NAME: ______________________________ (   )   CLASS: 4 (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2025

S4

ADDITIONAL MATHEMATICS

4049/01

Paper 1

22 August 2025
2 hours 15 minutes

Candidates answer on the Question Paper.
Additional Material: Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST
Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiners' Use

Question	Marks	Question	Marks		
1		8		Clarity / Logic	
2		9		Precision / Accuracy	
3		10			
4		11		Units	
5		12			
6		13		Total:	
7		14			90
Parent's Name & Signature:					
Date:					

This paper consists of **18** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Find the range of values of p for which $y = x^2 - x - 2$ lies entirely above the line $y = p(x+2)$. [4]

(b) Hence, deduce, without finding the discriminant, the number of intersection points when $p = 3$. [1]

2 By using substitution or otherwise, find the values of x for which $2^{2x-1} = 2^{x+3} - 24$, giving your answer where appropriate, to one decimal place. [5]

3 A closed circular cylinder has a volume of $\left(66\sqrt{2}+2\sqrt{3}\right)\pi$ cm^3, a radius of $\left(\sqrt{2}+2\sqrt{3}\right)$cm and a height h cm. Express h in the form of $p\sqrt{2}+q\sqrt{3}$ where p and q are integers. [4]

4 Express $\dfrac{2x^3-2x^2+7x-12}{x^3+4x}$ in partial fractions. [6]

5 Given the curve $y = kx^2 + 2kx - 3$, where k is a constant.

(a) By expressing y in the form $k(x+b)^2 - 1$, determine the value of k and of b. [3]

(b) Hence, show that the curve is always below the x-axis. [2]

(iii) State the turning point of the curve and determine the nature of this point. [2]

6 The binomial expansion of $(1+px)^n$ where $n > 0$, in ascending powers of x is

$$1-12x+28p^2x^2+qx^3+...$$

Find the value of p, of n and of q. [6]

7 **(a)** The surface area of a solid cube is increasing at $0.2\,\text{cm}^2/\text{s}$. Find the rate of increase of the volume when the length of a side is 1 cm. [4]

(b) A curve has the equation $y = e^{1-8x}(\cos 2x)$, where $0 \le x \le \frac{\pi}{4}$. Find the gradient of the curve at the point where $y = 0$, leaving your answer in exact form. [4]

8 The equation of a curve is $y=-\frac{5}{8}x+\frac{6}{x}$, where $x>0$. Given that the gradient of the normal at point P is 1, find

(a) the coordinates of P, [4]

(b) the equation of the normal at the curve at P. [2]

9 **(a)** Solve the equation $\log_2(x+2)-\log_{\sqrt{2}}(x-1)=1$. [4]

(b) It is given that $\lg a-\lg b=\lg(a+b)$.

(i) Express a in terms of b. [2]

(ii) State the range of b. [1]

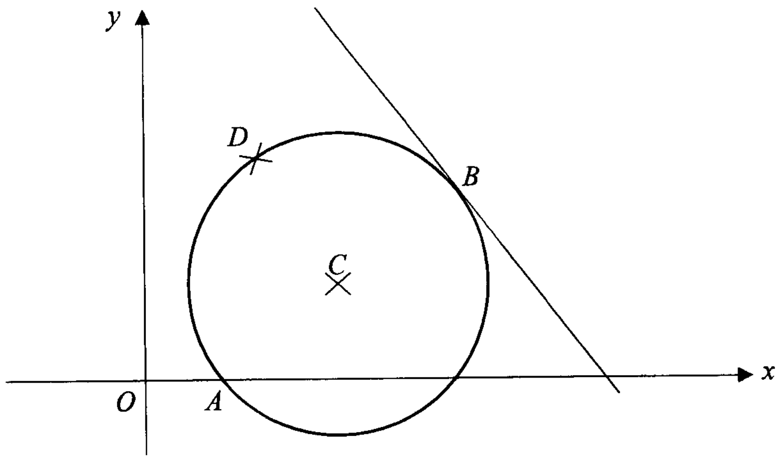
10 Given that $\sin A$ and $\cos B$ are in the same quadrant such that $\sin A = \frac{3}{5}$ and $\cos B = -\frac{5}{13}$.
Find, without using a calculator, the value of

(a) $\cos(A-B)$, [2]

(b) $\cos\frac{A}{2}$. [3]

11 Points A, $B(5,4)$ and D lie on a circle with centre C and AB is the diameter of the circle.

The line $y=-x+9$ is a tangent at point $B(5,4)$ on the circle.



