# AHMAD IBRAHIM SECONDARY SCHOOL
## GCE O-LEVEL PRELIMINARY EXAMINATION 2025

## SECONDARY 4 EXPRESS

| Name: | Class: | Register No.: |
|---|---|---|
| | | |

**ADDITIONAL MATHEMATICS** 
Paper 1

**4049/01**
**12 August 2025**

Candidates answer on the Question Paper.

**2 hours 15 minutes**

**READ THESE INSTRUCTIONS FIRST**

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

| For Examiner's Use |
|---|
| /90 |

This document consists of **19** printed pages.

## *Mathematical Formulae*

### 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

### 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for* $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 It is given that $P$, $Q$ and $R$ are the angles of a triangle.

**(a)** Show that $\cos P = -\cos(Q + R)$. [2]

**(b)** Given that $Q = 45°$ and $R = 60°$, find $\cos P$ in the form $\frac{1}{4}(\sqrt{a} - \sqrt{b})$, where $a$ and $b$ are integers. [3]

2 Baking powder is poured onto a flat surface at a constant rate of $2\pi$ $cm^3s^{-1}$ and formed a right circular cone. The radius of the cone is always $\frac{1}{18}$ of its height. Find the rate of change of the radius of the cone after 3 seconds of pouring. [5]

**3** **(a)** Determine the set of values of $m$ for which the equation $2x^2+4x+2m=6mx-2$ has real roots. [4]

**(b)** Hence state what can be deduced about the curve $y=2(x+1)^2$ and the line $y=6x-2$. Justify your statement. [2]

4 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(\cos x\right)\right) = -\tan x$. [2]

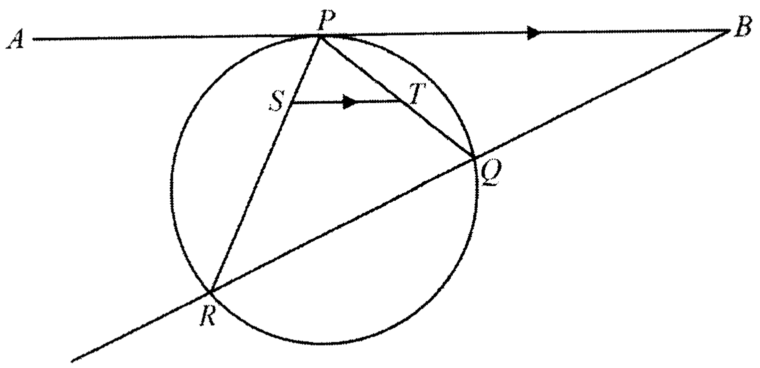
**(b)** Differentiate $x\tan x$ with respect to $x$. [2]

**(c)** Using the results from part **(a)** and **(b)**, find $\int x\sec^2 x \,\mathrm{d}x$ and hence show that $\int_0^{\frac{\pi}{4}} x\sec^2 x \,\mathrm{d}x = \frac{\pi}{4} - \frac{1}{2}\ln 2$. [4]

5 **(a)** In the expansion of $(2+x)^n$, where $n$ is a positive integer, the coefficient of $x^2$ is twice the coefficient of $x$. Find the value of $n$. [3]

**(b)** Find the value of the term that is independent of $x$ in the expansion of $\left(2x-\dfrac{1}{4x^4}\right)^{15}$. [4]

