BEDOK SOUTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025

4E5N

CANDIDATE NAME | CLASS | REGISTER NUMBER

ADDITIONAL MATHEMATICS 4049 / 01
Paper 1
2 hours 15 minutes

90

Answer **all** the questions.

1 Prove that $\sin 2x - \cos 2x \tan x = \tan x$. [4]

2 Show that the solution of $2^{3x+4} \times 5^{2x+1} = 5^{3(x+1)} \times 16^{x}$ is $2\lg\frac{4}{5}$. [4]

3 Giving your answer in the form $\frac{c+d\sqrt{2}}{3}$, solve, without using a calculator,

$$x\sqrt{18} = 3x + \sqrt{32}.$$ [5]

4 A square of area $(11+\sqrt{120})\text{ cm}^2$ has a length of $(\sqrt{a}+\sqrt{b})$ cm, where $a < b$.
Without using a calculator, find the values of *a* and *b*. [5]

5 The line $3x + 4y = 13$ intersects the curve $y = \frac{6x-10}{3x-1}$ at points *P* and *Q*.
Calculate the exact length of the line segment *PQ*. [5]

6 The coefficient of x^3 in the expansion of $(10+kx)\left(2-\frac{1}{2}x\right)^8$ is zero.
Find the value of the constant *k*. [5]

7 Solve the equation $\frac{5\cos x + \sin 2x}{\cos 2x + 5\sin x} = \cot x$ for $0 \le x \le 180°$. [6]

8 **(a)** Show that the derivative of $x(x-3)^4$ with respect to *x* is $(5x-3)(x-3)^3$. [2]

(b) Hence, given that $y = \frac{x(x-3)^4}{5x-3}$ and *y* is decreasing at a constant rate of 70 units/s, calculate the rate of change of *x* when $x = 1$. [4]

9 (a) Express $y=5-12x-3x^2$ in the form $y=a(x+b)^2+c$ and hence show that y can never be greater than 20. [3]

(b) Explain why there are no values of k for which the curve

$y=(k-1)x^2+2(k+2)x+k+3$ is always positive. [4]

10 A rectangular field has sides $(3x-5)$ m and $(x-10)$ m. Its area is at most 200 m^2.

(a) Find the range of values of x that satisfies the above sides and area conditions. [4]

(b) Justify whether a fence of 98 m is enough to enclose the field. [3]

11 (a) State the range of values of x for which the equation below is valid.

$$2\log_3(4-x)-\log_9(x-4)^2=2$$ [1]

(b) Express $2\log_3(4-x)-\log_9(x-4)^2=2$ as a quadratic equation $x^2+bx+c=0$ and explain why there is only one real solution. [6]

12 A quadratic curve is given by $y=hx^2-2x-9h+6$, where h is a constant.

(a) Show that the equation $y=0$ has real roots for all values of h except $h=0$. [3]

(b) State the value of h in the case where $y=0$ has two real and equal roots. [1]

(c) Given that the line $y=2x-h-12$ meets the curve $y=hx^2-2x-9h+6$, find the range of values of h. [5]

13 It is given that $f(x)=x^2e^{x+2}$.

(a) Show that the range of values of x for which $f(x)$ is a decreasing function is $-2<x<0$. [4]

(b) The gradient with the least value is in the range $-2<x<0$.
Find the value of this gradient, giving your answer in exact form. [5]

14 It is given that $f(x)=3x^3+2x^2+16$. The remainder when $f(x)$ is divided by $x-3a$, where a is a constant, is the same as the remainder when it is divided by $3x+2$.

(a) Find the possible values of a. [3]

(b) Show, with clear working, that $x=-2$ is a solution of $f(x)=0$. [1]

(c) Explain why $f(x)=0$ has only one real root. [5]

(d) Hence use your answers to parts (b) and (c) to solve the equation

$$16y^3+2y-3=0.$$ [2]

END OF PAPER

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

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CANDIDATE NAME ____________ CLASS ____ REGISTER NUMBER ____

ADDITIONAL MATHEMATICS
Paper 2

4049 / 02
2 hours 15 minutes

90

Answer **all** the questions.

1 **(a)** Solve the equation $2(4^x)-8=11(2^x)-2^{2x+3}$, giving your answer correct to 3 significant figures. [5]

(b) Show that the solution from **part (a)** may be written in the form $m-\log_n p$ where m, n and p are integers to be determined. [2]

2 The mass, M grams, of a radioactive substance, present at time t years after first being observed, is given by the formula $M=200e^{-kt}$, where k is a constant.
The mass of the substance was 123.7 g after being observed for 2 years.

(a) **(i)** State the initial mass of the substance. [1]

(ii) Show that k is approximately 0.240, correct to 3 significant figures. [1]

(iii) Find the mass of the substance when $t=5$, [1]

(iv) Find the value of t when the mass of the substance is 15% of its initial mass.
Give your answers correct to three significant figures. [2]

(b) Explain, with clear working, why the mass of the substance can never be more than 200 grams. [1]

(c) Sketch the graph of M against t. [2]

3 **(a)** It is given that $y=x^2\sqrt{2x-5}$, where $x \ge h$.

