中正中学 义顺

CHUNG CHENG HIGH SCHOOL (YISHUN)

2025 Preliminary Examination

Secondary Four Express / Five Normal Academic

CANDIDATE NAME

FORM CLASS / SUBJECT GROUP   /   INDEX NUMBER

ADDITIONAL MATHEMATICS 4049/01

Paper 1

21 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Up to 2 marks may be deducted for improper presentation.

The number of marks is given in brackets [ ] at the end of each question or part question.

Question Number	Marks Possible	Marks Obtained
1	4	
2	9	
3	7	
4	5	
5	6	
6	6	
7	10	
8	9	
9	9	
10	10	
11	7	
12	8	
Presentation Deduction		−1 / −2
TOTAL	90	

This document consists of **17** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGNOMETRY

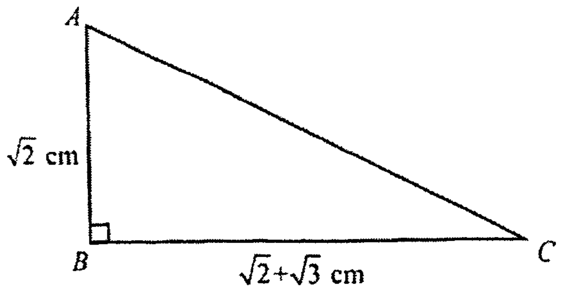
Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1



The diagram above shows a right-angled triangle ABC, where $AB = \sqrt{2}$ cm, and $BC = \sqrt{2} + \sqrt{3}$ cm.

(a) Find the exact value of $\tan \angle ACB$ in the form of $a + \sqrt{b}$, where a and b are integers. [2]

(b) Hence, find the exact value of $\sec^2 \angle ACB$ in the form $m + n\sqrt{k}$, where m and n are integers. [2]

2 A container has a capacity of 840 cm^3 and is initially filled completely with water. The volume, V cm^3, of water in the container is given by $V = h^2 + 2h$, where h cm is the height of the water level in the container. Due to leakage at the bottom of the container, the height of the water level in the container decreases at a rate of $\frac{5t}{3}$ cm/s, where t is the time in seconds.

(a) Find the initial height of the water level in the container. [2]

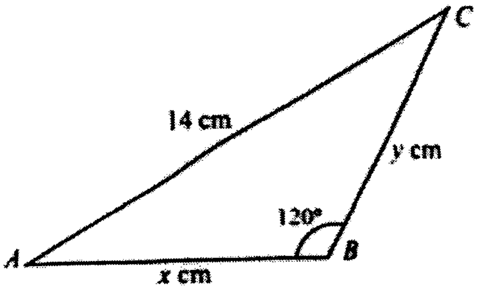
(b) Show that the height of the water level h can be expressed as $h = -\frac{5t^2}{6} + 28$. [2]

(c) Find the rate of decrease of volume when $t = 3$. [5]

3 (a) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(xe^{\frac{1}{2}x}\right)$. [2]

(b) Hence evaluate $\int_0^4 xe^{\frac{1}{2}x}\,\mathrm{d}x$, leaving your answer in the form $k\left(e^2+1\right)$, where k is a constant to be found. [5]

4 In triangle ABC below, $AB = x$ cm, $BC = y$ cm, $AC = 14$ cm and angle $ABC = 120°$.



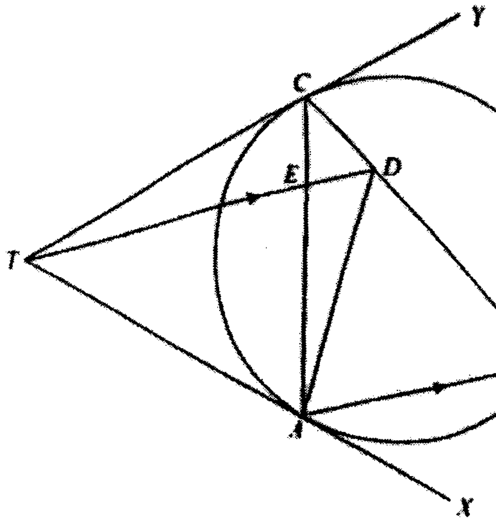
(a) Using cosine rule, form an equation involving x and y. [1]

(b) Given that the perimeter of triangle ABC is 30 cm, find the exact area of triangle ABC. [4]

5 (a) By considering the general term in the expansion of $\left(x^5+\frac{1}{x}\right)^7$, explain why there are no even powers of x in its expansion. [2]

(b) Given that the coefficient of x^6 in the expansion of $\left(x^5+\frac{1}{x}\right)^7+(kx+3)^7$ is 1344, where k is a positive constant, find the coefficient of x^4 in the expansion. [4]

6 In the diagram below, TAX and TCY are tangents to the circle at A and C respectively. AC meets TD at E. D is on BC such that TD is parallel to AB.