(a) Given that point P lies on BA extended and the x-coordinate of point P is -1, find the coordinates of P. [2]

It is given that the distance BP is three times the distance BC.

(b) **(i)** Show that centre C is $(3,2)$. [1]

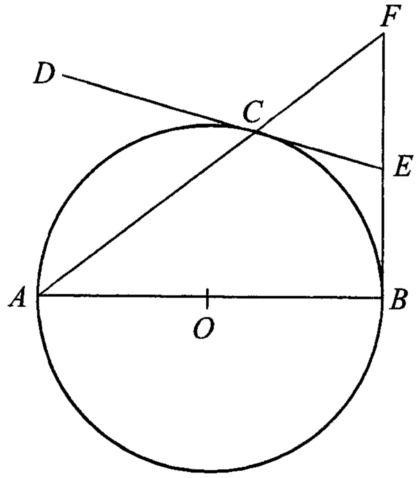
(ii) Hence, find the equation of the circle. [2]

(c) Given that triangle *BCD* is a right-angled triangle, find the coordinates of *D*. [3]

12 **(a)** Given that $y=\ln\left(\sin^2 x\right)$, show that $\frac{\mathrm{d}y}{\mathrm{d}x}=2\cot x$. [2]

(b) Hence, show that the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\cot x-\cos^2 2x\right)\mathrm{d}x=a+b\ln 2$, where a and b are constants. [4]

13 In the diagram, AB is a diameter of the circle with centre O. DE and BF are tangents to the circle at C and B respectively. DCE and BEF are straight lines. Prove that



(a) ΔABC and ΔBFC are similar, [3]

(b) hence or otherwise, show that ΔCFE is an isosceles triangle. [4]

14 The mass, m grams, of the decomposition of a radio-active substance is given by the formula $m = ae^{-bt}$, where a and b are constants, and t is the number of weeks after the initial measurement of the mass. Experimental values of t and m are given in the table below.

t	1	2	3	4	5
m	1.51	0.454	0.137	0.0183	0.0124

(i) What does the constant a refer to? [1]

(ii) By drawing a suitable straight line graph using the data provided, estimate the value of a and of b. [7]

(iii) It is discovered that one of the readings of the mass, m, is incorrect. Using your graph, estimate the correct value for this incorrect reading. [2]

NAME: ______________________________ (   )   CLASS: 4 (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2025

S4

ADDITIONAL MATHEMATICS **4049/02**
Paper 2 **26 August 2025**
2 hours 15 minutes

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1 **(a)** Show that $\frac{4}{\cot\theta - \tan\theta} = 2\tan 2\theta$. [4]

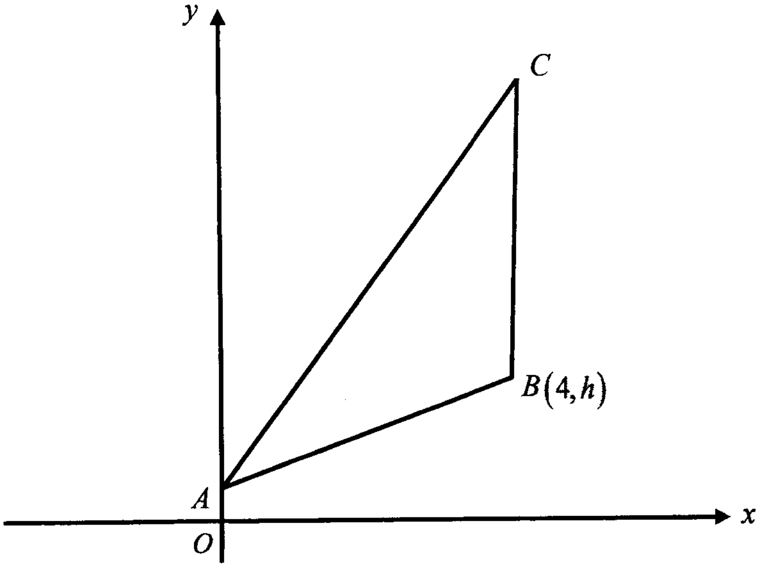
(b) Hence, solve the equation $\frac{4}{\cot 2x - \tan 2x} = 5$ for $0 < x < \pi$. [3]

2 The expression $4x^3 + ax + b$, where a and b are constants, has a factor of $x - 1$, and a remainder of -9 when divided by $x + 2$.

