x$ $x=0,\cos x=0$ $A(\frac{\pi}{2},0)$	M1 M1 A1
	(c)	$\int_0^{\frac{\pi}{2}} x\cos x \, dx$ $=\left[x\sin x+\cos x\right]_0^{\frac{\pi}{2}}$ $=\left(\frac{\pi}{2}-1\right)units^2$	M1 A1

6	(a) (i)	$2x^2 - 7x + k$ $b^2 - 4ac < 0$ $(-7)^2 - 4(2)(k) < 0$ $k > \frac{49}{8}$	M2 A1
	(ii)	$\frac{(3x-2)(x-4)}{2x^2 - 7x + 8} < 0$ $2x^2 - 7x + 8 > 0$ $(3x-2)(x-4) < 0$ $\frac{2}{3} < x < 4$	M1 M1 A1
	(b) (i)	$px^2 + x + p(x+1) = 0$ $px^2 + (1+p)x + p = 0$ $b^2 - 4ac > 0$ $(1+p)^2 - 4(p)(p) > 0$ $1 + 2p - 3p^2 > 0$ $-\frac{1}{3} < p < 1$	M1 M1 A1
	(ii)	$b^2 - 4ac = 0$ $1 + 2p - 3p^2 = 0$ $p = 1, -\frac{1}{3}$	B1[10]

7	(a)	Appropriate scale and correct axes used All points plotted correctly Best fit line	P1 P1 P1
	(b)	$\frac{1}{a} = 0.1(\pm 0.01)$ $a = 10$ $\frac{b}{a} = \frac{1.0 - 0.1}{0.8 - 0}(\pm 1)$ $b = \frac{90}{8}$ or 11.25	 B1 M1 A1
	(c)	$\frac{a}{y} = \frac{b}{x} + 1$ $\frac{xa}{y} = b + x$ $\frac{x}{y} = \frac{1}{a}x + \frac{b}{a}$ vertical axis= $\frac{x}{y}$	 M1 A1
	(d)	$\frac{a}{y} = \frac{b}{x} + 1$ $\frac{xa}{y} = b + x$ $\frac{x}{y} = \frac{1}{a}x + \frac{b}{a}$ $a = \frac{1}{gradient}$ $b = (\frac{x}{y}\text{ intercept})(\frac{1}{\text{gradient}})$	 B1 B1[10]

8	**(a)**	$L = VY + YX + XW + VW$ $= 12 + 20\cos\theta + (32\sin\theta - 20\sin\theta) + 32\cos\theta$ $= 12 + 52\cos\theta + 12\sin\theta$ $= 12\sin\theta + 52\cos\theta + 12(shown)$	M1 – for YX or VW M1 – for XW A1
	(b)	$12\sin\theta + 52\cos\theta = R\sin(\theta + \alpha)$ $R = \sqrt{52^2 + 12^2}$ $= 53.367$ $\theta = \tan^{-1}\frac{52}{12}$ $= 77.005$ $L = 12 + 53.4\sin(\theta + 77.0^o)$	B1 B1 B1
	(c)	$L = 12 + 53.367$ $= 65.367$ max value $L = 65.4$ $65.367 = 12 + 53.367\sin(\theta + 77.0^o)$ $1 = \sin(\theta + 77.0^o)$ $\theta = \sin^{-1}1 - 77.005^o$ $= 12.995$ $= 13.0^o$	B1 M1 A1
	(d)	$30 = 12 + 53.367\sin(\theta + 77.005^o)$ $0.33729 = \sin(\theta + 77.005^o)$ $\alpha = 19.712^o$ $\theta + 77.005^o = 19.712^o, 160.29^o$ $\theta = -57.3^o(rejected), 83.3^o$	M1 A1 A1[12]

9	(a)	$\frac{3}{t+1}-\frac{t+1}{3}=0$ $\frac{3}{t+1}=\frac{t+1}{3}$ $(t+1)^2=9$ $t+1=3,-3$ $t=2,-4(rejected)$	M1 A2
	(b)	$s=\int\frac{3}{t+1}-\frac{t+1}{3}\,dt$ $=3\ln(t+1)-\frac{(t+1)^2}{3(2)}+c$ $0=3\ln(1)-\frac{(0+1)^2}{3(2)}+c$ $c=\frac{1}{6}$ $s=3\ln(t+1)-\frac{(t+1)^2}{3(2)}+\frac{1}{6}$ $s=3\ln(2+1)-\frac{(2+1)^2}{3(2)}+\frac{1}{6}$ $=1.9625$ $=1.96m$ OR $s=\int\frac{3}{t+1}-\frac{t}{3}-\frac{1}{3}\,dt$ $=3\ln(t+1)-\frac{t^2}{6}-\frac{t}{3}+c$ $t=0,s=0$ $0=3\ln(1)-0-0+c$ $c=0$ $s=3\ln(t+1)-\frac{t^2}{6}-\frac{t}{3}$ $s=3\ln(2+1)-\frac{(2)^2}{6}-\frac{2}{3}$ $=1.96m$	M1 M1 M1 A1 A1 M1 M1 M1 A1 A1

