MARKING SCHEME
(Bedok South) 4E5N AM P1 (Prelim 2025)

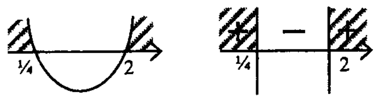
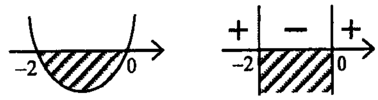
Qn	Solution	Marks	Testing
1	**Trigonometric Identities**		
	LHS		
	$= \sin 2x - \cos 2x \tan x$		
	$= 2\sin x\cos x - (2\cos^2 x - 1)\frac{\sin x}{\cos x}$	M2	Apply double angle for sine & cosine
	$= 2\sin x\cos x - 2\sin x\cos x + \frac{\sin x}{\cos x}$	M1	Expand & simplify
	$= \tan x$	A1	
	$= RHS$		
2	**Exponential Equations**		
	$2^{2(x+2)} \times 5 = 5^{x+3} \times 8^x$		
	$2^4(2^{2x}) \times 5 = 5^3(5^x) \times 2^{3x}$	M1	Split the powers
	$\frac{2^{2x}}{(2^{3x})(5^x)} = \frac{5^3}{2^4 \times 5}$	M1	Separate factors with x and non-x
	$\frac{1}{(2^x)(5^x)} = \frac{25}{16}$		
	$\frac{1}{10^x} = \left(\frac{5}{4}\right)^2$		Combine common index
	$10^x = \left(\frac{4}{5}\right)^2$		
	$x = \lg\left(\frac{4}{5}\right)^2$	M1	Express in logarithm
	$\mathbf{x = 2\lg\left(\frac{4}{5}\right)}$	A1	Power law
	OR		
	$2^{2(x+2)} \times 5 = 5^{x+3} \times 8^x$		
	$2^{2x+4} \times 5 = 5^{x+3} \times 2^{3x}$	M1	Convert to prime bases
	$\frac{2^{2x+4}}{2^{3x}} = \frac{5^{x+3}}{5}$	M1	Separate factors of different bases
	$\frac{2^4}{2^x} = 5^x5^2$		
	$10^x = \frac{2^4}{5^2}$		Combine common index
	$10^x = \frac{4^2}{5^2}$		
	$x = \lg\left(\frac{4}{5}\right)^2$	M1	Express in logarithm
	$\mathbf{x = 2\lg\left(\frac{4}{5}\right)}$	A1	Power law
3	**Surds Expressions**		
	$x\sqrt{18} = 3x + \sqrt{32}$		
	$3x\sqrt{2} - 3x = 4\sqrt{2}$		
	$3x(\sqrt{2} - 1) = 4\sqrt{2}$	M1	Grouping
	$3x = \frac{4\sqrt{2}}{\sqrt{2} - 1}$		
	$= \frac{4\sqrt{2}\ (\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$	M1	Rationalise denominator with conjugate
	$= \frac{8 + 4\sqrt{2}}{(\sqrt{2})^2 - (1)^2}$	M1	Expansion
		M1	Difference of squares
	$\mathbf{x = \frac{8 + 4\sqrt{2}}{3}}$	A1	
	OR		
	$x\sqrt{18} = 3x + \sqrt{32}$		
	$3\sqrt{2}x = 3x + 4\sqrt{2}$		
	$(3\sqrt{2} - 3)x = 4\sqrt{2}$	M1	Grouping
	$x = \frac{4\sqrt{2}}{3\sqrt{2} - 3}$		
	$= \frac{4\sqrt{2}\ (3\sqrt{2}+3)}{(3\sqrt{2}-3)(3\sqrt{2}+3)}$	M1	Rationalise denominator with conjugate
	$= \frac{24 + 12\sqrt{2}}{(3\sqrt{2})^2 - (3)^2}$	M1	Expansion
		M1	Difference of squares
	$= \frac{24 + 12\sqrt{2}}{9}$		
	$\mathbf{= \frac{8 + 4\sqrt{2}}{3}}$	A1	
4	**Surds (Application)**		
(a)	Area of square $= 11 + \sqrt{120}$		
	$(\sqrt{a} + \sqrt{b})^2 = 11 + 2\sqrt{30}$		
	$a + b + 2\sqrt{ab} = 11 + 2\sqrt{30}$	M1	Expand
	Equating the <u>rational</u> part,	M1	Equating rational & irrational parts separately
	$a + b = 11$		
	$b = 11 - a$......(1)		
	Equating the <u>irrational</u> part,		
	$2\sqrt{ab} = 2\sqrt{30}$		
	$ab = 30$ ......(2)		
	Subst. (1) into (2),		
	$a(11 - a) = 30$	M1	Substitution method
	$11a - a^2 = 30$		
	$a^2 - 11a + 30 = 0$		
	$(a - 5)(a - 6) = 0$		
	$a = 5$ or $a = 6$	M1	Solve quadratic equation
	Corresponding b values:		
	$b = 6$ or $b = 5$		
	Since $a < b$		
	$\mathbf{a = 5,\ b = 6}$	A1	Correct set of values