(a) Find the value of a and of b. [4]

(b) Hence solve the equation $4x^3 + ax + b = 0$ expressing the non-integer roots in the form of $\frac{c+\sqrt{d}}{2}$, where c and d are constants. [4]

3 The diagram shows an isosceles triangle ABC, where $AB = BC$ and the line segment BC is parallel to the y-axis. Point A lies on the y-axis and the equation of line AC is $y = 2x + 3$. Point B is $(4, h)$, where h is a positive constant.



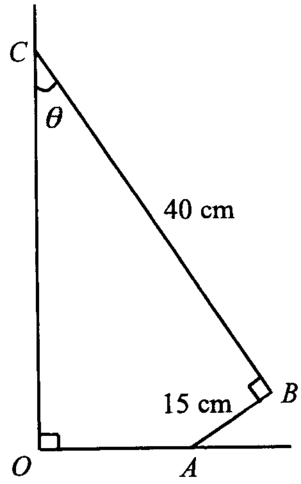
(a) Find the coordinates of A, B and C. [4]

Given that the point D is such that $ABCD$ is a kite, and the line segment CD is parallel to the x-axis.

(b) Find the coordinates of D. [4]

(c) Find the area of the kite $ABCD$. [2]

4 In the diagram, it is given that $AB = 15$ cm, $BC = 40$ cm and $\angle OCB = \theta$. The vertical line OC is perpendicular to OA and AB is perpendicular to BC.



(a) Show that $OC = 15\sin\theta + 40\cos\theta$. [2]

(b) Express OC in the form $R\sin(\theta + \alpha)$, where R is a positive constant and α is an acute angle in degrees. [3]

(c) Given that OC is at the maximum, determine the shortest distance of B to OC. [3]

5 **(a)** Show $\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\tan 3x\right)\right]=\frac{k}{\sin 6x}$, stating the value of the constant k. [4]

(b) Differentiate $\ln\sqrt{\frac{7x}{x^2-1}}$ with respect to x. [3]

6 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}''(x) = 2e^{x} + e^{-2x}$.

The y-axis is a normal to the curve at the point Q, where $y = 1$.

(a) Show that Q is a stationary point. [2]

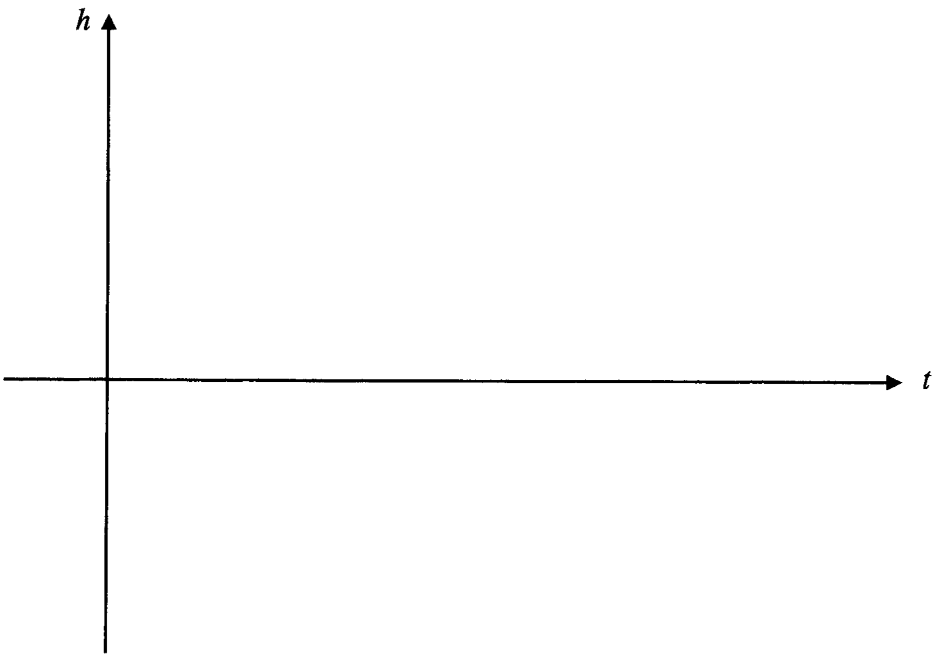
(b) Hence, find an expression for $\mathrm{f}(x)$. [6]

7 A waterwheel rotates at 1 revolution per 12 seconds. Initially, Point A, marked on the highest point of the rim of the wheel is 6 m above the water surface. It reaches 2 m below the water surface 6 seconds later.