6



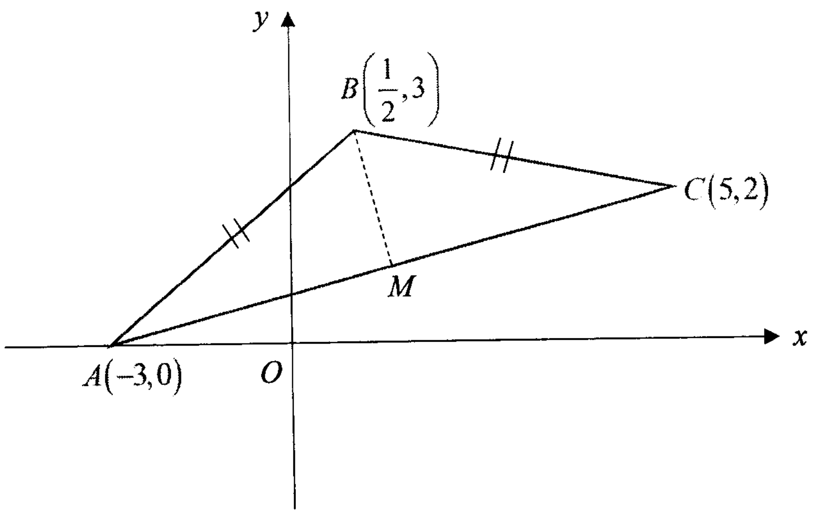

The diagram shows a circle passing through the points $P$, $Q$ and $R$. The point $Q$ lies on the line $RB$. $AB$ is a tangent to the circle at $P$. The points $S$ and $T$ lie on $PR$ and $PQ$ respectively. Given that $AB$ is parallel to $ST$, prove that

**(a)** triangle $PST$ is similar to triangle $PQR$, [3]

**(b)** $PQ \times PT = PR \times PS$, [2]

**(c)** Determine if *STQR* is a cyclic quadrilateral. [4]

**7**




The diagram shows an isosceles triangle $ABC$ in which $A(-3,0)$, $B\left(\frac{1}{2},3\right)$ and $C(5,2)$. $M$ is the foot of perpendicular from $B$ to $AC$.

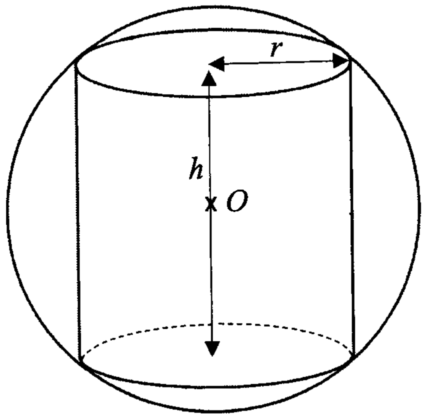
**(a)** Find the coordinates of $M$. [1]

**(b)** Find the equation of the perpendicular bisector of $AC$. [2]

**(c)** Given that $ABCD$ is a kite with $BM = \frac{2}{7}BD$ , find the coordinates of $D$. [3]

**(d)** Find the area of the kite $ABCD$. [2]

8




A prototype consists of a cylindrical container of height $h$ cm and radius $r$ cm inscribed in a hollow sphere with centre $O$.

The sphere has a surface area of $6400\pi$ cm$^2$ and both the sphere and container have negligible thickness.

**(a)** Show that the volume of the cylinder container, $V$ cm$^3$, is given by [3]

$$V = 2\pi r^2\sqrt{1600 - r^2}\,.$$

**(b)** Given that $r$ can vary, find the value of $r$ for which the volume $V$ is stationary. [5]

**(c)** A scientist plans to launch this prototype into outer space carrying as much fuel as possible. Explain whether the prototype can satisfy the scientist's requirement. [2]

9 It is given that $f(x) = 4 + \cos\left(\frac{x}{2}\right)$ and $g(x) = -2\sin x$.

**(a)** State the period and amplitude of f($x$). [2]

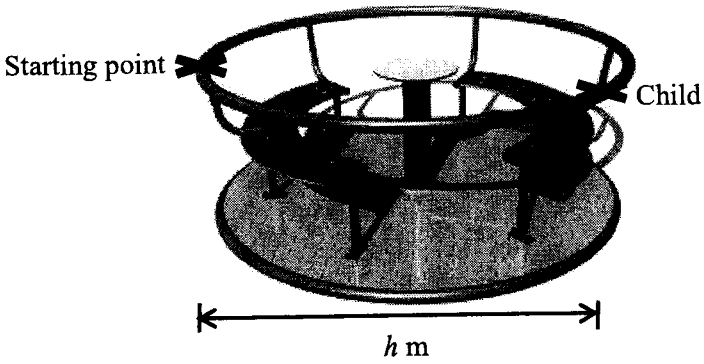
**(b)** State the period and amplitude of g($x$). [1]

