

1 It is given that P , Q and R are the angles of a triangle.

(a) Show that $\cos P = -\cos(Q + R)$.

[2]

$$\cos P = \cos [180^\circ - (Q + R)]$$

[M1 – for replacing]

$$= \cos 180^\circ \cos(Q + R) + \sin 180^\circ \sin(Q + R)$$

$$= -\cos(Q + R) + 0$$

[A1 – apply addition formula &

$$= -\cos(Q + R)$$

[evaluate to arrive at result]

Accept supplementary angles:

$$\cos P = -\cos(180^\circ - P) \quad \text{M1}$$

$$= -\cos(Q + R) \quad \text{A1}$$

(b) Given that $Q = 45^\circ$ and $R = 60^\circ$, find $\cos P$ in the form $\frac{1}{4}(\sqrt{a} - \sqrt{b})$,

where a and b are integers.

[3]

$$\cos P = -\cos(45^\circ + 60^\circ)$$

$$= -\cos 45^\circ \cos 60^\circ + \sin 45^\circ \sin 60^\circ$$

[M1 – correct use of formula expansion]

$$= -\frac{\sqrt{2}}{2} \left(\frac{1}{2}\right) + \frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2}\right)$$

[M1 – correct special angles trigo ratios]

$$= \frac{1}{4}(\sqrt{6} - \sqrt{2})$$

[A1]

- 2 Baking powder is poured onto a flat surface at a constant rate of $2\pi \text{ cm}^3 \text{ s}^{-1}$ and formed a right circular cone. The radius of the cone is always $\frac{1}{18}$ of its height. Find the rate of change of the radius of the cone after 3 seconds of pouring. [5]

$$\begin{aligned} \text{Vol. of cone, } V &= \frac{1}{3} \pi r^2 (18r) \\ &= 6\pi r^3 \end{aligned}$$

$$\frac{dV}{dr} = 18\pi r^2 \quad [\text{B1}]$$

After 3 seconds, $V = 6\pi$

$$6\pi r^3 = 6\pi \quad [\text{M1 - finding corresponding } r]$$

$$r = 1 \quad [\text{A1}]$$

$$\frac{dV}{dt} = \left. \frac{dV}{dr} \right|_{r=1} \times \left. \frac{dr}{dt} \right|_{r=1} \quad [\text{M1 - connected rate of change}]$$

$$2\pi = 18\pi(1)^2 \times \left. \frac{dr}{dt} \right|_{r=1}$$

$$\left. \frac{dr}{dt} \right|_{r=1} = \frac{1}{9}$$

The rate of change required is $= \frac{1}{9} \text{ cm/s.}$ [A1 o.e.]

- 3 (a) Determine the set of values of m for which the equation $2x^2 + 4x + 2m = 6mx - 2$ has real roots. [4]

$$2x^2 + (4 - 6m)x + 2m + 2 = 0$$

$$b^2 - 4ac \geq 0 \quad \text{[M1 - correct Discriminant]}$$

$$(4 - 6m)^2 - 4(2)(2m + 2) \geq 0$$

$$16 - 48m + 36m^2 - 8(2m + 2) \geq 0$$

$$36m^2 - 64m \geq 0 \quad \text{[M1 - simplification in factors]}$$

$$m(9m - 16) \geq 0$$

$$m \leq 0 \quad \text{or} \quad m \geq \frac{16}{9} \quad \text{[A2, minus 1 mark if inequality sign is wrong due to earlier wrong D sign]}$$

- (b) Hence state what can be deduced about the curve $y = 2(x + 1)^2$ and the line $y = 6x - 2$. Justify your statement. [2]

$$2x^2 + 4x + 2 = 6x - 2$$

By comparing with (a), $m = 1$ [B1 - correct m value]

When $m = 1$, it is not within the set of values of m for which there will be real roots, hence the curve will not meet the line/ the curve will not cut the line. [B1]

- 4 (a) Show that $\frac{d}{dx}(\ln(\cos x)) = -\tan x$. [2]

$$\begin{aligned} \frac{d}{dx}(\ln(\cos x)) &= \frac{1}{\cos x} \times \frac{d}{dx}(\cos x) && \text{[M1 – show working]} \\ &= \frac{-\sin x}{\cos x} = -\tan x && \text{[A1 – show fraction]} \end{aligned}$$

- (b) Differentiate $x \tan x$ with respect to x . [2]

$$\begin{aligned} \frac{d}{dx} x \tan x &= x \sec^2 x + \tan x && \text{[M1 – show product rule]} \\ &&& \text{[A1 – correct ans for both]} \end{aligned}$$

- (c) Using the results from part (a) and (b), find $\int x \sec^2 x \, dx$ and hence show that $\int_0^{\frac{\pi}{4}} x \sec^2 x \, dx = \frac{\pi}{4} - \frac{1}{2} \ln 2$. [4]

$$\begin{aligned} \text{From (b), } \int (x \sec^2 x + \tan x) \, dx &= x \tan x + C && \text{[M1 – use part (b), 'C' must be seen]} \\ \int x \sec^2 x \, dx + \int \tan x \, dx &= x \tan x + C \\ \int x \sec^2 x \, dx &= x \tan x - \int \tan x \, dx + C && \text{[M1 – proper integration and final correct ans, 'C' must be seen]} \\ &= x \tan x + \ln(\cos x) + C \end{aligned}$$

$$\int_0^{\frac{\pi}{4}} x \sec^2 x \, dx = \left[x \tan x + \ln(\cos x) \right]_0^{\frac{\pi}{4}} \quad \text{[minus 1 mark if "C" is not seen]}$$

$$= \frac{\pi}{4} \tan \frac{\pi}{4} + \ln \left(\cos \frac{\pi}{4} \right) - 0 - \ln 1$$

$$= \frac{\pi}{4} + \ln \left(\frac{1}{\sqrt{2}} \right) \quad \text{[M1 – proper evaluation]}$$

$$= \frac{\pi}{4} + \ln 2^{-\frac{1}{2}}$$

$$= \frac{\pi}{4} - \frac{1}{2} \ln 2 \quad \text{[A1]}$$

- 5 (a) In the expansion of $(2+x)^n$, where n is a positive integer, the coefficient of x^2 is twice the coefficient of x . Find the value of n . [3]

$$(2+x)^n = 2^n + \binom{n}{1}2^{n-1}x + \binom{n}{2}2^{n-2}x^2 + \dots$$

$$\binom{n}{2}2^{n-2} = 2\binom{n}{1}2^{n-1} \quad \text{[M1 - correct coeff]}$$

$$\frac{n(n-1)}{2}(2^{n-2}) = 2n(2^{n-1})$$

$$\frac{n(n-1)}{2}(2^n \times 2^{-2}) = 2n(2^n \times 2^{-1}) \quad (n \neq 0) \quad \text{[M1 - simplify]}$$

$$\frac{n-1}{2(4)} = 1$$

$$n-1 = 8$$

$$n = 9 \quad \text{[A1 - with rejection]}$$

- (b) Find the value of the term that is independent of x in the expansion of $\left(2x - \frac{1}{4x^4}\right)^{15}$. [4]

general term

$$= \binom{15}{r}(2x)^{15-r} \left(-\frac{1}{4x^4}\right)^r$$

$$= \binom{15}{r}2^{15-r}x^{15-r} \left(-\frac{1}{4}\right)^r x^{-4r} \quad \text{[M1]}$$

$$= \binom{15}{r}2^{15-r} \left(-\frac{1}{4}\right)^r x^{15-5r} \quad \text{[M1 - gather } x \text{ and let power} = 0]$$