Qn	Solution	Marks	Testing
5	**Simultaneous Equations**		
	$4x - y = 5 \quad \ldots\ldots [1]$ $y = 2x^2 - 6x + 7 \quad \ldots\ldots [2]$ Subst (2) into (1) $4x - (2x^2 - 6x + 7) = 5$	M1	Substitution or Elimination Method
	$4x - 2x^2 + 6x - 7 = 5$ $2x^2 - 10x + 12 = 0$ $x^2 - 5x + 6 = 0$ $(x-2)(x-3) = 0$ $x = 2 \ \ or \ \ x = 3$	M1	Factorisation or Quadratic Formula
	Subst respective x values into [1], $y = 4(2) - 5$, $y = 3$ | $y = 4(3) - 5$, $y = 7$	M1	Find values of 2nd variable
	$P(2,3)$ and $Q(3,7)$ $PQ = \sqrt{(3-2)^2 + (7-3)^2}$	M1	Apply Distance formula
	$\boldsymbol{PQ = \sqrt{17}}$	A1	
6	**Binomial Theorem**		
	General term of expansion $\left(2 - \frac{1}{2}x\right)^8$ is $T_{r+1} = \binom{8}{r}(2)^{8-r}\left(-\frac{x}{2}\right)^r$ $= \binom{8}{r}(-1)^r(2)^{8-2r}x^r$	M1	General term of binomial expansion
	$T_3 = \binom{8}{2}(-1)^2(2)^{8-2(2)}x^2$ $= 448x^2$	A1	Extract specific terms
	$T_4 = \binom{8}{3}(-1)^3(2)^{8-2(3)}x^3$ $= -224x^3$	A1	
	$(10 + kx)\left(2 - \frac{1}{2}x\right)^8$ $= (10 + kx)(\ldots 448x^2 - 224x^3 \ldots)$ Coeff. of $x^3 = 0$ $(10)(-224) + k(448) = 0$ $448k = 2240$	M1	Equating coefficient
	$\boldsymbol{k = 5}$	A1	
7	**Trigonometric Equations**		
	$\frac{5\cos x + \sin 2x}{\cos 2x + 5\sin x} = \cot x$ $\frac{5\cos x + \sin 2x}{\cos 2x + 5\sin x} = \frac{\cos x}{\sin x}$ $\sin x(5\cos x + \sin 2x)$ $= \cos x(\cos 2x + 5\sin x)$	M1	Convert to $\sin x$ & $\cos x$
	$\sin x(5\cos x + 2\sin x\cos x)$ $= \cos x(\cos^2 x - \sin^2 x + 5\sin x)$	M1	Apply double angle formula $\sin 2x$ & $\cos 2x$
	$5\sin x\cos x + 2\sin^2 x\cos x$ $= \cos^3 x - \sin^2 x\cos x + 5\sin x\cos x$ $2\sin^2 x\cos x = \cos^3 x - \sin^2 x\cos x$ $3\sin^2 x\cos x - \cos^3 x = 0$	M1	Expand & simplify
	$\cos x(3\sin^2 x - \cos^2 x) = 0$ $\cos x = 0$, $x = 90°$ | $3\sin^2 x - \cos^2 x = 0$	M1	Zero product rule
	$3\sin^2 x = \cos^2 x$ $3\tan^2 x = 1$ $\tan^2 x = \frac{1}{3}$ $\tan x = \pm\frac{1}{\sqrt{3}}$ $x = 30°,\ 150°$	A1	Either solution correct
	$\boldsymbol{\therefore x = 30°,\ 90°,\ 150°}$	A1	Final solution
8	**Differentiation (Product Rule)**		
(a)	$\frac{d}{dx}[x(x-3)^4]$ $= (x-3)^4 + 4x(x-3)^3$	M1	Apply Product Rule
	$= (x-3)^3(x - 3 + 4x)$ $\boldsymbol{= (5x-3)(x-3)^3}$	A1	
	Differentiation (Quotient Rule)		
(b)	$y = \frac{x(x-3)^4}{5x-3}$ $\frac{dy}{dx} = \frac{(5x-3)(x-3)^3(5x-3) - 5x(x-3)^4}{(5x-3)^2}$	M1	Apply Quotient Rule
	$= \frac{(x-3)^3((5x-3)^2 - 5x(x-3))}{(5x-3)^2}$ $= \frac{(x-3)^3(25x^2 - 30x + 9 - 5x^2 + 15x)}{(5x-3)^2}$ $\boldsymbol{= \frac{(x-3)^3(20x^2 - 15x + 9)}{(5x-3)^2}}$	M1	Factorise & simplify
	When $x = 1$, $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $-70 = \frac{(x-3)^3(20x^2 - 15x + 9)}{(5x-3)^2} \times \frac{dx}{dt}$ $= \frac{(1-3)^3(20 - 15 + 9)}{(5-3)^2} \times \frac{dx}{dt}$ $-70 = -28 \times \frac{dx}{dt}$	M1	Rate of change
	$\boldsymbol{= 2.5}$ **units/s**	A1	

Qn	Solution	Marks	Testing
9	**Quadratic Functions**		
(a)	$y = 5 - 12x - 3x^2$		
	$= -3x^2 - 12x + 5$		
	$= -3(x^2 + 4x) + 5$		
	$= -3\{(x+2)^2 - (2)^2\} + 5$	M1	Completing the Square
	$= -3(x+2)^2 + 12 + 5$		
	$\mathbf{y = -3(x+2)^2 + 17}$	A1	
	Since $(x+2)^2 \geq 0$		
	$-3(x+2)^2 \leq 0$	B1	
	$-3(x+2)^2 + 17 \leq 17$		
	$\max y = 17 < 20$,		
	$\therefore$ for all values of y, it can never be greater than 20.		
(b)	Given		
	$y = (k-1)x^2 + 2(k+2)x + k + 3$		
	For y to be always positive,		
	(i) Coeff of $x^2 > 0$, and		
	(ii) Discriminant < 0.		2 conditions for $y > 0$
	From (i), $k - 1 > 0$		
	$k > 1$	D1	
	From (ii),		
	$\{2(k+2)\}^2 - 4(k-1)(k+3) < 0$	M1	
	$4(k+2)^2 - 4(k-1)(k+3) < 0$		
	$(k+2)^2 - (k-1)(k+3) < 0$		$D = b^2 - 4ac$
	$(k^2 + 4k + 4) - (k^2 + 2k - 3) < 0$		
	$2k + 7 < 0$		
	$k < -\frac{7}{2}$	D1	
	From the two inequalities,		
	$k > 1$ and $k < -\frac{7}{2}$, there are no values of k for which $y > 0$.	A1	
10	**Quadratic Inequalities (Word Problem)**		
(a)	Since $Side > 0$,		
	$3x - 5 > 0$ and $x - 10 > 0$		Form Side inequalities
	$x > \frac{5}{3}$ and $x > 10$ ......(1)	M1	
	$Area\ of\ rectangle \leq 200$		
	$(3x-5)(x-10) \leq 200$	M1	Form Area inequality
	$3x^2 - 35x + 50 \leq 200$		
	$3x^2 - 35x - 150 \leq 0$		
	$(3x+10)(x-15) \leq 0$	M1	Factorisation
	$-\frac{10}{3} \leq x \leq 15$ ......(2)		
	From (1) & (2),		
	$\mathbf{10 < x \leq 15}$	A1	

Qn	Solution	Marks	Testing
(b)	To enclose the field,		
	$Perimeter \leq 98$		Form Perimeter inequality
	$2(3x-5) + 2(x-10) \leq 98$	M1	
	$2(4x-15) \leq 98$		
	$4x - 15 \leq 49$		
	$x \leq 16$ ......(3)	A1	
	From part (a) $10 < x \leq 15$ and (3) $x \leq 16$, it is **enough** to enclose the field.	B1	
11	**Logarihemic Equations**		
(a)	For a valid logarithm,		
	$4 - x > 0$		
	$\therefore\ x < 4$	B1	
(b)	$2\log_3(4-x) - \log_9(x-4)^2 = 2$		
	$2\log_3(4-x) - \frac{\log_3(x-4)^2}{\log_3 9} = 2$	M1	Change-base law
	$\log_3(4-x)^2 - \frac{\log_3(x-4)^2}{\log_3 3^2} = 2$	M1	Power law
	$\log_3(4-x)^2 - \frac{\log_3(x-4)^2}{2} = 2$		Identity property
	$2\log_3(4-x)^2 - \log_3(x-4)^2 = 4$		
	$\log_3(4-x)^4 - \log_3(x-4)^2 = 4$		
	$\log_3\frac{(4-x)^4}{(x-4)^2} = 4$	M1	Quotient Law
	$\frac{(4-x)^4}{(x-4)^2} = 3^4$	M1	Convert Logarithmic form to Exponential form
	$(4-x)^4 = 81(x-4)^2$		
	$(x-4)^4 - 81(x-4)^2 = 0$		
	$(x-4)^2((x-4)^2 - 81) = 0$		Solve factorised equation
	$(x-4)^2 = 0$ or $(x-4)^2 = 81$		
	$x - 4 = 0$ or $x - 4 = \pm 9$		
	$x = 4$ or $x = -5$ or $x = 13$	A1	
	From (a), since $x < 4$,		
	$\therefore x = -5$ is the only one real solution	B1	
	OR		
	$2\log_3(4-x) - \log_9(x-4)^2 = 2$	M1	Change-base law
	$2\log_3(4-x) - \frac{\log_3(x-4)^2}{\log_3 9} = 2$		
	$\log_3(4-x)^2 - \frac{\log_3(x-4)^2}{\log_3 3^2} = 2$	M1	Power law
	$\log_3(4-x)^2 - \frac{\log_3(x-4)^2}{2} = 2$		Identity property
	$\frac{\log_3(4-x)^2}{2} = 2$		
	$\log_3(4-x)^2 = 4$		
	$(4-x)^2 = 3^4$	M1	Convert Logarithmic form to