	(c)	$s=3\ln(t+1)-\frac{(t+1)^2}{3(2)}+\frac{1}{6}$ When $t=4$, $s=3\ln(4+1)-\frac{(4+1)^2}{3(2)}+\frac{1}{6}$ $=0.82831$ When $t=5$ $s=3\ln(5+1)-\frac{(5+1)^2}{3(2)}+\frac{1}{6}$ $=-0.45805$ $\therefore$ particle is again at O at some instant during the fifth second.	M1 M1 A1
	(d)	$v=\frac{3}{t+1}-\frac{t+1}{3}$ $\frac{dv}{dt}=-3(t+1)^{-2}-\frac{1}{3}$ $=-\frac{5}{12}m/s^2$	M1 A1[13]
10	(a)	Method 1 $x^2+y^2-8x-16y+55=0.$ $(x-4)^2+(y-8)^2=-55+16+64$ $(x-4)^2+(y-8)^2=5^2$ $C(4,8)$ radius $=5$ units Method 2 $2g=-8$ $g=-4$ $2f=-16$ $f=-8$ $C(4,8)$ $r=\sqrt{f^2+g^2-c}$ $=\sqrt{(-8)^2+(-4)^2-55}$ $=5units$	M1 A1 A1 B1 M1 A1

	(b)	gradient $CA = \frac{12-8}{7-4}$ $= \frac{4}{3}$ gradient of tangent $= -\frac{3}{4}$ $12 = -\frac{3}{4}(7) + c$ $c = 17\frac{1}{4}$ $y = -\frac{3}{4}x + 17\frac{1}{4}$ *OR* $y = -\frac{3}{4}x + \frac{69}{4}$ *OR* $y = -0.75x + 17.25$	M1 M1 A1
	(c)	$0 = -\frac{3}{4}x + 17\frac{1}{4}$ $x = 23$ $B(23, 0)$	B1

	(d)	Method 1 Since $\angle CAB$ is 90° (rad $\perp$ tan) BC is the diameter of S_2 (rt $\angle$ in semicircle) D is the midpoint of $CB = \left(\frac{4+23}{2}, \frac{8+0}{2}\right)$	M1
		centre of $S_2 = D\left(\frac{27}{2}, 4\right)$	B1
		Radius of $S_2 = \sqrt{\left(23-\frac{27}{2}\right)^2 + (0-4)^2}$	M1
		$= \frac{5\sqrt{17}}{2}$ OR $\sqrt{106.25}$	A1
		Therefore, equation of circle is $\left(x-\frac{27}{2}\right)^2 + (y-4)^2 = \frac{425}{4}$ *OR* $(x-13.5)^2 + (y-4)^2 = 106.25$	A1
		Method 2 Gradient of perpendicular bisector of $AB = \frac{4}{3}$ Midpoint $AB =$ (15,6) Eqn of perpendicular bisector $6 = \frac{4}{3}(15) + c$ $y = \frac{4}{3}x - 14$	M1
		Gradient of perpendicular bisector of $CB = \frac{19}{8}$ Midpoint $CB =$ (13.5,4) Eqn of perpendicular bisector $4 = \frac{19}{8}(13.5) + c$ $y = \frac{19}{8}x - 28\frac{1}{16}$	M1
		OR Gradient of perpendicular bisector of $AC = -\frac{3}{4}$ Midpoint $AC =$ (5.5,10) Eqn of perpendicular bisector	

		$10 = -\frac{3}{4}(5.5) + c$ $y = -\frac{3}{4}x + \frac{113}{8}$	M1
		$y = \frac{4}{3}x - 14$ $y = \frac{19}{8}x - 28\frac{1}{16}$ $centre\left(\frac{27}{2}, 4\right)$	A1
		$radius = \sqrt{\left(\frac{27}{2} - 23\right)^2 + (4-0)^2}$ $= \sqrt{106.25}$ $\left(x - \frac{27}{2}\right)^2 + (y-4)^2 = 106.25$	M1 A1
	(e)	$C(4, 8)$, $radius = 5$ distance $CD = \sqrt{\left(\frac{27}{2} - 4\right)^2 + (4-8)^2}$	M1
		$= 10.308$ > 5 D lies outside	A1[14]