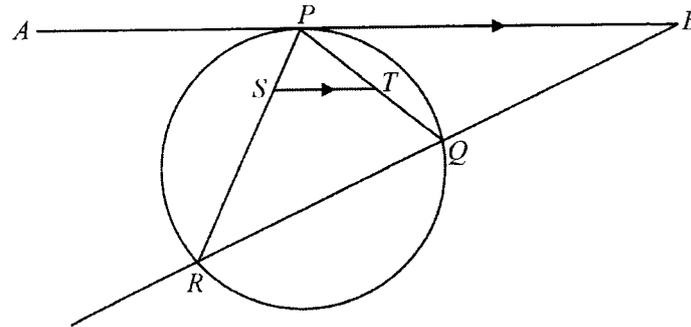
$$15 - 5r = 0$$

$$r = 3 \quad \text{[M1 for } r \text{ value]}$$

$$\text{value} = \binom{15}{3}2^{12} \left(-\frac{1}{4}\right)^3 = -29120 \quad \text{[A1]}$$

Max 2 marks if able to gather $15 - 5r$ for exponent of ' x ' and equate to zero with correct r value.

6



The diagram shows a circle passing through the points P , Q and R . The point Q lies on the line RB . AB is a tangent to the circle at P . The points S and T lie on PR and PQ respectively. Given that AB is parallel to ST , prove that

(a) triangle PST is similar to triangle PQR , [3]

$$\angle SPT = \angle QPR \text{ (common angle)} \quad [M1]$$

$$\begin{aligned} \angle PST &= \angle SPA \text{ (alt. } \angle\text{s, } AB \parallel ST) & \text{OR} & \quad \angle PTS = \angle TPB \text{ (alt. } \angle\text{s, } AB \parallel ST) \\ &= \angle PQR \text{ (Alt. Segment Thm)} & & \quad = \angle PRQ \text{ (Alt. Segment Thm)} \quad [M1] \end{aligned}$$

\therefore triangle PST is similar to triangle PQR . [A1]

(2 pairs of corresponding angles are equal)

(b) $PQ \times PT = PR \times PS$, [2]

$$\text{From above result, } \frac{PQ}{PS} = \frac{PR}{PT} \quad [M1]$$

$$\Rightarrow PQ \times PT = PR \times PS \quad [A1]$$

(c) Determine if $STQR$ is a cyclic quadrilateral.

[4]

$$\angle QTS = 180^\circ - \angle PTS \text{ (adj. } \angle\text{s on a st. line)}$$

$$= 180^\circ - \angle QRS \text{ (from (a) result)} \quad [\text{M1}]$$

$$\text{So } \angle QTS + \angle QRS = 180^\circ \quad [\text{M1}]$$

$$\angle RST + \angle RQT = 360^\circ - (\angle QTS + \angle QRS) \text{ (}\angle \text{ sum of a quadrilateral)}$$

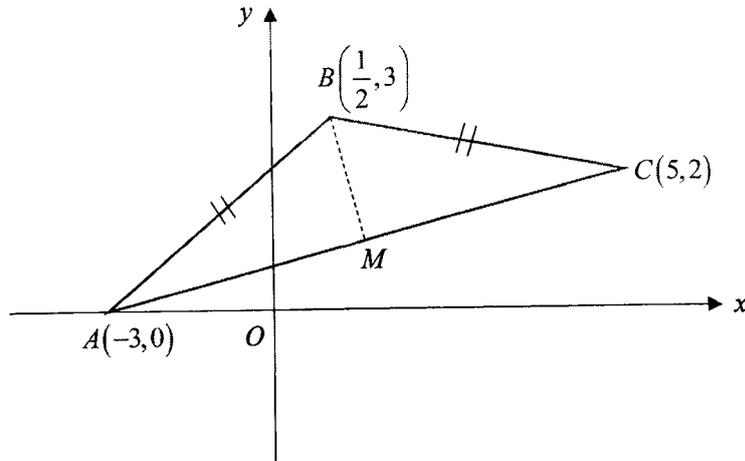
$$= 360^\circ - 180^\circ$$

$$= 180^\circ \quad [\text{M1}]$$

By converse of angles in opposite segment, $STQR$ is a cyclic quadrilateral and all four vertices lie on the circumference of a circle.

[A1 – with correct reason, accept even if no mention of four vertices]

7



The diagram shows an isosceles triangle ABC in which $A(-3, 0)$, $B\left(\frac{1}{2}, 3\right)$ and $C(5, 2)$. M is the foot of perpendicular from B to AC .

(a) Find the coordinates of M .

[1]

$$M = \left(\frac{-3+5}{2}, \frac{0+2}{2} \right) = (1, 1) \quad [\text{B1}]$$

(b) Find the equation of the perpendicular bisector of AC .

[2]

$$\text{Gradient of } AC = \frac{2-0}{5-(-3)} = \frac{1}{4}$$

$$\text{Gradient of perpendicular } BM = -4$$

[M1 – use $m \times m_{\perp} = -1$]

$$\text{or } m_{BM} = \frac{3-1}{\frac{1}{2}-1} = -4$$

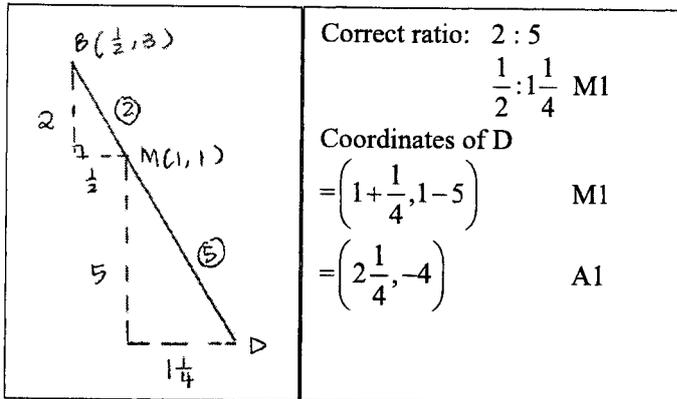
Equation of perpendicular bisector:

$$y-1 = -4(x-1)$$

$$y = -4x + 5$$

[A1]