The motion of point A can be modelled using the trigonometric equation $h = b\cos(kt) + 2$, where the height of point A above the water surface, h, is a function of time t, in seconds.

(a) Show that $b = 4$ and $k = \frac{\pi}{6}$. [2]

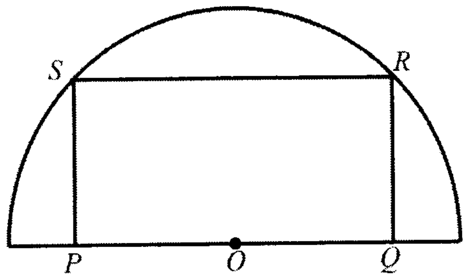
(b) On the axes below, sketch the graph of h, for $0 \le t \le 18$. [3]



A similar waterwheel at a different location has a point, B, marked on its rim. The motion of point B is modelled by the equation $h_B = 4\cos(\frac{\pi}{6}t) + 4$.

(c) By drawing a suitable straight line on the same axes above, determine the number of time(s) when point B touches the water surface from $0 \le t \le 18$. [2]

8 In the figure, $PQRS$ is a rectangle which fits inside a semicircle of radius 8 cm and centre O.

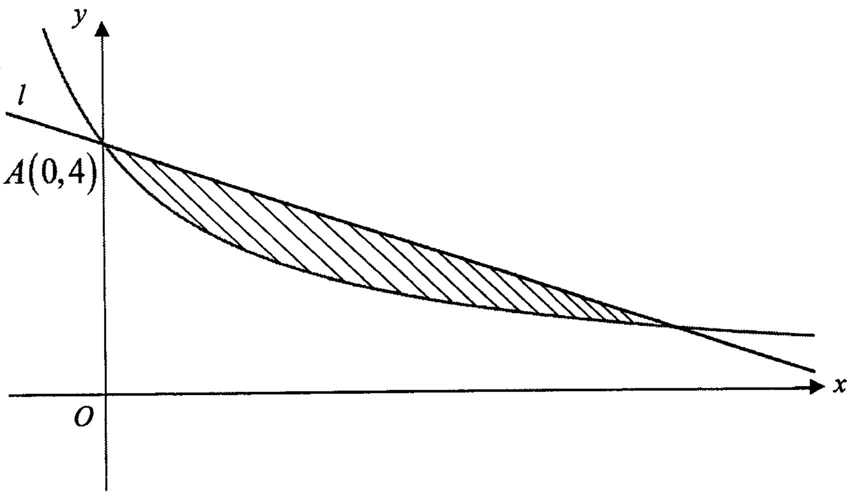


If $PQ = x$ cm and $QR = y$ cm,

(a) Show that the area of the rectangle, A cm^2, is given by $A = \frac{x}{2}\sqrt{256 - x^2}$. [3]

(b) The maximum area is obtained when the value of x, which can vary, gives a stationary value of A. Find the maximum value of A. [6]

9 The diagram shows part of the curve $y = \dfrac{8}{2+3x}$. A line, l, passes through the curve at point $A(0,4)$ and is parallel to the tangent to the curve at $x = -2$.



(a) Show that the equation of line l is $y = -\dfrac{3}{2}x + 4$. [3]

(b) Find the area of the shaded region and leave your answer in the form $a+b\ln 2$. [6]

10 A particle travels in a straight line such that at time t seconds after leaving a point A, which has a displacement of 2 metres from a fixed point O, its velocity, v ms^{-1} is given by

$v = 3t^2 - 21t + 18$.

Find,

(a) the minimum velocity of the particle, [3]

(b) the time(s) at which the particle is instantaneously at rest, [1]

(c) the total distance travelled in the first 7 seconds. [6]

11 **(a)** Given the function $y = \frac{m - 2x}{x^2 - 3}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(b) Hence, find the range of values of m such that the function $y = \frac{m - 2x}{x^2 - 3}$ is always increasing. [4]

END OF PAPER