Qn	Solution	Marks	Testing
	$x^2 - 8x + 16 = 81$		Exponential form
	$x^2 - 8x - 65 = 0$	M1	
	$(x+5)(x-13) = 0$		Writing $\log_3(x-4)$ is not correct as $4 - x > 0$, i.e. $x - 4 < 0$
	$x = -5$ or $x = 13$	A1	
	From (a), since $x < 4$,		
	$\therefore x = -5$ is the only one real solution	B1	
	OR		
	$2\log_3(4-x) - \log_9(x-4)^2 = 2$	M1	Change-base law
	$2\log_3(4-x) - \dfrac{\log_3(x-4)^2}{\log_3 9} = 2$		
	$\log_3(4-x)^2 - \dfrac{\log_3(x-4)^2}{\log_3 3^2} = 2$	M1	Power law
	$\log_3(4-x)^2 - \dfrac{\log_3(x-4)^2}{2} = 2$		Identity property
	$\log_3(4-x)^2 - \log_3(4-x) = 2$		
	$\log_3 \dfrac{(4-x)^2}{(4-x)} = 2$	M1	Quotient law
	$\dfrac{(4-x)^2}{(4-x)} = 3^2$	M1	Convert Logarithmic form to Exponential form
	$x^2 - 8x + 16 = 9(4-x)$		
	$x^2 - 8x + 16 = 36 - 9x$		
	$x^2 + x - 20 = 0$		Writing $\log_3(x-4)$ is not correct as $4 - x > 0$, i.e. $x - 4 < 0$
	$(x+5)(x-4) = 0$		
	$x = -5$ or $x = 4$	A1	
	From (a), since $x < 4$,		
	$\therefore x = -5$ is the only one real solution	B1	
12	**Quadratic Equations**		
(a)	When $y = 0$		
	$hx^2 - 2x - 9h + 6 = 0$		
	$hx^2 - 2x + (6 - 9h) = 0$		
	For a quadratic curve,		
	coeff of $x^2 \neq 0$,		
	$h \neq 0$ … [1]	M1	
	$D = (-2)^2 - 4h(6 - 9h)$	M1	$D = b^2 - 4ac$
	$= 4 - 4h(6 - 9h)$		
	$= 4(1 - 3h(2 - 3h))$		
	$= 4(9h^2 - 6h + 1)$		
	$= 4(3h-1)^2$ or $(6h-2)^2$	M1	
	$D \geq 0$ …[2]		
	$\therefore$ from [1] & [2], $y = 0$ has real roots for all values of h except $h = 0$.		

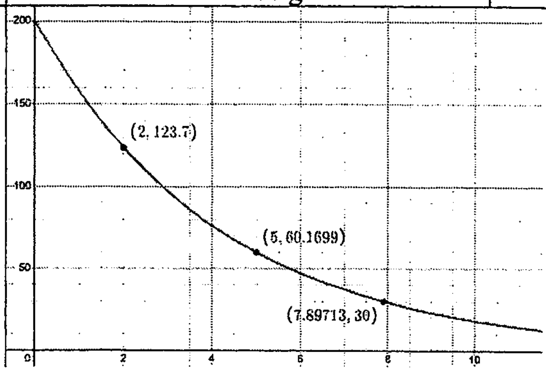
Qn	Solution	Marks	Testing
(b)	For 2 real and equal roots,		
	$D = 0$		
	$4(3h-1)^2 = 0$		
	$h = \dfrac{1}{3}$	A1	
(c)	For line meeting the curve,		
	$hx^2 - 2x - 9h + 6 = 2x - h - 12$		
	$hx^2 - 4x + 18 - 8h = 0$	M1	
	$D \geq 0$		
	$(-4)^2 - 4h(18 - 8h) \geq 0$	M1	
	$16 - 8h(9 - 4h) \geq 0$		
	$2 - h(9 - 4h) \geq 0$		
	$4h^2 - 9h + 2 \geq 0$		
	$(4h-1)(h-2) \geq 0$	M1	
			
	$h \leq \frac{1}{4}$ or $h \geq 2$ ……[3]	A1	
	From [1] & [3],		
	$h \leq \frac{1}{4}$ **or** $h \geq 2$ **and** $h \neq 0$	A1	
13	**Increasing / Decreasing Fns**		
(a)	$f(x) = x^2 e^{(x+2)}$		
	$f'(x) = 2xe^{(x+2)} + x^2 e^{(x+2)}$	M1	Apply Product Rule
	$= xe^{(x+2)}(2 + x)$		
	$= x(x+2)e^{(x+2)}$	A1	
	For decreasing function,		
	$f'(x) < 0$		
	$x(x+2)e^{(x+2)} < 0$	M1	Decreasing function
	since $e^{(x+2)} > 0$		Must explain exponential function is positive for all values of x
	$x(x+2) < 0$	M1	
			
	$-2 < x < 0$		
	Maxima & Minima		
(b)	$f'(x) = (x^2 + 2x)e^{(x+2)}$		
	$f''(x)$		
	$= (2x + 2)e^{(x+2)} + (x^2 + 2x)e^{(x+2)}$	M1	Apply Product Rule
	$f''(x) = (x^2 + 4x + 2)e^{(x+2)}$		
	For the least gradient,		
	$f''(x) = 0$		