- (c) Given that $ABCD$ is a kite with $BM = \frac{2}{7}BD$, find the coordinates of D . [3]



Alternative: using vectors

$$\overline{BM} = \frac{2}{7}\overline{BD}$$

$$\begin{pmatrix} 0.5 \\ -2 \end{pmatrix} = \frac{2}{7}(\overline{OD} - \overline{OB}) \quad [\text{M1 - position vectors}]$$

$$\overline{OD} = 3.5 \begin{pmatrix} 0.5 \\ -2 \end{pmatrix} + \begin{pmatrix} 0.5 \\ 3 \end{pmatrix} = \begin{pmatrix} 2.25 \\ -4 \end{pmatrix} \quad [\text{M1}]$$

$$D = \left(2\frac{1}{4}, -4\right) \quad [\text{A1}]$$

- (d) Find the area of the kite $ABCD$. [2]

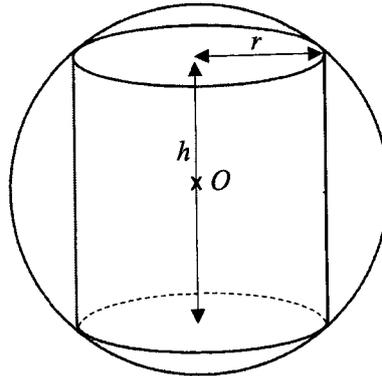
$$\text{Area of } ABCD = \frac{1}{2} \begin{vmatrix} -3 & 2\frac{1}{4} & 5 & \frac{1}{2} & -3 \\ 0 & -4 & 2 & 3 & 0 \end{vmatrix}$$

M1 (ecf 1 – correct method)

$$= \frac{1}{2}(31.5 + 28) \quad (\text{anti-clockwise})$$

$$= 29.75 \text{ units}^2 \quad (\text{or } \frac{119}{4}) \quad \text{A1}$$

(accept 29.7 with 3 s.f. if using other method such as Pythagoras)



A prototype consists of a cylindrical container of height h cm and radius r cm inscribed in a hollow sphere with centre O .

The sphere has a surface area of 6400π cm² and both the sphere and container have negligible thickness.

- (a) Show that the volume of the cylinder container, V cm³, is given by [3]

$$V = 2\pi r^2 \sqrt{1600 - r^2}.$$

$$4\pi R^2 = 6400\pi \quad [\text{M1} - \text{find radius } R]$$

$$R = 40 \quad (\text{radius } R \text{ of sphere})$$

By Pythagoras' Theorem,

$$r^2 + \left(\frac{h}{2}\right)^2 = 40^2 \quad [\text{M1}]$$

$$h^2 = 4(1600 - r^2)$$

$$h = \sqrt{4(1600 - r^2)}$$

$$h = 2\sqrt{1600 - r^2}$$

$$V = \pi r^2 h \quad [\text{A1}]$$

$$= 2\pi r^2 \sqrt{1600 - r^2}$$

9 It is given that $f(x) = 4 + \cos\left(\frac{x}{2}\right)$ and $g(x) = -2\sin x$.

(a) State the period and amplitude of $f(x)$. [2]

Period = 720° or 4π [B1]
 Amplitude = 1 [B1]

(b) State the period and amplitude of $g(x)$. [1]

Period = 360° or 2π
 Amplitude = 2 [B1 for both]

(c) Sketch, on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0^\circ \leq x \leq 360^\circ$. [3]

	$f(x)$	$g(x)$
Shape	cosine	negative sine
Amplitude	1	2
No. of cycle	Half	1
Shift	4	-

[B1 – for starts and ends at ‘zero’ for $g(x)$]
 [B1 – for starts ‘5’ and ends at ‘3’ for $f(x)$]
 [B1 – fully correct graphs]

- 10 (a) Express $\frac{1-3x-3x^2}{x(x+1)^2}$ in partial fractions. [5]

$$\frac{1-3x-3x^2}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$1-3x-3x^2 = A(x+1)^2 + Bx(x+1) + Cx \quad \text{[M1]}$$

$$\text{When } x = -1, \quad C = -1 \quad \text{[M1]}$$

$$\text{When } x = 0, \quad A = 1 \quad \text{[M1]}$$

$$\text{When } x = 1, \quad B = -4 \quad \text{[M1]}$$

$$\frac{1-3x-3x^2}{x(x+1)^2} = \frac{1}{x} - \frac{4}{x+1} - \frac{1}{(x+1)^2} \quad \text{[A1]}$$

- (b) Hence find $\int \frac{1-3x-3x^2}{2x(x+1)^2} dx$. [4]

$$\int \frac{1-3x-3x^2}{2x(x+1)^2} dx = \frac{1}{2} \int \frac{1-3x-3x^2}{x(x+1)^2} dx$$

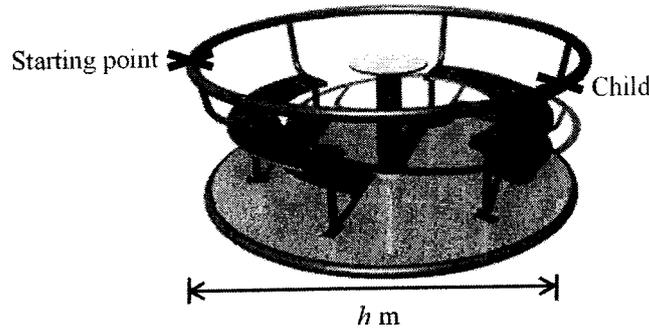
$$= \frac{1}{2} \int \frac{1}{x} - \frac{4}{x+1} - \frac{1}{(x+1)^2} dx \quad \text{[M1 - take out 0.5]}$$

$$= \frac{1}{2} \ln x - 2 \ln(x+1) + \frac{1}{2(x+1)} + C \quad \text{[A1 for each term, must have constant C]}$$

$$\text{Accept } = \ln \sqrt{x} - \ln(x+1)^2 + \frac{1}{2(x+1)} + C$$

$$\text{or } \ln \frac{\sqrt{x}}{(x+1)^2} + \frac{1}{2(x+1)} + C$$

11



The horizontal distance of a child on a carousel, h m, from the starting point is modelled by the equation, $h = 2(1 - \cos kt)$, where k is a constant and t is the time in seconds after the child leaves the starting point. The time to complete one revolution is 20 seconds.