Show that $\frac{dy}{dx}=\frac{kx(x-2)}{\sqrt{2x-5}}$, where k is an integer and determine the value of h and of k. [4]

(b) Hence evaluate $\int_3^7 \left\{\frac{10x(x-2)}{\sqrt{2x-5}}+6\right\} dx$. [4]

4 **(a)** **(i)** Factorise completely x^3+64. [2]

(ii) **Hence**, express $\frac{3x^2}{x^3+64}$ in partial fractions. [5]

(b) Using the results in **part (a)**, or otherwise, find

$$\int \frac{2x^3+3x^2+128}{x^3+64}\,dx.$$ [3]

5 A particle moves in a straight line so that, at time t seconds after passing a fixed point O, its velocity is v m/s, where $v = 4 + 8\cos 2t$. Find

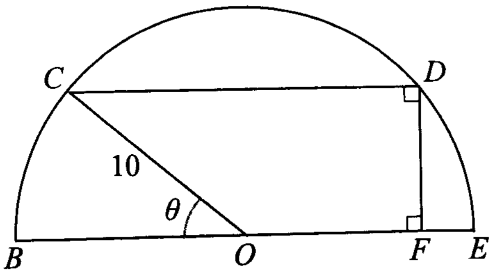
(a) the velocity of the particle at the instant it passes O, [1]

(b) the least value of the particle's acceleration, [1]

(c) the values of t, in terms of π, when the particle is at rest for $0 \le t \le 3$, [4]

(d) the distance travelled in the first 2 seconds. [4]

6



The diagram shows right angled trapezium $OCDF$ inside a semicircle with centre O and radius 10 cm such that angle BOC is θ radians, and angle CDF and angle OFD are right angles.

(a) Show that the perimeter, P cm, of trapezium $OCDF$ is given by

$$P = 10 + 30\cos\theta + 10\sin\theta$$ [2]

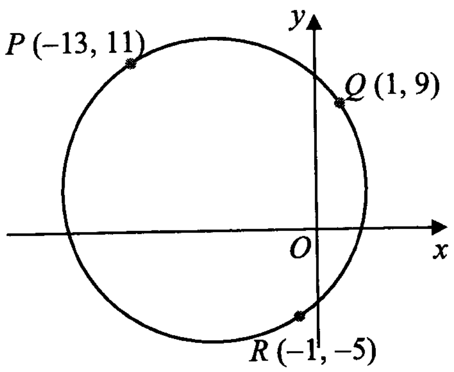
(b) Find the value of R when $10\sin\theta + 30\cos\theta$ is expressed as $R\cos(\theta - \alpha)$, where R and α are constants, and hence state the maximum perimeter of the trapezium. [3]

(c) Show that the area, A cm^2, of trapezium $OCDF$ is given by

$$A = 75\sin 2\theta$$ [2]

(d) The area of the trapezium varies with the value of θ. Find the value of θ for which the area has a stationary value and determine whether this area is a maximum or a minimum. [4]

7 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram, P, Q and R are points on the circle.

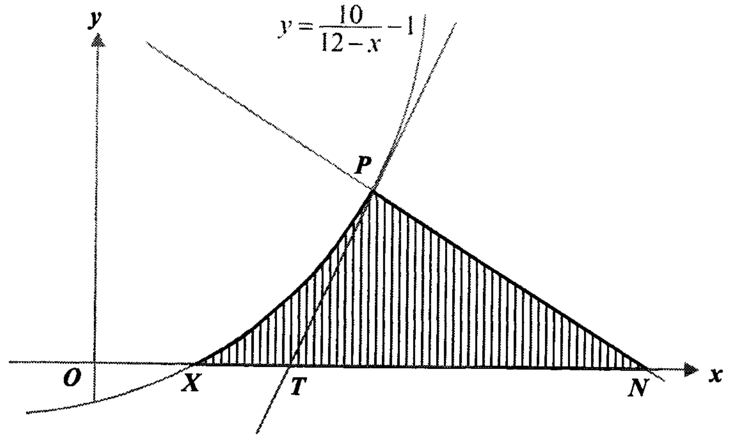
(a) Explain, with geometrical reason, why PR is the diameter of the circle. [3]

(b) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are integers. [3]

(c) Find the equation of the perpendicular bisector of PQ. [3]

(d) The point S lies on the circle such that it is furthest from the point Q.
Show that the coordinates of S are $(-15, -3)$ and hence calculate the area of the quadrilateral $PQRS$. [3]

8



The diagram shows part of the curve $y = \dfrac{10}{12-x} - 1$ passing through the point $P(2k, k-1)$, where k is a constant. The curve meets the x-axis at the point X. The tangent and normal at P meet the x-axis at the points T and N respectively.

(a) Find the equation of the normal at P. [6]

(b) Find the **exact** area of the shaded region. [6]

9 The table below shows experimental values of two variables x and y.

x	1	2	3	4	5	6
y	2.20	1.74	1.71	1.83	1.87	1.96

The variables x and y are related by the equation $\dfrac{y}{a} = \dfrac{1}{x} + b\sqrt{x}$, where a and b are constants.
One value of y has been recorded incorrectly.

(a) Show how $\dfrac{y}{a} = \dfrac{1}{x} + b\sqrt{x}$ is transformed to plot a graph of xy against $x\sqrt{x}$. [1]

(b) Draw a straight line graph of xy against $x\sqrt{x}$ for the given data. [2]

(c) Using your graph,

(i) find an approximate value of y to replace the incorrect value, [2]

(ii) estimate the value of a and of b, [3]

(iii) find the value of y when $x = 2.52$. [2]

(d) A pair of values of x and y is considered acceptable only if the xy value is within 2% vertical difference from the straight line. A student recorded a pair of values such that $x = 4.50$ and $y = 1.86$.
Verify whether the recorded values by the student are acceptable. [2]

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