Qn	Solution	Marks	Testing
	$(x^2+4x+2)e^{(x+2)}=0$	M1	
	$x^2+4x+2=0$		Quadratic formula or Completing the Square
	$x=\dfrac{-4\pm\sqrt{4^2-4(1)(2)}}{2(1)}$	M1	
	$=\dfrac{-4\pm\sqrt{8}}{2}$		
	$=\dfrac{-4\pm2\sqrt{2}}{2}$		
	$=-2\pm\sqrt{2}$	A1	
	Since the least gradient lies in $-2<x<0$, $x=-2+\sqrt{2}$		
	Least gradient $f'(x)=x(x+2)e^{(x+2)}$		
	$f'(-2+\sqrt{2})$		
	$=(-2+\sqrt{2})(-2+\sqrt{2}+2)e^{(-2+\sqrt{2}+2)}$		
	$=(-2+\sqrt{2})\sqrt{2}e^{\sqrt{2}}$		
	$=(2-2\sqrt{2})e^{\sqrt{2}}$	A1	
14	**Remainder & Factor Thm**		
(a)	Given $f(x)=3x^3+2x^2+16$.		
	$f(3a)=f\left(-\frac{2}{3}\right)$		
	$3(3a)^3+2(3a)^2+16$ $=3\left(-\frac{2}{3}\right)^3+2\left(-\frac{2}{3}\right)^2+16$	M1	Substitute values correctly
	$3(27a^3)+2(9a^2)$ $=3\left(-\frac{8}{27}\right)+2\left(\frac{4}{9}\right)$		
	$81a^3+18a^2=-\frac{8}{9}+\frac{8}{9}$		
	$9a^2(9a+2)=0$	M1	Factorisation
	$\boldsymbol{a=0}$ **or** $\boldsymbol{a=-\frac{2}{9}}$	A1	
(b)	Subst $x=-2$ into $f(x)$,		
	$f(-2)=3(-2)^3+2(-2)^2+16$		
	$=-24+8+16$		Apply Factor Theorem
	$=0$	A1	
	$\therefore\ x=-2$ is a solution of $f(x)$.		

Qn	Solution	Marks	Testing
(c)	From (b), $x+2$ is a factor of $f(x)$.		
	By inspection,		
	$f(x)=3x^3+2x^2+16$ $=(x+2)(3x^2+kx+8)$	M1	
	Equating coefficient of		
	x^2 : $k+6=2$, $k=-4$	M1	Comparing coefficients
	or		or
	x : $2k+8=0$, $k=-4$		Long Division
	$f(x)=3x^3+2x^2+16$ $=(x+2)(3x^2-4x+8)$	M1	Factorisation
	For $f(x)=0$		
	$(x+2)(3x^2-4x+8)=0$		
	$x+2=0$		
	$x=-2$		Solution for linear equation
	or $3x^2-4x+8=0$		
	$D=(4)^2-4(3)(8)$	M1	Use of discriminant or Quadratic formula
	$D=-80$		
	Since $D<0$, there is no real roots for the quadratic equation	B1	
	$\therefore\ f(x)=0$ has only one real root $x=-2$.		
(d)	$16y^3+2y-3=0$		
	$16+\frac{2}{y^2}-\frac{3}{y^3}=0$		
	$3\left(-\frac{1}{y}\right)^3+2\left(-\frac{1}{y}\right)^2+16=0$	M1	Transform into the required format
	Let $x=-\frac{1}{y}$		
	$3x^3+2x^2+16=0$		
	$x=-2$		
	$-\frac{1}{y}=-2$		
	$\boldsymbol{y=\frac{1}{2}}$	A1	

MARKING SCHEME

(Bedok South) 4E5N AM P2

Qn	Solution	Testing
1.	**Exponential Equations**	
(a)	$2(4^x) - 8 = 11(2^x) - 2^{2x+3}$ $2(2^{2x}) - 8 = 11(2^x) - (2^{2x})2^3$ $2(2^x)^2 - 8 = 11(2^x) - 8(2^x)^2$ Let $u = 2^x$ $2u^2 - 8 = 11u - 8u^2$ $10u^2 - 11u - 8 = 0$ $(2u+1)(5u-8) = 0$ $u = -\frac{1}{2}$ or $u = \frac{8}{5}$ $2^x = -\frac{1}{2}$ (reject) $2^x = \frac{8}{5}$ $\lg 2^x = \lg\frac{8}{5}$ $x = \frac{\lg\frac{8}{5}}{\lg 2}$ $= 0.678072$ $= \mathbf{0.678}$ (to 3 sig. fig.)	Convert to base 2 Apply substitution correctly Solving for u Apply logarithm appropriately
(b)	From (a), $2^x = \frac{8}{5}$ $\log_2 2^x = \log_2 \frac{8}{5}$ $x\log_2 2 = \log_2 8 - \log_2 5$ $x = \log_2 2^3 - \log_2 5$ $= 3\log_2 2 - \log_2 5$ $= 3 - \log_2 5$ $= m - \log_n p$ $\mathbf{m = 3,\ n = 2,\ p = 5}$	Apply $\log_2$ onto equation Power law Quotient law Identity property
2.	**Exponential Functions (Word Problems)**	
(a) (i)	When $t = 0$, $M = 200e^{-kt}$ $M_0 = 200e^{-k(0)}$ $= \mathbf{200\ g}$	
(a) (ii)	When $t = 2, M = 123.7$ $M = 200e^{-kt}$ $123.7 = 200e^{-k(2)}$ $\frac{123.7}{200} = e^{-2k}$ $\ln\left(\frac{123.7}{200}\right) = -2k$ $k = -\frac{1}{2}\ln\left(\frac{123.7}{200}\right)$ $= 0.240229$ $= \mathbf{0.240}$ (to 3 sig. fig.)	Take natural logarithm on both sides of equation
(a) (iii)	When $t = 5$, For $k = 0.24$: $M = 200e^{-kt}$ $= 200e^{-0.24(5)}$ $= 60.23884$ For $k = 0.240229$: $M = 200e^{-kt}$ $= 200e^{-0.240229(5)}$ $= 60.16991$ $= \mathbf{60.2\ g}$ (to 3 sig. fig.)	
(a) (iv)	$M = M_0e^{-kt}$ $\frac{M}{M_0} = e^{-kt}$ $15\% = e^{-kt}$ $0.15 = e^{-kt}$ For $k = 0.24$: $0.15 = e^{-0.24t}$, $\ln 0.15 = -0.24t$, $t = -\frac{\ln 0.15}{0.24}$ $= 7.90467$ For $k = 0.240229$: $0.15 = e^{-0.240229t}$, $\ln 0.15 = -0.240229t$, $t = -\frac{\ln 0.15}{0.240229}$ $= 7.89713$ $t = \mathbf{7.90\ years}$ (to 3 sig. fig.)	Take natural logarithm on both sides of equation
(b)	Since for $t \geq 0$, $e^{kt} \geq 1$ $\frac{1}{e^{kt}} \leq 1$ $e^{-kt} \leq 1$ $200e^{-kt} \leq 200$ $M \leq 200$ $\therefore$ mass of substance can never be more than 200 g.	
(c)		

G1 – Initial state (0, 200)
G1 – Smooth curve decreasing towards asymptote t-axis