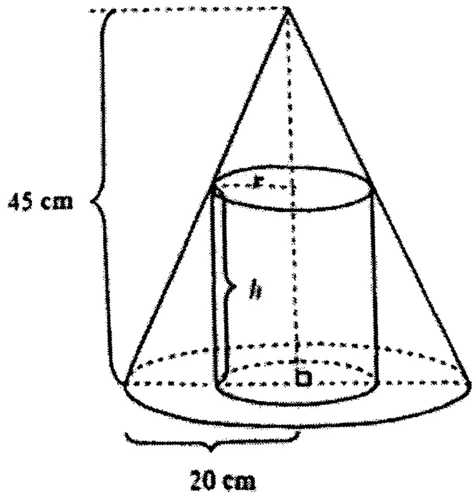


(a) Prove that angle ACB is equal to angle ATD. [2]

(b) Explain why a circle can be drawn passing through the points T, A, D and C. [1]

(c) Hence prove that $CE \times AE = DE \times TE$. [3]

7 The diagram shows a solid cylinder of radius r cm and height h cm inscribed in a hollow cone of height 45 cm and base radius 20 cm. The cylinder rests on the base of the cone and the circumference of the top surface of the cylinder touches the curved surface of the cone.



(a) Show that the volume, V cm^3, of the cylinder is given by $V = 45\pi r^2 - \frac{9}{4}\pi r^3$. [3]

(b) Given that r can vary, find the maximum volume of the cylinder, leaving your answer in terms of π. [5]

(c) Hence show that the cylinder occupies at most $\frac{4}{9}$ of the volume of the cone. [2]

8 (a) Find the value of a and of b for which $\{x : x < -3.5 \text{ or } x > 2\}$ is the solution set of $b < x^2 + ax$. [3]

(b) Find the range of values of p for which $y = x^2 - px - 3x - p$ is always positive. [3]

(c) Explain whether the line $y = -5x - 2$ intersects the curve $y = kx^2 + 3$ where $k < 1$. [3]

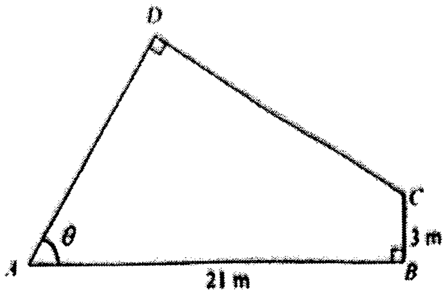
9 (a) Express $5\sin^2 x - \cos^2 x - 1$ in the form $1 - k\cos 2x$. [2]

(b) State the amplitude and period, in radians, of $5\sin^2 x - \cos^2 x - 1$. [2]

(c) Sketch the graph of $y = 5\sin^2 x - \cos^2 x - 1$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [3]

(d) By drawing the line $y = 1 - \frac{3x}{2\pi}$ on the same axes, state the number of solutions to the equation $2\pi - 3x = 2\pi\left(5\sin^2 x - \cos^2 x - 1\right)$ in the range $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [2]

10



The diagram shows a quadrilateral field *ABCD*, where $AB = 21$ m and $BC = 3$ m.
Angle ABC = angle $ADC = 90°$. Angle $BAD = \theta$, for $0° < \theta < 90°$, and can vary.
The perimeter of the fencing around the quadrilateral field *ABCD* is *P* m.

(a) Show that $P = 24 + 18\cos\theta + 24\sin\theta$. [3]

(b) Express *P* in the form $R\sin(\theta + \alpha) + 24$, where $R > 0$ and $0° < \alpha < 90°$. [3]

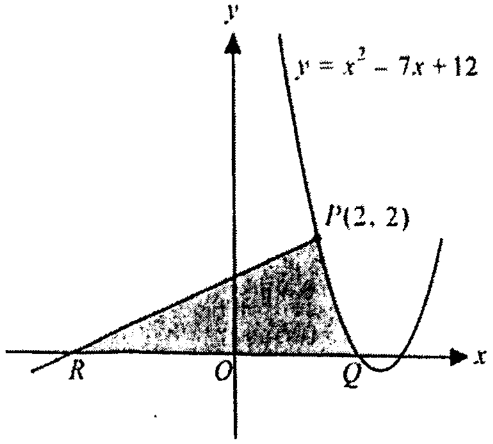
(c) Given that the total perimeter of the fencing is 53 m, find the value(s) of θ. [2]

(d) Explain why the total length of the fencing will never exceed a certain value and state this value. [2]

11 (a) Express $\frac{11x+12}{x^2(x+4)}$ in partial fractions. [5]

(b) Hence find $\int \frac{11x+12}{x^2(x+4)}\,dx$. [2]

12 The diagram shows part of the curve $y = x^2 - 7x + 12$, cutting the x-axis at Q. The normal to the curve at $P(2, 2)$ meets the x-axis at R.