- (a) Explain why this model suggests that the diameter of the carousel is 4 m. [1]

$h = 2(1 - \cos kt)$.
 Since the diameter of the carousel = max value of h , $h = 2(1 - (-1)) = 4$ m
 [B1 – must relate diameter to h , do not accept amplitude method as question ask on the model equation]

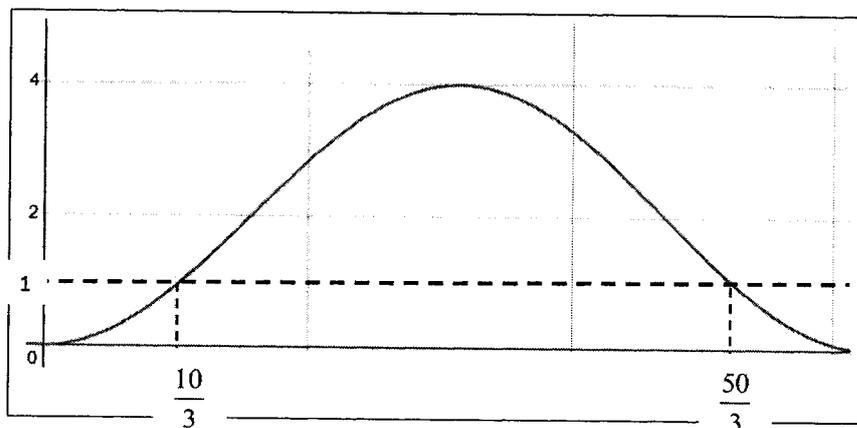
- (b) Show that the value of k is $\frac{\pi}{10}$ radians per second. [2]

<p>Period = 20 s</p> <p>$\frac{2\pi}{k} = 20$ [M1]</p> <p>$k = \frac{\pi}{10} \text{ rad/s}$ (shown) [A1]</p>	<p>accept 2π rad in 20 s $\frac{2\pi}{20}$ rad in 1 s $\frac{2\pi}{20}t$ rad in t s $k = \frac{\pi}{10}$</p>
	<p>accept $h = 4$ when $t = 10$ to find k value</p>

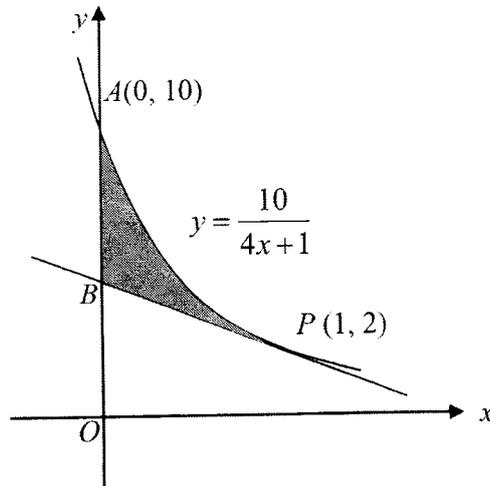
- (c) As the carousel turns, it is possible for the child on the carousel to view a landmark, provided that the horizontal distance of the child is within 1 m from the starting point.

Find the duration of time for which the child will not be able to view the landmark during one revolution. [5]

$1 = 2(1 - \cos \frac{\pi}{10} t)$	accept	[M1]
$\frac{1}{2} = \cos \frac{\pi}{10} t$	$2(1 - \cos \frac{\pi}{10} t) > 1$	
$\text{basic } \angle = \frac{\pi}{3}$	(solution in 1st and 4th quad)	
$\frac{\pi}{10} t = \frac{\pi}{3}, \frac{5\pi}{3}$	accept	[M1]
$t = \frac{10}{3}, \frac{50}{3}$	$t > \frac{10}{3} \quad t < \frac{50}{3}$	[M1]
Not able to view: $\frac{50}{3} - \frac{10}{3} = \frac{40}{3} s$ or 13.3 s (3s.f.)		[M1, A1]



12



The diagram shows part of the curve $y = \frac{10}{4x+1}$ intersecting the y -axis at $A(0, 10)$. The tangent to the curve at the point $P(1, 2)$ intersects the y -axis at B .

(a) Show that the coordinates of B is $(0, 3.6)$.

[4]

$$y = \frac{10}{4x+1} = 10(4x+1)^{-1}$$

$$\frac{dy}{dx} = -10(4x+1)^{-2}(4) \quad [\text{M1}]$$

$$\frac{dy}{dx} = -40(4x+1)^{-2}$$

$$\text{When } x=1 \quad \frac{dy}{dx} = -40(4(1)+1)^{-2}$$

$$\frac{dy}{dx} = -1.6 \quad [\text{M1}]$$

$$\frac{y-2}{0-1} = -1.6 \quad [\text{M1}]$$

$$y-2 = 1.6$$

$$y = 3.6$$

$$\text{Coordinate of } B \text{ is } (0, 3.6) \quad [\text{A1}]$$

(b) Find the exact area of the shaded region.

[5]

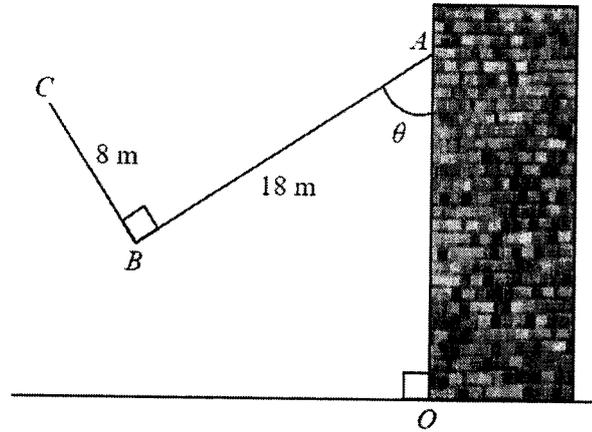
$$\begin{aligned}
 \text{Area} &= \int_0^1 \frac{10}{4x+1} dx - \frac{1}{2}(3.6+2)(1) && \text{[M1], [M1]} \\
 \text{Area} &= \left[\frac{10 \ln(4x+1)}{4} \right]_0^1 - 2.8 && \text{[M1]} \\
 \text{Area} &= \left[\frac{10 \ln(4+1)}{4} - \frac{10 \ln(1)}{4} \right] - 2.8 && \text{[M1]} \\
 \text{Area} &= \frac{5}{2} \ln 5 - 2.8 \text{ unit}^2 && \text{[A1]}
 \end{aligned}$$