**(c)** Sketch, on the same axes, the graphs of $y$ = f($x$) and $y$ = g($x$) for $0° \le x \le 360°$. [3]

**10** **(a)** Express $\frac{1-3x-3x^2}{x(x+1)^2}$ in partial fractions. [5]

**(b)** Hence find $\int \frac{1-3x-3x^2}{2x(x+1)^2}\,dx$. [4]

11




The horizontal distance of a child on a carousel, $h$ m, from the starting point is modelled by the equation, $h = 2(1 - \cos kt)$, where $k$ is a constant and $t$ is the time in seconds after the child leaves the starting point. The time to complete one revolution is 20 seconds.

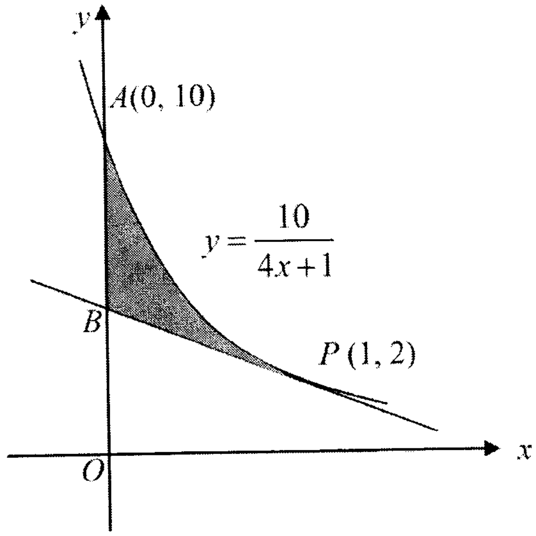
**(a)** Explain why this model suggests that the diameter of the carousel is 4 m. [1]

**(b)** Show that the value of $k$ is $\frac{\pi}{10}$ radians per second. [2]

**(c)** As the carousel turns, it is possible for the child on the carousel to view a landmark, provided that the horizontal distance of the child is within 1 m from the starting point.

Find the duration of time for which the child will not be able to view the landmark during one revolution. [5]

12




The diagram shows part of the curve $y = \frac{10}{4x+1}$ intersecting the $y$-axis at $A(0, 10)$. The tangent to the curve at the point $P(1, 2)$ intersects the $y$-axis at $B$.

**(a)** Show that the coordinates of $B$ is $(0, 3.6)$. [4]

**(b)** Find the **exact** area of the shaded region. [5]

**END OF PAPER**

# AHMAD IBRAHIM SECONDARY SCHOOL
## GCE O-LEVEL PRELIMINARY EXAMINATION 2025

## SECONDARY 4 EXPRESS

| Name: | Class: | Register No.: |
| --- | --- | --- |
| | | |

**ADDITIONAL MATHEMATICS**
Paper 2

**4049/01**
**13 August 2025**

Candidates answer on the Question Paper.

**2 hours 15 minutes**

**READ THESE INSTRUCTIONS FIRST**

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

| For Examiner's Use |
| --- |
| /90 |

***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 A curve has the equation $y = \dfrac{\sin 2x}{2 - \cos 2x}$.

**(a)** Show that the gradient function can be expressed in the form $\dfrac{k\cos 2x - 2}{(2 - \cos 2x)^2}$, where $k$ is a constant. [3]