Show that the area of the shaded region bounded by the x-axis, the line PR and the curve is $6\frac{5}{6}$ units^2. [8]

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2025 Preliminary Examination
Secondary Four Express / Five Normal Academic

CANDIDATE NAME		INDEX NUMBER	
FORM CLASS / SUBJECT GROUP	/	DATE	

ADDITIONAL MATHEMATICS — 4049/02

Paper 2

28 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

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1 In 2020, there were an estimated 7 million insects of a particular species in a certain country. In 2021, the numbers were estimated to have fallen to 5 million. Scientists believe that the number of these insects, N million, can be modelled by the formula $N = 3 + ae^{-kt}$, where t is the time in years after 2020.

(a) Find the exact values of a and of k. [4]

(b) What is the approximate size of the population for large values of t? [1]

2 The equation of a curve is $y = px^2 - (p+2)x + 1$, where p is a constant.

(a) Given that $p > 0$, by completing the square, find the minimum value of y.

Express your answer in the form $\frac{a - p^2}{bp}$, where a and b are integers. [3]

(b) Given instead that $p < 0$, find the coordinates of the maximum point in terms of p. [2]

3 (a) Show that $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$. [2]

(b) A curve has equation $y = e^{-ax} \cot x$, where a is a positive constant. There is exactly one point in the interval $-\frac{1}{2}\pi < x < 0$ at which the tangent is parallel to the x-axis.
Find the value of a and state the exact x-coordinate of this point. [5]

4 It is given that $f(x) = 3x^3 + ax^2 - 7x + 3$, where a is a constant, has a factor of $x+1$.

(a) Find the value of a and factorise $f(x)$ completely. [4]

(b) Solve the equation $f(\cot^2\theta) = 0$ for $-90° < \theta < 90°$. [3]

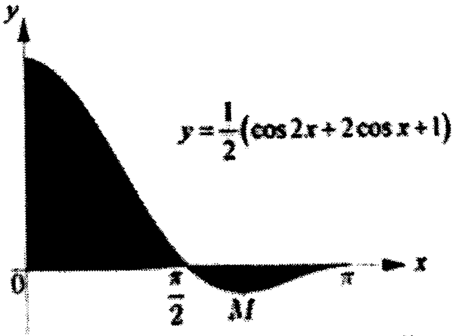
5 (a) Prove the identity $\sin 2x(a\cot x + b\tan x) = a + b + (a-b)\cos 2x$, where a and b are constants. [4]

(b) Hence solve the equation $\sin\frac{1}{2}\theta\left(3\cot\frac{1}{4}\theta + 5\tan\frac{1}{4}\theta\right) = 7$ for $0 \le \theta \le 4\pi$. [3]

6 (a) Solve the equation $\log_5(3y+1) - \frac{1}{\log_y 5} + \log_2 8 = 4$. [4]

(b) By expressing the equation $\ln\left(6e^{x}+1\right)+x=0$ as a quadratic equation in e^{x}, solve the equation $\ln\left(6e^{x}+1\right)+x=0$, giving values of x in logarithmic form. [4]

7 The diagram shows the curve $y=\frac{1}{2}(\cos 2x+2\cos x+1)$ for $0 \le x \le \pi$ radians.



The point M is the minimum point of the curve, where the x-coordinate of M lies in the interval $\frac{\pi}{2} < x < \pi$.

(a) Find the exact coordinates of M. [5]