END OF PAPER

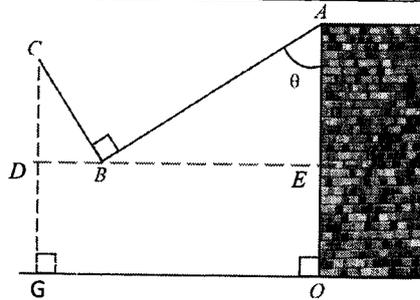
1	A curve has the equation $y = \frac{\sin 2x}{2 - \cos 2x}$.
	<p>(a) Show that the gradient function can be expressed in the form $\frac{k \cos 2x - 2}{(2 - \cos 2x)^2}$, where k is a constant. [3]</p>
	$y = \frac{\sin 2x}{2 - \cos 2x}$ $\frac{dy}{dx} = \frac{(2 - \cos 2x)2 \cos 2x - \sin 2x(2 \sin 2x)}{(2 - \cos 2x)^2}$ $= \frac{4 \cos 2x - 2 \cos^2 2x - 2 \sin^2 2x}{(2 - \cos 2x)^2}$ $= \frac{4 \cos 2x - 2(\cos^2 2x + \sin^2 2x)}{(2 - \cos 2x)^2}$ $= \frac{4 \cos 2x - 2}{(2 - \cos 2x)^2}$ <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 5px;">M1: correct quotient/pdt rule</div> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 5px;">M1: differentiate sin 2x and cos 2x correctly</div> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 5px;">A1: use of identity to reach answer</div>
	<p>(b) Find the acute angle between the tangent to the curve at $x = \frac{\pi}{12}$ and the line $y = 0$. [3]</p>
	$\text{gradient of tangent} = \frac{4 \cos \frac{\pi}{6} - 2}{\left(2 - \cos \frac{\pi}{6}\right)^2} = 1.1386$ <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 5px;">M1: correct gradient value</div> $\text{Angle required} = \tan^{-1}(1.1386) = 48.7^\circ \text{ (1 dp)}$ <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 5px;">M1, A1 (accepts 0.850 rad)</div>

2	<p>(a) Factorise $x^3 + 27k^3$ as a product of a linear and a quadratic factor. [2]</p>
	$x^3 + 27k^3 = (x + 3k)(x^2 - 3kx + 9k^2)$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> <p>M1: $(x+3k)$ A1</p> </div>
	<p>(b) Hence solve $x^3 + 27 = (x+3)(x+10)$, expressing non-integer roots in surd form. [3]</p>
	$k=1: x^3 + 27 = (x+3)(x^2 - 3x + 9)$ $(x+3)(x^2 - 3x + 9) = (x+3)(x+10)$ $x+3=0 \text{ or } x^2 - 3x + 9 = x+10$ $x=-3 \text{ or } x^2 - 4x - 1 = 0$ $x = \frac{4 \pm \sqrt{16 - 4(-1)}}{2}$ $= 2 \pm \sqrt{5}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> <p>B1: identify $x=-3$ as a root from Hence M1: apply quad formula correctly A1</p> </div>
	<p>(c) Find the value of k given that $x^3 + 27k^3$ leaves a remainder of 351 when divided by $x-2$. [2]</p>
	<p>Let $f(x) = x^3 + 27k^3$ $f(2) = 2^3 + 27k^3 = 351$ $k = \sqrt[3]{\frac{351-8}{27}} = \frac{7}{3}$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> <p>M1: applying remainder thm correctly A1</p> </div>

- 3 The diagram shows a L -shaped rod ABC where AB and BC have length 18 m and 8 m respectively and angle ABC is 90° . The rod is hinged to a wall at A so as to rotate in a vertical plane. The rod AB makes an acute angle θ with the vertical wall surface OA .



- (a) Given that G is a point directly below C , show that $OG = p \cos \theta + q \sin \theta$, where p and q are constants to be found. [2]



Using triangle BCD : Angle $DBC = \theta$

$$\cos \theta = \frac{BD}{8}$$

$$BD = 8 \cos \theta$$

B1: either correctly establishing BD or BE

Using triangle ABE : Angle $BAE = \theta$

$$\sin \theta = \frac{BE}{18}$$

$$BE = 18 \sin \theta$$

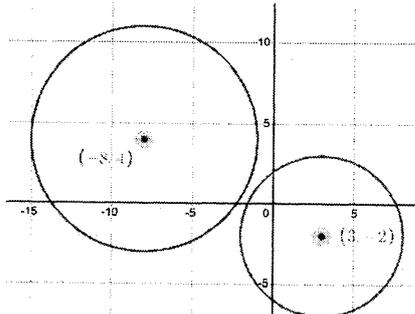
$$OG = BD + BE$$

$$= 8 \cos \theta + 18 \sin \theta, \quad p = 8, \quad q = 18$$

B1: showing the other component and clear indication that OG is a sum of the 2 values

- (b) Express OG in the form $R \cos(\theta - \alpha)$ where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$. [3]

	$OG = 8 \cos \theta + 18 \sin \theta$ $= \sqrt{8^2 + 18^2} \cos \left(\theta - \tan^{-1} \frac{18}{8} \right)$ $= \sqrt{388} \cos \left(\theta - \tan^{-1} \frac{9}{4} \right)$ $= 19.7 \cos(\theta - 66.0^\circ)$	<div style="border: 1px solid black; padding: 5px;"> <p>M1: $\sqrt{8^2 + 18^2}$</p> <p>M1: $\tan^{-1} \frac{18}{8}$</p> <p>A1: correct evaluation and form, accept $\sqrt{388}$</p> </div>
	<p>(c) Find the length of OG and the corresponding value of θ if G is at maximum displacement from O. [3]</p>	
	<p>max $OG = \sqrt{388}$ $= 19.7 \text{ m}$</p> <p>when $\cos \left(\theta - \tan^{-1} \frac{9}{4} \right) = 1$</p> $\theta - \tan^{-1} \frac{9}{4} = 0$ $\theta = \tan^{-1} \frac{9}{4} = 66.0^\circ$	<div style="border: 1px solid black; padding: 5px;"> <p>A1: $\sqrt{388}$ or 19.7m</p> <p>M1: $\theta - \tan^{-1} \frac{9}{4} = 0$</p> <p>A1: 66.0°</p> </div>

4	A circle C_1 has equation $x^2 + y^2 - 6x + 4y = 12$.
	(a) Find the radius and the coordinates of the centre of C_1 . [3]
	$x^2 - 6x + y^2 + 4y = 12$ $(x-3)^2 + (y+2)^2 = 12 + 3^2 + 2^2$ $(x-3)^2 + (y+2)^2 = 5^2$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: completing the square or using formula</div> <div style="display: flex; justify-content: space-between; margin-top: 10px;"> <div style="width: 45%;"> Radius = 5 units Centre = (3, -2) </div> <div style="border: 1px solid black; padding: 5px; width: 45%;"> A1 B1 </div> </div>
	(b) Find the equation of the tangent to the circle at the point $P(7, -5)$. [3]
	$\text{Gradient of normal} = \frac{-2 - (-5)}{3 - 7} = -\frac{3}{4}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px auto;">M1</div> $\text{Gradient of tangent} = \frac{4}{3}$ $\text{Eqn of tangent: } y + 5 = \frac{4}{3}(x - 7)$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px auto;">M1 award for correct pts and their -1/m used</div> $y = \frac{4}{3}x - \frac{43}{3}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px auto;">A1, or $3y = 4x - 43$</div>
	(c) Another circle C_2 has centre $(-8, 4)$ and radius 7 cm. Find the shortest distance between the 2 circles. [2]
	$\text{Distance between centres of circle} = \sqrt{(3+8)^2 + (-2-4)^2} = \sqrt{157}$ $\text{Shortest distance} = \sqrt{157} - 7 - 5 = 0.530 \text{ cm (3 s.f.)}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1 A1</div> 