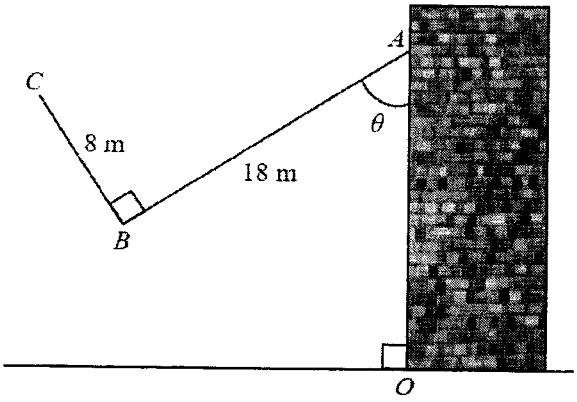
**(b)** Find the acute angle between the tangent to the curve at $x = \dfrac{\pi}{12}$ and the line $y = 0$. [3]

**2** **(a)** Factorise $x^3 + 27k^3$ as a product of a linear and a quadratic factor. [2]

**(b)** Hence solve $x^3 + 27 = (x+3)(x+10)$, expressing non-integer roots in surd form. [3]

**(c)** Find the value of $k$ given that $x^3 + 27k^3$ leaves a remainder of 351 when divided by $x - 2$. [2]

3 The diagram shows a $L$-shaped rod $ABC$ where $AB$ and $BC$ have length 18 m and 8 m respectively and angle $ABC$ is 90°. The rod is hinged to a wall at $A$ so as to rotate in a vertical plane. The rod $AB$ makes an acute angle $\theta$ with the vertical wall surface $OA$.




**(a)** Given that $G$ is a point directly below $C$, show that $OG = p\cos\theta + q\sin\theta$, where $p$ and $q$ are constants to be found. [2]

**(b)** Express $OG$ in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0° \leq \alpha \leq 90°$. [3]

**(c)** Find the length of $OG$ and the corresponding value of $\theta$ if $G$ is at maximum displacement from $O$. [3]

**4** A circle $C_1$ has equation $x^2+y^2-6x+4y=12$.

**(a)** Find the radius and the coordinates of the centre of $C_1$. [3]

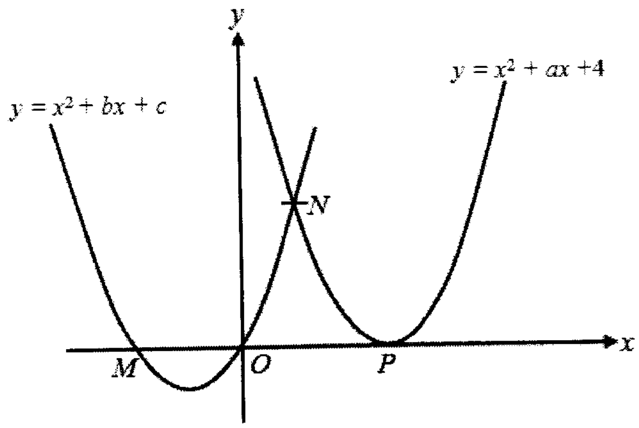
**(b)** Find the equation of the tangent to the circle at the point $P(7,-5)$. [3]

**(c)** Another circle $C_2$ has centre $(-8, 4)$ and radius 7 cm. Find the shortest distance between the 2 circles. [2]

**5** **(a)** Prove the identity $\dfrac{(\sin A - \cos A)(1 + \sin A \cos A)}{\cos^3 A} = \tan^3 A - 1$. [4]

**(b)** Hence solve $(\sin A - \cos A)(1 + \sin A \cos A) + 2\cos^3 A = 0$ exactly, for $-\pi \le A \le \pi$ radians. [4]

6 The diagram shows the graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$. The graph of $y = x^2 + ax + 4$ touches the $x$-axis at $P$. Points $M$ and $O$ are the $x$-intercepts of the graph of $y = x^2 + bx + c$. The origin $O$ is the mid-point of $MP$.