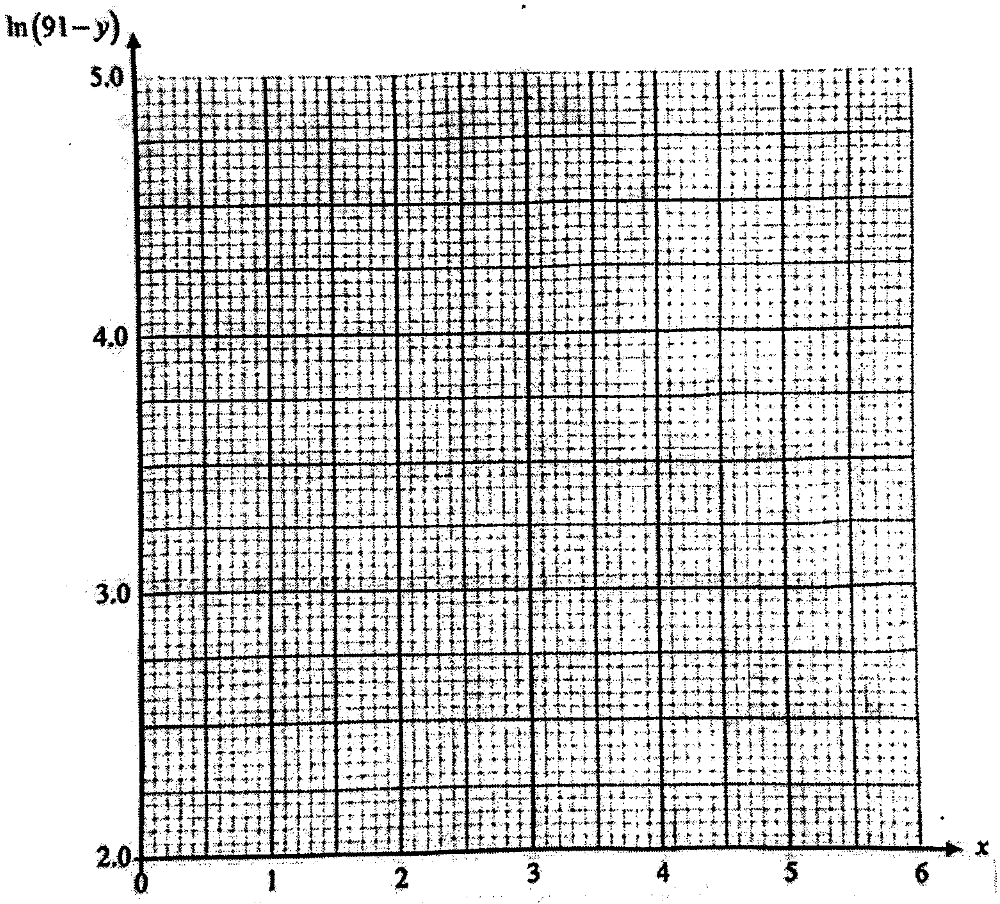
(b) Find the exact total area of the shaded regions bounded by the curve and the axes. [4]

8 The table shows Andy's marks for his Mathematics practice papers each week.

Week x	1	2	3	4	5	6
Marks y	45	59	68	74	71	82

He believed that these figures can be modelled by the formula $y = 91 - Ae^{kx}$, where A and k are constants, by excluding one datapoint that does not follow the trend.

(a) On the grid below, by ignoring the datapoint that does not follow the trend, plot $\ln(91-y)$ against x and draw a straight line graph. [2]



(b) Use the graph to estimate the value of each of the constants A and k. [5]

(c) Suggest a possible reason why one of the marks does not seem to follow the trend. [1]

(d) From your straight line graph, estimate the expected marks for the datapoint that was excluded. [2]

9 The equation of a circle is $x^2+y^2-8x-4y+15=0$.

(a) Find the radius and coordinates of the centre of the circle. [3]

Two points are given by $A(5,-2)$ and $B(9,2)$. The perpendicular bisector of AB cuts the circle at point C and D.

(b) Find the coordinates of C and of D. [6]

(c) Find the shortest distance from the origin O to the line segment CD, giving your answer in the form $k\sqrt{2}$, where k is a constant to be determined. [3]

10 A particle travels in a straight line so that its displacement, s m, from O at time t seconds, where $t \geq 0$, is modelled by $s = t^3 - \frac{9}{2}t^2 + 6t + 1$.

(a) Find the values of t for which the particle is instantaneously at rest. [3]

(b) The particle's acceleration is 15 m/s² at T seconds. Find the total distance travelled by the particle in the interval $t = 0$ to $t = T$. [4]

(c) Explain clearly why the answer found in part (b) should not be obtained by finding the value of s when $t = T$. [2]

11 (a) A curve $y = f(x)$ is such that $f'(x) = ax^2 - 6x + b$, where a and b are constants.

(i) Given that the curve is always increasing, what conditions must apply to a and b? [3]

It is now given that $a = 5$.

(ii) The curve intersects the y-axis at $(0, 4)$ and passes through the points $(-3, -80)$ and $(3, k)$. Find the value of k. [5]

This question is not related to part **(a)**.

(b) It is given that $y = g(x)$ is a solution of the equation $e^{-2x}\left(\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y\right) = -e^k$, where k is a constant. The point $(1, 0)$ is a stationary point on the curve $y = g(x)$.
Find the nature of this stationary point. [3]