Qn	Solution	Testing
3.	**Differentiation (Product Rule)**	
(a)	For square root to be defined, $2x-5\geq 0 \quad \Rightarrow \quad x\geq \frac{5}{2}$ $\therefore\ \boldsymbol{h=\frac{5}{2}}$	Square root criterion
	$y=x^2\sqrt{2x-5}$ $y=x^2\ (2x-5)^{\frac{1}{2}}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=2x(2x-5)^{\frac{1}{2}}$ $+x^2(2)\frac{1}{2}(2x-5)^{-\frac{1}{2}}$	Apply Product rule
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2x\sqrt{2x-5}+\frac{x^2}{\sqrt{2x-5}}$ $=\frac{2x(2x-5)+x^2}{\sqrt{2x-5}}$ $=\frac{2x(2x-5)+x^2}{\sqrt{2x-5}}$ $=\frac{5x^2-10x}{\sqrt{2x-5}}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5x(x-2)}{\sqrt{2x-5}}$ $\therefore\ \boldsymbol{k=5}$	Combine to single fraction
	Integration (Reverse of Differentiation)	
(b)	$\int_3^7\left\{\frac{10x(x-2)}{\sqrt{2x-5}}+6\right\}\mathrm{d}x$ $=2\int_3^7\frac{5x(x-2)}{\sqrt{2x-5}}\mathrm{d}x+\int_3^7 6\,\mathrm{d}x$	Split into 2 integrals
	$=2\left[x^2\sqrt{2x-5}\right]_3^7+6[x]_3^7$	Integration as reversed differentiation
	$=2\left\{7^2\sqrt{2(7)-5}-3^2\sqrt{2(3)-5}\right\}$ $+6(7-3)$ $=2\{49\sqrt{9}-9\sqrt{1}\}+24$ $=276+24$ $\boldsymbol{=300}$	Substitution of boundaries
	OR $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5x(x-2)}{\sqrt{2x-5}}$ $\int_3^7\frac{5x(x-2)}{\sqrt{2x-5}}\mathrm{d}x=\left[x^2\sqrt{2x-5}\right]_3^7$	Integration as reversed differentiation

Qn	Solution	Testing
	$\int_3^7\frac{10x(x-2)}{\sqrt{2x-5}}\mathrm{d}x=2\left[x^2\sqrt{2x-5}\right]_3^7$ $\int_3^7\frac{10x(x-2)}{\sqrt{2x-5}}\mathrm{d}x+\int_3^7 6\,\mathrm{d}x$	Transform to required integral
	$=2\left\{7^2\sqrt{2(7)-5}-3^2\sqrt{2(3)-5}\right\}$ $+\int_3^7 6\,\mathrm{d}x$ $\int_3^7\left\{\frac{10x(x-2)}{\sqrt{2x-5}}+6\right\}\mathrm{d}x$ $=2\{49\sqrt{9}-9\sqrt{1}\}+6[x]_3^7$ $=276+6(7-3)$ $\boldsymbol{=300}$	Substitution of boundaries
4.	**Cubic Identities**	
(a) (i)	x^3+64 $=(x)^3+(4)^3$ $=(x+4)((x)^2-(x)(4)+(4)^2)$ $\boldsymbol{=(x+4)(x^2-4x+16)}$	Apply sum of cubes identity
	Partial Fractions	
(a) (ii)	$\frac{3x^2}{x^3+64}$ $=\frac{3x^2}{(x+4)(x^2-4x+16)}$ $=\frac{A}{x+4}+\frac{Bx+C}{x^2-4x+16}$ $A(x^2-4x+16)$ $+(Bx+C)(x+4)=3x^2$	Express proper fraction in partial fractions with unknown coefficients
	Let $x=-4$, $A((-4)^2-4(-4)+16)$ $=3(-4)^2$ $A(16+16+16)=3(16)$ $A=1$ Let $x=0$, $A(16)+C(4)=0$ $1(16)+4C=0$ $16+4C=0$ $C=-4$ Let $x=1$, $A(1-4+16)$ $+(B+C)(1+4)=3$ $1(13)+(B-4)(5)=3$ $13+5B-20=3$	Use Substitution and/or Equating coefficients methods to find unknowns Award 1 mark for at least 2 correct values

Qn	Solution	Testing
	$B = 2$	
	OR Equating coefficient of x^2 : $A + B = 3$ $B = 3 - A \cdots (1)$ x : $-4A + 4B + C = 0 \cdots (2)$ x^0 : $16A + 4C = 0$ $C = -4A \cdots (3)$ Subst (1) & (3) into (2), $-4A + 4(3 - A) + (-4A) = 0$ $-4A + 12 - 4A - 4A = 0$ $12A = 12$ $A = 1$ $B = 3 - 1 = 2$ $C = -4(1) = -4$	
	$\therefore \frac{3x^2}{x^3 + 64}$ $= \mathbf{\frac{1}{x + 4} + \frac{2x - 4}{x^2 - 4x + 16}}$	
	Indefinite Integral	
(b)	$\int \frac{2x^3 + 3x^2 + 128}{x^3 + 64} \mathrm{d}x$ $= \int \frac{2(x^3 + 64)}{x^3 + 64} \mathrm{d}x + \int \frac{3x^2}{x^3 + 64} \mathrm{d}x$ $= \int 2 \, \mathrm{d}x + \int \frac{3x^2}{x^3 + 64} \mathrm{d}x$ $= \mathbf{2x + \ln(x^3 + 64) + c}$	Split into 2 integrals Correct integration
5.	**Kinematics (Trigonometric Functions)**	
(a)	When particle passes O, $t = 0$, Hence, $v = 4 + 8\cos 2(0)$ $= \mathbf{12\ m/s}$	
(b)	$v = 4 + 8\cos 2t$ $\frac{dv}{dt} = -(2)8\sin 2t$ $a = -16\sin 2t$ Least acceleration $= \mathbf{-16\ m/s^2}$	
(c)	$0 \le t \le 3$ $0 \le 2t \le 6$ When particle is at rest, $v = 0$ m/s,	

Qn	Solution	Testing
	$4 + 8\cos 2t = 0$ $\cos 2t = -\frac{1}{2}$ Basic angle $\alpha = \cos^{-1}\left(\frac{1}{2}\right) = \frac{1}{3}\pi$ For $\cos 2t < 0$, $2t$ lies in Q2, Q3 $2t = \pi - \frac{1}{3}\pi, \ \pi + \frac{1}{3}\pi$ $= \frac{2}{3}\pi, \ \frac{4}{3}\pi$ $t = \frac{1}{3}\pi, \ \frac{2}{3}\pi$	Correct Substitution Basic Angle Angles in correct quadrants
(d)	$s = \int_0^{\frac{\pi}{3}} (4 + 8\cos 2t) \, \mathrm{d}t$ $s = 4t + 4\sin 2t + c$	Correct Integration

t	s
0	0
$\frac{1}{3}\pi$	7.65289
2	4.97279

Distance travelled in the first 2 sec
$= (7.65289 - 0)$
$+ (7.65289 - 4.97279)$
$= 10.33299$
$= \mathbf{10.3\ m}$ (to 3 sig. fig.)