5	<p>(a) Prove the identity $\frac{(\sin A - \cos A)(1 + \sin A \cos A)}{\cos^3 A} = \tan^3 A - 1$. [4]</p>
	$LHS = \frac{(\sin A - \cos A)(1 + \sin A \cos A)}{\cos^3 A}$ $= \frac{\sin A - \cos A + \sin^2 A \cos A - \sin A \cos^2 A}{\cos^3 A} \quad \text{M1: correct expansion of terms}$ $= \frac{\sin A - \cos A + (1 - \cos^2 A) \cos A - \sin A (1 - \sin^2 A)}{\cos^3 A} \quad \text{M1: applying identity}$ $= \frac{\sin A - \cos A + \cos A - \cos^3 A - \sin A + \sin^3 A}{\cos^3 A}$ $= \frac{\sin^3 A - \cos^3 A}{\cos^3 A} \quad \text{M1: expand \& simplify}$ $= \frac{\sin^3 A}{\cos^3 A} - \frac{\cos^3 A}{\cos^3 A} \quad \text{A1: manipulation to RHS}$ $= \tan^3 A - 1$ <p>OR</p> $LHS = \frac{(\sin A - \cos A)(1 + \sin A \cos A)}{\cos^3 A}$ $= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{\cos^3 A} \quad \text{M1: applying identity}$ $= \frac{\sin^3 A + \sin A \cos^2 A + \sin^2 A \cos A - \sin^2 A \cos A - \cos^3 A - \sin A \cos^2 A}{\cos^3 A} \quad \text{M1: correct expansion of terms}$ $= \frac{\sin^3 A - \cos^3 A}{\cos^3 A} \quad \text{M1: simplify}$ $= \frac{\sin^3 A}{\cos^3 A} - \frac{\cos^3 A}{\cos^3 A} \quad \text{A1: manipulation to RHS}$ $= \tan^3 A - 1$

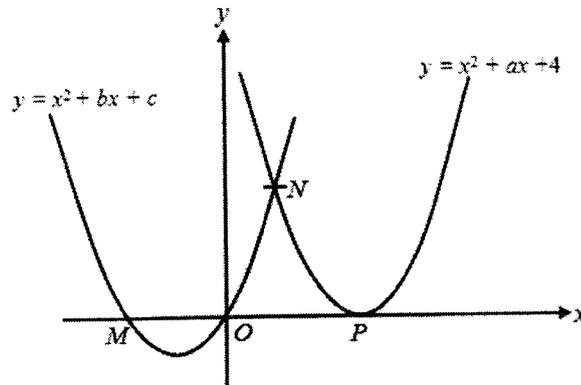
	<p>(b) Hence solve $(\sin A - \cos A)(1 + \sin A \cos A) + 2 \cos^3 A = 0$ exactly, for $-\pi \leq A \leq \pi$ radians. [4]</p>
	$(\sin A - \cos A)(1 + \sin A \cos A) = -2 \cos^3 A$ $\frac{(\sin A - \cos A)(1 + \sin A \cos A)}{\cos^3 A} = -2$ $\tan^3 A - 1 = -2$ $\tan^3 A = -1$ $\tan A = -1$ <p>Basic angle = $\frac{\pi}{4}$</p> <p>Quad: Q2, Q4</p> $A = \frac{3\pi}{4}, -\frac{\pi}{4}$

M1: simplification to single trigo equation

M1: correct basic angle – must be acute

A1, A1

- 6 The diagram shows the graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$. The graph of $y = x^2 + ax + 4$ touches the x -axis at P . Points M and O are the x -intercepts of the graph of $y = x^2 + bx + c$. The origin O is the mid-point of MP .



(a) Find the values of a , b and c .

[4]

Since origin $(0, 0)$ is on $y = x^2 + bx + c$, $c = 0$

B1

Discriminant of $y = x^2 + ax + 4 = 0$ since curve intersects x -axis once only:

$$a^2 - 4(1)(4) = 0$$

$$a = \pm 4$$

Since P is on positive x -axis, $a < 0$: $a = -4$

M1: using discriminant or the equation must be a perfect square

A1

OR

$$y = x^2 + ax + 4 = \left(x + \frac{a}{2}\right)^2 - \frac{a^2}{4} + 4.$$

Since curve intersects x -axis once only:

$$4 - \frac{a^2}{4} = 0$$

$$a = \pm 4$$

Since P is on positive x -axis, $a < 0$: $a = -4$

M1: using discriminant or the equation must be a perfect square

A1

$$\text{Coor of } P = \left(-\frac{a}{2}, 0\right) = (2, 0)$$

$$\text{Coor of } M = (-b, 0) = (-2, 0)$$

$$b = 2$$

B1

	<p>(b) The graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$ intersects at N. Find the coordinates of N. [2]</p>
	$x^2 - 4x + 4 = x^2 + 2x$ $6x = 4$ $x = \frac{2}{3}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: equating and solving correct x (allows ecf)</div> $y = \left(\frac{2}{3}\right)^2 + 2\left(\frac{2}{3}\right) = \frac{16}{9}$ $\text{Coordinates of } N = \left(\frac{2}{3}, \frac{16}{9}\right)$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">A1</div>
	<p>(c) The graph $y = px^2 + qx + r$ has its turning point at N and passes through point P. Find the values of p, q and r, where $r > 0$. [3]</p>
	<p>Graph with turning point at N and passes P (downward opening):</p> $y = -p\left(x - \frac{2}{3}\right)^2 + \frac{16}{9}, p < 0$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: correct completed square form</div> <p>At $P(2, 0)$, $0 = -p\left(2 - \frac{2}{3}\right)^2 + \frac{16}{9}$</p> $p = -1$ $y = -\left(x - \frac{2}{3}\right)^2 + \frac{16}{9}$ $= -\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) + \frac{16}{9}$ $= -x^2 + \frac{4}{3}x + \frac{4}{3}$ $p = -1, q = r = \frac{4}{3}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">A1: correct p A1: correct q & r</div> <p>OR (longer method): form 3 equations with coor of N, P and either 2nd x-intercept or derivative and solve</p> $y = px^2 + qx + r$ $\frac{dy}{dx} = 2px + q$ <p>Turning point at $\left(\frac{2}{3}, \frac{16}{9}\right)$: $2p\left(\frac{2}{3}\right) + q = 0$</p> $4p = -3q \text{ ---- (1)}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: establishing 3 equations correctly</div> <p>Graph passes through P: $0 = p(2)^2 + q(2) + r$</p> $4p + 2q + r = 0$