**(a)** Find the values of $a$, $b$ and $c$. [4]

**(b)** The graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$ intersects at $N$. Find the coordinates of $N$. [2]

**(c)** The graph $y = px^2 + qx + r$ has its turning point at $N$ and passes through point $P$. Find the values of $p$, $q$ and $r$, where $r > 0$. [3]

7 A particle $P$, travels in a straight line, so that its displacement, $s$ m, from $O$ at time $t$ seconds, is modelled by $s = \frac{1}{3}t^3 - 5t^2 - 3$.

**(a)** Find the value of $t$ when particle $P$ returns to its initial position. [2]

**(b)** Find the minimum velocity of particle $P$. [3]

**(c)** Another particle, $Q$, travels in a straight line from $O$ such that its velocity, $v$ m/s, at time $t$ seconds after passing $O$ is given by $v = 24\left(e^{-\frac{t}{6}} - e^{-1}\right)$.

Find the value of $t$ at which the particle $Q$ is instantaneously at rest. [2]

**(d)** Find the total distance travelled by particle $Q$ for the first 9 seconds. [4]

**8** The height, $h$ cm, of a plant is modelled by $h = \dfrac{80}{1+10e^{-0.4t}}$, where $t$ is the number of months after the first observation.

**(a)** Show that $h$ is an increasing function. [3]

**(b)** Find the value of $t$ when the height of the plant first exceeds four times its initial observation. [3]

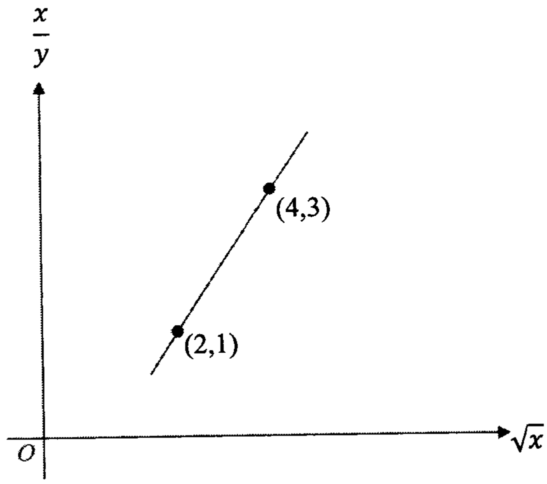
**(c)** The height, $y$ cm, of another species of plant, $t$ months after the first observation is given by $y = \frac{1}{a + be^{-t}}$, where $a$ and $b$ are constants. Explain clearly how a straight line graph can be drawn to represent this relationship. You should state which variables should be plotted on each axis and explain how the values of $a$ and $b$ can be calculated. [4]

**9 (a)** Without using a calculator, solve the equation $x\sqrt{15}+\sqrt{5}=x\sqrt{2}+\sqrt{6}$. Leave your answer in the form $p\sqrt{10}+q\sqrt{3}$, where $p$ and $q$ are fractions. [4]

**(b)** Without using a calculator, solve the equation $\log_2 x-\log_x 16=0$. [4]

**(c)** Solve the equation $3^{x+2} - 2\left(3^{-x}\right) = 17$. [4]

**10** **(a)** The diagram shows part of a straight line graph which passes through $(2,1)$ and $(4,3)$.




Find the equation of the straight line in the form $y = \dfrac{x}{a + b\sqrt{x}}$, where $a$ and $b$ are constants. [3]

BLANK PAGE

**(b)** The table below shows the experimental values of two variables $x$ and $y$.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $y$ | 63 | 127 | 258 | 510 | 1000 | 2100 |

It is known that $x$ and $y$ are related by an equation of the form $y = \frac{b^x}{10^a}$, where $a$ and $b$ are constants.

**(i)** On the grid next page, plot $\lg y$ against $x$ and draw a straight line graph. [3]

**(ii)** Use your graph to estimate the value of $a$ and of $b$. [3]

**(iii)** Explain how would you use the graph to find the value of $x$ for which $(10b)^x = 10^{a+1}$. [2]

**END OF PAPER**