Qn	Solution	Testing
(d)	Distance travelled in the first 2 sec $= \left\lvert \int_0^{\frac{\pi}{3}} (4 + 8\cos 2t) \, \mathrm{d}t \right\rvert$ $+ \left\lvert \int_{\frac{\pi}{3}}^{2} (4 + 8\cos 2t) \, \mathrm{d}t \right\rvert$ $= \left\lvert [4t + 4\sin 2t]_0^{\frac{\pi}{3}} \right\rvert$ $+ \left\lvert [4t + 4\sin 2t]_{\frac{\pi}{3}}^{2} \right\rvert$ $= \left\lvert 4\left(\frac{\pi}{3}\right) + 4\left(\sin\frac{2\pi}{3}\right) \right\rvert$ $+ \left\lvert 4\left(2 - \frac{\pi}{3}\right) + 4\left(\sin 4 - \sin\frac{2\pi}{3}\right) \right\rvert$ $= 7.65289 + 2.6801$ $= 10.33299$ $= \mathbf{10.3\ m}$ (to 3 sig. fig.)	Correct Expressions Correct Integration Correct Substitution

Qn	Solution	Testing
6.	**R-Formulae**	
(a)	Draw a line $CG \perp BE$. In ΔCGO, $\angle CGO = \frac{\pi}{2}, \ OC = 10$ $CG = 10\sin\theta$ $GO = 10\cos\theta$ $DF = CG = 10\sin\theta$ $CD = GF = 2GO$ $= 20\cos\theta$ $FO = GO = 10\cos\theta$ $P = OC + CD + DF + FO$ $= 10 + 20\cos\theta + 10\sin\theta + 10\cos\theta$ $P = 10 + 30\cos\theta + 10\sin\theta$	
(b)	$30\cos\theta + 10\sin\theta$ $= R\cos(\theta - \alpha)$ $= \sqrt{30^2 + 10^2}\cos(\theta - \alpha)$ $= 10\sqrt{10}\cos(\theta - \alpha)$ where $R = \mathbf{10\sqrt{10}}$ $= 31.6227$ $= \mathbf{31.6}$ (to 3 s.f.) $Max\ P = 10 + R$ $= \mathbf{41.6\ cm}$ (to 3 s.f.)	
(c)	$A = \frac{1}{2}DF(CD + FO)$ $= \frac{1}{2}(10\sin\theta)(20\cos\theta + 10\cos\theta)$ $A = 5\sin\theta\,(30\cos\theta)$ $A = 150\sin\theta\cos\theta$ $A = 75\sin 2\theta$	Area $= \frac{1}{2} \times base \times ht$ Double angle formula
	Stationary Values	
(d)	$\frac{dA}{d\theta} = 150\cos 2\theta$ $\frac{d^2A}{d\theta^2} = -300\sin 2\theta$ For stationary value, $\frac{dA}{d\theta} = 0$ $150\cos 2\theta = 0$ $2\theta = \frac{\pi}{2}$	

Qn	Solution	Testing
	$\theta = \frac{\pi}{4}$ $\frac{d^2A}{d\theta^2} = -300\sin 2\theta$ $= -300\sin\frac{\pi}{2}$ $= -300$ $\frac{d^2A}{d\theta^2} < 0$ $A = 75\sin 2\theta$ $= 75\sin\frac{\pi}{2}$ $= 75$ A has a maximum area (of 75 cm^2) when $\theta = \frac{\pi}{4}$.	Value of A is not required
7.	**Circles & Coordinate Geometry**	
(a)	$P(-13, 11)$ $Q(1, 9)$ $R(-1, -5)$ Gradient of PQ: $m_{PQ} = \frac{11 - 9}{-13 - 1} = -\frac{1}{7}$ Gradient of QR: $m_{QR} = \frac{9 - (-5)}{1 - (-1)} = 7$ Since $m_{PQ} \times m_{QR} = -\frac{1}{7} \times 7 = -1$ $\Rightarrow PQ$ is perpendicular to QR $\Rightarrow \angle PQR = 90°$ **(right angle in semicircle)** $\Rightarrow PR$ is the diameter of circle.	Applying $m_1 \times m_2 = -1$
	OR $PQ^2 = (-13 - 1)^2 + (11 - 9)^2$ $= 200$ $QR^2 = (1 - (-1))^2 + (9 - (-5))^2$ $= 200$ $PR^2 = (-13 + 1)^2 + (11 + 5)^2$ $= 400$ Since $PR^2 = PQ^2 + QR^2 = 400$, by converse of Pythagoras' Theorem, $\angle PQR = 90°$. **(right angle in semicircle)** $\Rightarrow PR$ is the diameter of circle.	

Qn	Solution	Testing
	OR Midpoint of PR, $M=\left(\frac{-13-1}{2},\ \frac{11-5}{2}\right)$ $=(-7,\ 3)\ldots[1]$ $PM=\sqrt{(-13+1)^2+(11-3)^2}$ $=10$ $QM=\sqrt{(1+7)^2+(9-3)^2}$ $=10$ $RM=(-1+7)^2+(-5-3)^2=10$ $\therefore PM=QM=RM=10\ldots[2]$ From [1] and [2], **midpoint of chord *PR* is equidistant to *P*, *Q* & *R*** $\Rightarrow PR$ is the diameter of circle.	
(b)	As centre is midpoint of diameter, centre $=\left(\frac{-13-1}{2},\frac{11-5}{2}\right)$ $=(-7,3)$ radius $=\frac{1}{2}PR$ $=\frac{1}{2}\sqrt{(-13-(-1))^2+(11-(-5))^2}$ $=\frac{1}{2}\sqrt{400}$ $=10$ Equation of circle $(x-(-7))^2+(y-3)^2=10^2$ $x^2+14x+49+y^2-6y+9=100$ $\boldsymbol{x^2+y^2+14x-6y-42=0}$	Award 1 mark for either centre or radius correct Correct substitution
(c)	Let $T(x,y)$ be a general point on the perpendicular bisector of PQ. $PT=QT$ $\sqrt{(x-(-13))^2+(y-11)^2}$ $=\sqrt{(x-1)^2+(y-9)^2}$ $(x+13)^2+(y-11)^2$ $=(x-1)^2+(y-9)^2$ $x^2+26x+169+y^2-22y+121$ $=x^2-2x+1+y^2-18y+81$ $4y=28x+208$ $\boldsymbol{y=7x+52}$	
	OR Perpendicular bisector passes through the centre. Using centre $(-7,\ 3)$ and $m_{\perp PQ}=7$	