	<p>Subst (1): $q = r$ ----- (2)</p> <p>Graph passes through N: $\frac{16}{9} = p\left(\frac{2}{3}\right)^2 + q\left(\frac{2}{3}\right) + r$</p> $4p + 6q + 9r = 16$ <p>Subst (1) and (2):</p> $-3q + 6q + 9q = 16$ $12q = 16$ $q = \frac{4}{3}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;">A1: correct p</div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;">A1: correct q & r</div>
7	<p>A particle P, travels in a straight line, so that its displacement, s m, from O at time t seconds, is modelled by $s = \frac{1}{3}t^3 - 5t^2 - 3$.</p>
	<p>(a) Find the value of t when particle P return to its initial position. [2]</p> <p>Initial position when $t = 0$, $s = -3$</p> $\frac{1}{3}t^3 - 5t^2 - 3 = -3$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;">M1: setting $s = -3$</div> $\frac{1}{3}t^3 - 5t^2 = 0$ $t^2\left(\frac{1}{3}t - 5\right) = 0$ $t = 0 \text{ or } t = 15s$ <p>Particle returns to initial position at $t = 15s$. A1</p>
	<p>(b) Find the minimum velocity of particle P. [3]</p> $v = t^2 - 10t$ $= (t - 5)^2 - 25$ <p>Minimum velocity = -25 m/s^2</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;">M1: $v = t^2 - 10t$</div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;">M1: completing the square</div> <p>OR</p> $v = t^2 - 10t$ <p>At stationary v,</p> $\frac{dv}{dt} = 2t - 10 = 0$ $t = 5$ $v = (5)^2 - 10(5) = -25 \text{ m/s}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;">A1</div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;">M1: $v = t^2 - 10t$</div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;">M1: solving for $t = 5$</div> $\frac{d^2v}{dt^2} = 2 > 0, \text{ minimum } v \text{ at } -25 \text{ m/s}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;">A1: includes check for minimum</div>

	<p>(c) Another particle, Q, travels in a straight line from O such that its velocity, v m/s, at time t seconds after passing O is given by $v = 24\left(e^{\frac{t}{6}} - e^{-1}\right)$.</p> <p>Find the value of t at which the particle Q is instantaneously at rest. [2]</p>
	$v = 24\left(e^{\frac{t}{6}} - e^{-1}\right) = 0$ $e^{\frac{t}{6}} = e^{-1}$ $t = 6s$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;"> <p>M1: Setting $v=0$</p> <p>A1</p> </div>
	<p>(d) Find the total distance travelled by particle Q for the first 9 seconds. [4]</p>
	$s = 24\left(\frac{e^{\frac{t}{6}}}{\frac{1}{6}} - te^{-1}\right) = -24\left(6e^{\frac{t}{6}} + te^{-1}\right) + C$ <p>When $t=0, s=0: s = -24(6) + C = 0$ $C = 144$</p> $s = 144 - 24\left(6e^{\frac{t}{6}} + te^{-1}\right)$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;"> <p>M1: correct integration (award even if without +C)</p> </div> <p>When $t=6, s = 144 - 24(6e^{-1} + 6e^{-1}) = 38.051m$ (5 sf)</p> <p>When $t=9, s = 144 - 24(6e^{\frac{9}{6}} + 9e^{-1}) = 32.407m$ (5 sf)</p> <p>Total distance travelled = $s = 38.051 + (38.051 - 32.407) = 43.7m$ (3 sf)</p>
	<p>OR</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;"> <p>A1</p> </div>
	<p>Displacement covered for 1st 6 sec =</p> $\int_0^6 24\left(e^{\frac{t}{6}} - e^{-1}\right) dt = 24\left[\frac{e^{\frac{t}{6}}}{\frac{1}{6}} - te^{-1}\right]_0^6$ $= -24\left[6e^{\frac{t}{6}} + te^{-1}\right]_0^6$ $= -24\left[6e^{\frac{6}{6}} + 6e^{-1} - 6e^{\frac{0}{6}} - 0\right]$ $= 38.051m$ (3sf) <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;"> <p>M1: correct integration</p> </div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px; margin-top: 20px;"> <p>M1</p> </div> <p>Displacement covered for next 3 sec =</p>

	$\int_6^9 24 \left(e^{-\frac{t}{6}} - e^{-1} \right) dt = -24 \left[6e^{-\frac{t}{6}} + te^{-1} \right]_6^9$ $= -24 \left[6e^{-\frac{9}{6}} + 9e^{-1} - 6e^{-\frac{6}{6}} - 6e^{-\frac{6}{6}} \right]$ $= -5.6434m \text{ (5sf)}$	<div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 0 auto;">M1</div>
	Total distance = $s = 38.051 + (38.051 - 32.407) = 43.7m$	<div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 0 auto;">A1</div>

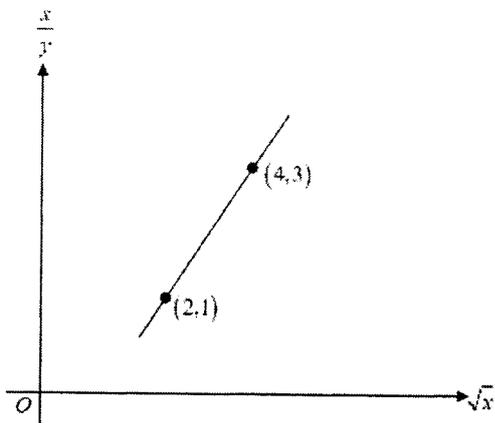
8	The height, h cm, of a plant is modelled by $h = \frac{80}{1+10e^{-0.4t}}$, where t is the number of months after the first observation.
	(a) Show that h is an increasing function. [3]
	$\frac{dh}{dt} = 80 \frac{d}{dt} (1+10e^{-0.4t})^{-1}$ $= 80(-1)(1+10e^{-0.4t})^{-2} (10(-0.4)e^{-0.4t})$ $= \frac{320e^{-0.4t}}{(1+10e^{-0.4t})^2}$ <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 0 auto;">M1: differentiate $1+10e^{-0.4t}$ correctly</div> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 0 auto;">M1: simplification</div> <p>Since $e^{-0.4t} > 0$ for $t > 0$, $320e^{-0.4t} > 0$ and $(1+10e^{-0.4t})^2 > 0$</p> $\frac{dh}{dt} = \frac{320e^{-0.4t}}{(1+10e^{-0.4t})^2} > 0$ <p>Therefore, h is an increasing function.</p> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 0 auto;">A1: Clear explanation on why derivative is positive and therefore h is increasing</div>
	(b) Find the value of t when the height of the plant first exceeds four times its initial observation. [3]
	<p>Since h is increasing,</p> $\frac{80}{1+10e^{-0.4t}} > \frac{4 \times 80}{1+10e^0}$ $1+10e^{-0.4t} < \frac{11}{4}$ $e^{-0.4t} < \frac{\frac{11}{4} - 1}{10} = 0.175$ $-0.4t < \ln 0.175$ $t > 4.3574 \text{ (5 s.f.)}$ $t = 4.36 \text{ (3 s.f.)}$ <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 0 auto;">M1: correct set up of equation (accepts equal if students round up at the end, preferably with explanation that h is an increasing function)</div> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 0 auto;">M1: simplification to exponential eqn</div> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 0 auto;">A1 (accepts 5, though context need not be integer)</div>

	<p>(c) The height, y cm, of another species of plant, t months after the first observation is given by $y = \frac{1}{a + be^{-t}}$, where a and b are constants. Explain clearly how a straight line graph can be drawn to represent this relationship. You should state which variables should be plotted on each axis and explain how the values of a and b can be calculated. [4]</p>
	$y = \frac{1}{a + be^{-t}}$ $\frac{1}{y} = a + be^{-t}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: simplification to $Y=mX + c$ form</div> <p>Plot $\frac{1}{y}$ on the vertical axis against e^{-t} on the horizontal axis to obtain a straight line graph. [A1] The gradient of the graph would give the value of b [A1] and the y-intercept will give the value of a [A1].</p> <p>Note: no award of full A1s if M1 not awarded.</p>

9	<p>(a) Without using a calculator, solve the equation $x\sqrt{15} + \sqrt{5} = x\sqrt{2} + \sqrt{6}$. Leave your answer in the form $p\sqrt{10} + q\sqrt{3}$, where p and q are fractions. [4]</p>
	$x\sqrt{15} + \sqrt{5} = x\sqrt{2} + \sqrt{6}$ $x(\sqrt{15} - \sqrt{2}) = \sqrt{6} - \sqrt{5}$ $x = \frac{\sqrt{6} - \sqrt{5}}{\sqrt{15} - \sqrt{2}}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: $x = \frac{\sqrt{6} - \sqrt{5}}{\sqrt{15} - \sqrt{2}}$</div> $= \frac{\sqrt{6} - \sqrt{5}}{\sqrt{15} - \sqrt{2}} \times \frac{\sqrt{15} + \sqrt{2}}{\sqrt{15} + \sqrt{2}}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: correct rationalisation fraction</div> $= \frac{\sqrt{90} - \sqrt{10} - \sqrt{75} + \sqrt{12}}{15 - 2}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: expansion of numerator and denominator</div> $= \frac{3\sqrt{10} - \sqrt{10} - 5\sqrt{3} + 2\sqrt{3}}{13}$ $= \frac{2\sqrt{10} - 3\sqrt{3}}{13}$ $= \frac{2}{13}\sqrt{10} - \frac{3}{13}\sqrt{3}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">A1: in $p\sqrt{10} + q\sqrt{3}$, where p and q are fractions</div>

	<p>(b) Without using a calculator, solve the equation $\log_2 x - \log_x 16 = 0$. [4]</p> <p>$\log_2 x - \log_x 16 = 0$</p> <p>$\log_2 x - \frac{\log_2 16}{\log_2 x} = 0$ M1: correct change of base</p> <p>$(\log_2 x)^2 = 4$ M1: simplification of $\log_2 16$ and reduced to a solvable quadratic equation</p> <p>$\log_2 x = \pm 2$</p> <p>$x = 2^2$ or $x = 2^{-2}$</p> <p>$x = 4$ or $x = \frac{1}{4}$ A1, A1</p>
	<p>(c) Solve the equation $3^{x+2} - 2(3^{-x}) = 17$. [4]</p> <p>$3^{x+2} - 2(3^{-x}) = 17$</p> <p>$9(3^x) - \frac{2}{3^x} = 17$ M1: correct application of indices rules and rewriting into quadratic eqn in 3^x</p> <p>$9(3^x)^2 - 17(3^x) - 2 = 0$</p> <p>Let $u = 3^x$</p> <p>$9u^2 - 17u - 2 = 0$</p> <p>$(u - 2)(9u + 1) = 0$ M1: factoring or formula to solve quadratic eqn</p> <p>$u = 2$ or $u = -\frac{1}{9}$ (rej since $3^x > 0$) A1: Solving of 3^x with rejection</p> <p>$3^x = 2$</p> <p>$x = \frac{\ln 2}{\ln 3} = 0.631$ (3 s.f.) A1: do not accept $x = \log_3 2$ as no way of computing</p>

- 10 (a) The diagram shows part of a straight line graph which passes through (2,1) and (4,3).



Find the equation of the straight line in the form $y = \frac{x}{a+b\sqrt{x}}$, where a and b are constants. [3]

gradient of straight line: $\frac{3-1}{4-2} = 1$

equation:

$$Y - 1 = 1(X - 2)$$

$$Y = X - 1$$

$$\frac{x}{y} = \sqrt{x} - 1$$

$$y = \frac{x}{\sqrt{x} - 1}$$

M1: gradient of line

M1: eqn of line

A1

- (b) The table below shows the experimental values of two variables x and y .

x	1	2	3	4	5	6
y	63	127	258	510	1000	2100

It is known that x and y are related by an equation of the form $y = \frac{b^x}{10^a}$, where a and b are constants.

- (i) On the grid next page, plot $\lg y$ against x and draw a straight line graph. [3]

- (ii) Use your graph to estimate the value of a and of b . [3]

$$y = \frac{b^x}{10^a} \Rightarrow$$

	$\lg y = \lg \left(\frac{b^x}{10^a} \right)$ $\lg y = \lg b^x - \lg 10^a$ $\lg y = x \lg b - a \quad \text{M1}$ $-a = 1.5$ $a = -1.5 \quad \text{A1 (accept -1.54 to -1.46)}$ $\lg b = 0.3$ $b = 2.00 \quad \text{A1 (accept 1.82 to 2.19)}$
	<p>(iii) Explain how you would use the graph to find the value of x for which $(10b)^x = 10^{a+1}$. [2]</p>
	$(10b)^x = 10^{a+1}$ $10^x b^x = 10^a \times 10$ $\frac{b^x}{10^a} = \frac{10}{10^x} \quad \text{M1}$ $y = 10^{1-x}$ $\lg y = \lg(10^{1-x})$ $\lg y = 1 - x$ <p>Draw the line $\lg y = 1 - x$ and find the x-coordinate of the point of intersection. A1</p> <p>OR</p> $(10b)^x = 10^{a+1}$ $x \lg(10b) = (a+1) \lg(10)$ $x \lg(10) + x \lg(b) = a+1$ $x \lg(b) - a = x+1$ $\lg y = x+1$