Qn	Solution	Testing
	Equation of perpendicular bisector of QR is $y-3=7(x-(-7))$ $\boldsymbol{y=7x+52}$	
	OR Perpendicular bisector passes through the midpoint of chord. Midpoint of PQ $=\left(\frac{-13+1}{2},\ \frac{11+9}{2}\right)$ $=(-6,\ 10)$ $m_{\perp PQ}=7$ Equation of perpendicular bisector of QR is $y-10=7(x-(-6))$ $\boldsymbol{y=7x+52}$	
(d)	The furthest two points on a circle is the diameter of the circle. $\therefore$ centre $(-7,3)$ is midpoint of diameter QS, $\frac{x_Q+x_S}{2}=-7$; $1+x_S=-14$; $x_S=-15$ $\frac{y_Q+y_S}{2}=3$; $9+y_s=6$; $y_S=-3$ $\boldsymbol{\therefore S(-15,-3)}$	Diameter is the longest chord Mid-point
	Area of quadrilateral $PQRS$ $=\frac{1}{2}\begin{vmatrix} P & Q & R & S & P \\ -13 & 1 & -1 & -15 & -13 \\ 11 & 9 & -5 & -3 & 11 \end{vmatrix}$ $=\frac{1}{2}\begin{vmatrix}(-117-5+3-165) \\ -(11-9+75+39)\end{vmatrix}$ $\boldsymbol{=200\ \text{units}^2}$	Shoelace method
8.	**Tangent & Normal**	
(a)	$y=\frac{10}{12-x}-1\ldots\ldots[1]$ Subst $P(2k,4)$ into [1] $4=\frac{10}{12-2k}-1$ $5=\frac{5}{6-k}$ $6-k=1$ $k=5$ $P(10,4)$ $\frac{dy}{dx}=(-10)(12-x)^{-2}(-1)$ $\frac{dy}{dx}=\frac{10}{(12-x)^2}\ldots\ldots[2]$	Chain rule

Qn	Solution	Testing
	Subst $P(10,4)$ into [2] $\frac{dy}{dx} = \frac{10}{(12-10)^2} = \frac{5}{2}$ $m_{PT} = \frac{5}{2}$ and $m_{PN} = -\frac{2}{5}$ Using $P(10,4)$ and $m_{PN} = -\frac{2}{5}$, **Equation of normal at *P*** $y - 4 = -\frac{2}{5}(x - 10)$ $y - 4 = -\frac{2}{5}x + 4$ $\boldsymbol{y = -\frac{2}{5}x + 8}$ ...[3]	
	Area under Curve	
(b)	Subst $X(x_X, 0)$ into [1] $0 = \frac{10}{12 - x_X} - 1$ $\frac{10}{12 - x_X} = 1$ $10 = 12 - x_X$ $x_X = 2$ Subst $N(x_N, 0)$ into [3] $0 = -\frac{2}{5}x_N + 8$ $\frac{2}{5}x_N = 8$ $x_N = 20$ Area of shaded region $\int_2^{10}\left(\frac{10}{12-x} - 1\right)dx + \frac{1}{2}(20-10)(4)$	
	$= [-10\ln(12-x) - x]_2^{10} + 20$ $= -[10\ln(12-x) + x]_2^{10} + 20$ $= [10\ln(12-x) + x]_{10}^{2} + 20$	Integration
	$= 10\ln\left(\frac{12-2}{12-10}\right) + \{2 - 10\} + 20$ $= 10\ln 5 + 12$	Substitution of boundaries
	OR Area of shaded region $\int_2^{10}\left(\frac{10}{12-x} - 1\right)dx + \int_{10}^{20}\left(-\frac{2}{5}x + 8\right)dx$ $= [-10\ln(12-x) - x]_2^{10} +$ $\left[-\frac{1}{5}x^2 + 8x\right]_{10}^{20}$	Integration

Qn	Solution	Testing
	$= -10\ln\left(\frac{12-10}{12-2}\right) - \{10 - 2\} -$ $\frac{1}{5}\{20^2 - 10^2\} + 8\{20 - 10\}$ $= 10\ln 5 + 12$	Substitution of boundaries
9.	**Linear Law**	
(a)	$\frac{y}{a} = \frac{1}{x} + b\sqrt{x}$ Multiply equation with ax, $xy = a + abx\sqrt{x}$ $xy = ab(x\sqrt{x}) + a$	Linearise equation $Y = mX + C$
(b)	see table below	Tabulate values for Y vs. X

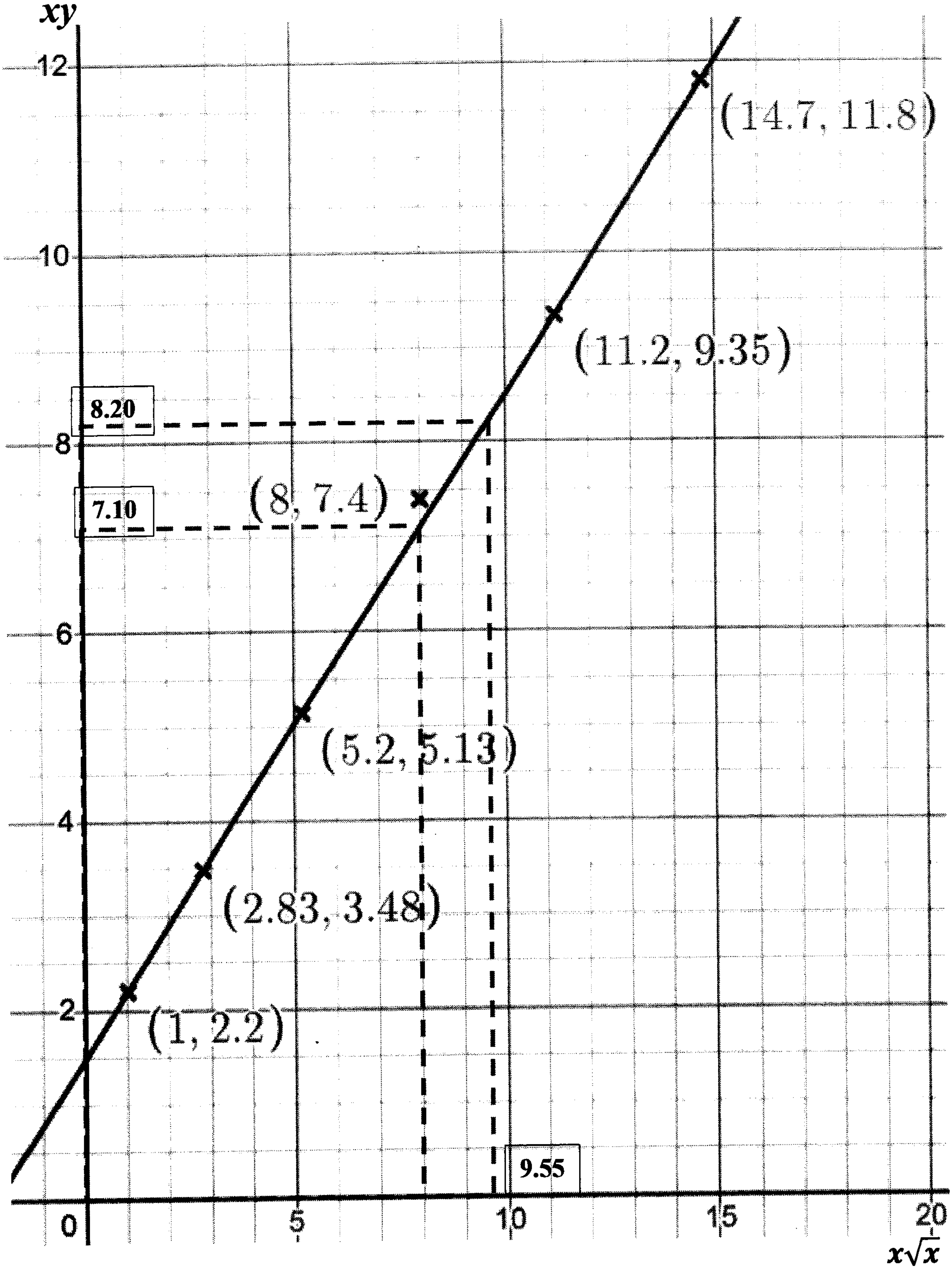
x	1	2	3	4	5	6
y	2.20	1.74	1.71	1.85	1.87	1.96
$x\sqrt{x}$	1.00	2.83	5.20	8.00	11.2	14.7
xy	2.20	3.48	5.13	7.40	9.35	11.8

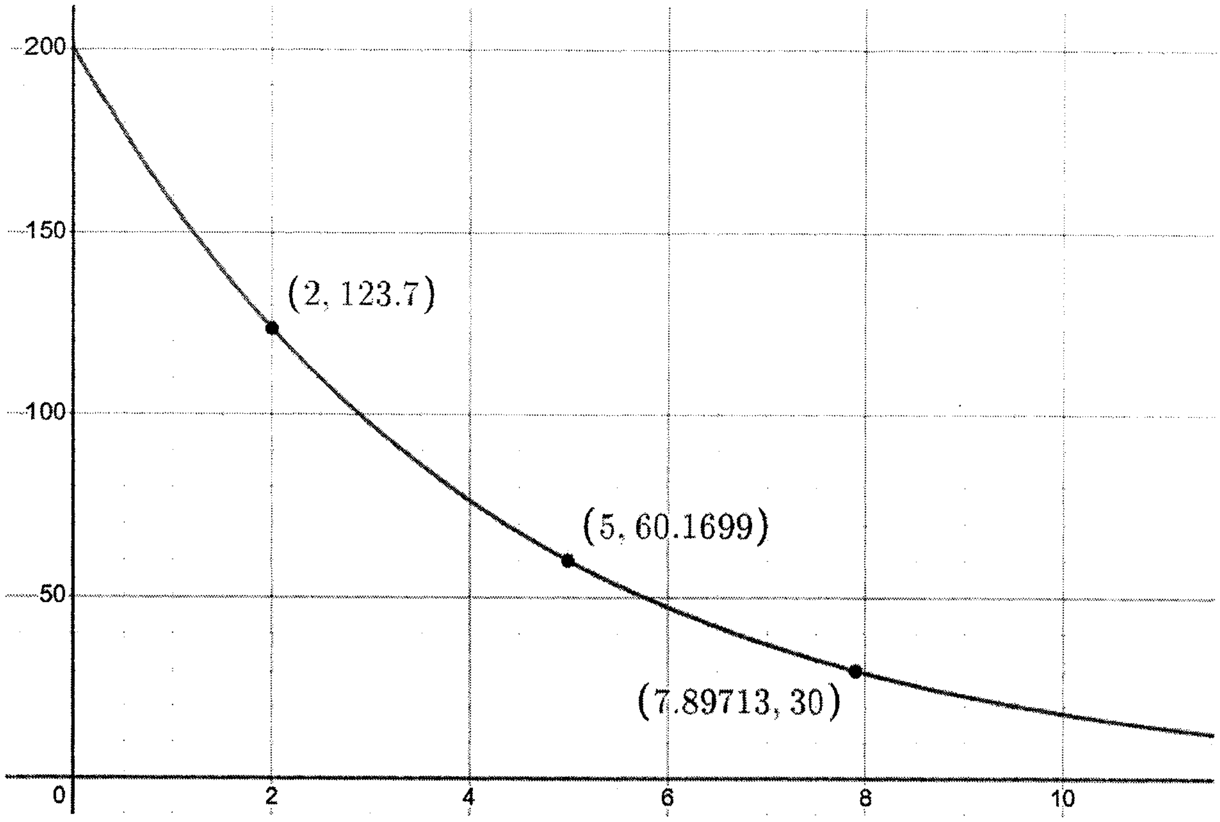
G1 – Plotted points
G1 – Best-fit line

Qn	Solution	Testing
(c) (i)	From the graph, the incorrect point is (8, 7.40). Corresponding to $x = 4$ (in the table), For $x\sqrt{x} = 8$ Correct $xy = 7.05$ *to* 7.15 $4y = 7.05$ *to* 7.15 $y = 1.7625$ *to* 1.7875 $\boldsymbol{y = 1.76}$ ***to*** $\boldsymbol{1.79}$ (to 3 sig. fig.)	Read from graph
(c) (ii)	From the graph, xy intercept $= 1.5$ $\boldsymbol{a = 1.50}$ (to 3 sig. fig.) Gradient $= \frac{11.8 - 2.20}{14.7 - 1.00}$ $ab = 0.7007$ $1.50\,b = 0.7007$ $b = 0.467153$ $\boldsymbol{b = 0.467}$ (to 3 sig. fig.)	Read from graph
(c) (iii)	When $x = 2.52$, for $x\sqrt{x} = 2.52\sqrt{2.52}$ $= 4.00037$ From the graph, for $x\sqrt{x} = 4.00$ $xy = 4.3$ $2.52\,y = 4.3$ $y = 1.70634$ $\boldsymbol{y = 1.71}$ (to 3 sig. fig.)	Read from graph

Qn	Solution	Testing
(d)	From readings of experiment, $x = 4.50$ and $y = 1.86$. $x\sqrt{x} = 9.5459$ $xy = (4.50)(1.86)$ $= 8.37$ From the graph, for $x\sqrt{x} = 9.5459$ $xy = 8.20$ Vertical difference $= \dfrac{8.37 - 8.20}{8.20} \times 100\%$ $= 2.073\% > 2\%$ Since vertical difference is **more than 2%**, the experimental readings are **unacceptable**.	Read from graph

Qn	Solution	Testing
	OR Acceptable range $= 8.20 \times (100 \pm 2)\%$ $= 8.036\ to\ 8.364$ Since $xy = 8.37$ is **out of** the acceptable **range**, the experimental readings are **unacceptable**.	


xy
12
10
8
6
4
2
0
5
10
15
20
x√x
(14.7, 11.8)
(11.2, 9.35)
8.20
7.10
(8, 7.4)
(5.2, 5.13)
(2.83, 3.48)
(1, 2.2)
9.55


200
150
100
50
0
2
4
6
8
10
(2, 123.7)
(5, 60.1699)
(7.89713, 30)