Answer

1a) -0.41 b) $(3x-1)(9x^2+12x+7)$

2b) $x=1, x=-\frac{2}{3}, x=-3$ c) $y=3$, $y=\frac{4}{3}, y=-1$

3b) $\frac{2}{3}$

4b) $AB = 13\cos(\theta - 67.4^0)$ c) Maximum $AB = 13$ cm, $\theta = 67.4^0$, Radius = 6.5 cm

5a) $k<-2$ b(i) 8, 24 (ii) $m>6$

6a) $x^5+5kx^2+\frac{10k^2}{x}+\frac{10k^3}{x^4}+\ldots$ b) $k=\frac{1}{5}$ or $k=-\frac{1}{2}$ (reject) c) $n=3r$, 6 & 9

7a) (4, 5), 5 units b) $y=-\frac{3}{4}x+\frac{57}{4}$

8a) ln 2 b) 4 m c) -2 m/s

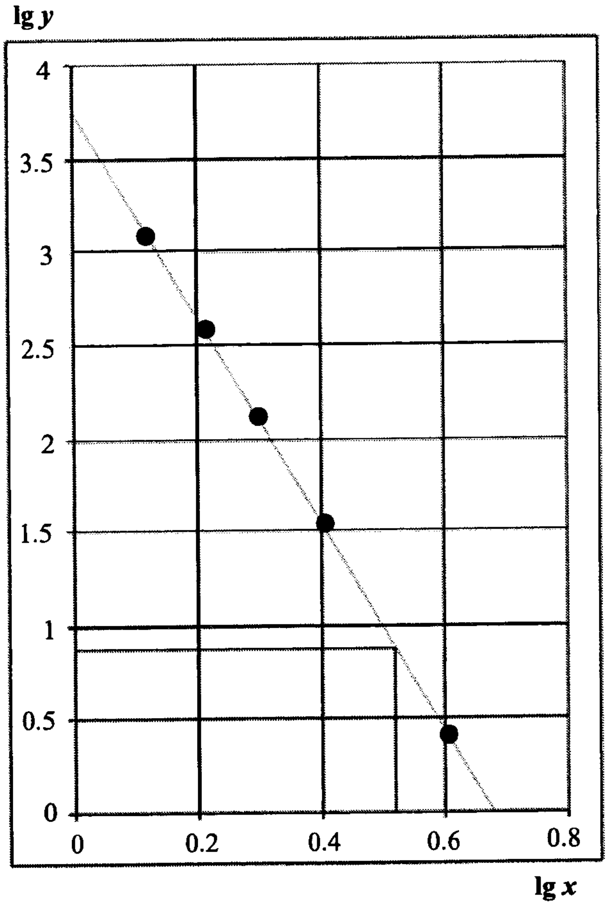
9a)

	Y	X	m	c
$py = xy + 1$	y	xy	$\frac{1}{p}$	$\frac{1}{p}$
$xy = py - 1$	xy	y	p	-1

9b)i)

$\lg x$	0.117	0.212	0.294	0.405	0.605
$\lg y$	3.10	2.60	2.12	1.55	0.420

$\lg y = -a \lg x + \lg b$



b)(ii) $a = 5.51$ $b = 6310\ (5012 - 7943)$

10(a)(ii) 4.04 (b)(i) 0°, 180° (c) $-\frac{527}{625}$

Name: Solution	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR 2025
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4049/01**
Paper 1 **29 August 2025**

Candidates answer on the Question Paper.
No Additional Materials are required. **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST
Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Marks														

Table of Penalties		Qn. No.		
Presentation	–1			
Significant Figures / Units	–1		Parent's/ Guardian's Signature	90

This document consists of 23 printed pages and 1 blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 The curve $x^2+xy-6y^2+9=0$ and line $y=x+1$ intersect at the points R and S.
Find the x-coordinate of the midpoint of RS. [3]

Sub $y=x+1$ into $x^2+xy-6y^2+9=0$

$x^2+x(x+1)-6(x+1)^2+9=0$

$x^2+x^2+x-6(x^2+2x+1)+9=0$

$2x^2+x-6x^2-12x-6+9=0$

$4x^2+11x-3=0$

$(4x-1)(x+3)=0$

$x=\frac{1}{4}$ or $x=-3$

x-coordinate of the midpoint of $RS = \dfrac{\frac{1}{4}+(-3)}{2} = -1\frac{3}{8}$.

2 Integrate $\dfrac{4}{x^2}-\dfrac{3}{1-5x}+\sin 5x-\pi$ with respect to x. [5]

$= \int 4x^{-2}-\dfrac{3}{1-5x}+\sin 5x-\pi \, dx$

$= \dfrac{4x^{-1}}{-1}-\dfrac{3\ln(1-5x)}{-5}-\dfrac{\cos 5x}{5}-\pi x+c$

$= -\dfrac{4}{x}+\dfrac{3}{5}\ln(1-5x)-\dfrac{1}{5}\cos 5x-\pi x+c$,

c is an arbitrary constant

3 A cuboid has base area $\left(4+2\sqrt{5}\right)$ cm^2 and a volume of $\left(9+5\sqrt{5}\right)$ cm^3.

Find, **without using a calculator**, the height of the cuboid, in cm, giving your answer in the form $\frac{1}{2}\left(a+b\sqrt{5}\right)$, where a and b are integers. [3]

$$\text{Height} = \frac{9+5\sqrt{5}}{4+2\sqrt{5}} \cdot \frac{4-2\sqrt{5}}{4-2\sqrt{5}}$$

$$= \frac{36-18\sqrt{5}+20\sqrt{5}-50}{16-20}$$

$$= \frac{-14+2\sqrt{5}}{-4}$$

$$= \frac{7}{2}-\frac{1}{2}\sqrt{5}$$

$$= \frac{1}{2}\left(7-\sqrt{5}\right) \text{ cm}$$

4 **(a)** Express $-4x^2+12x+2$ in the form $a(x+b)^2+c$ and state the coordinates of

the turning point of the curve $y=-4x^2+12x+2$. [3]

$$-4x^2+12x+2=-4\left(x^2-3x\right)+2$$

$$=-4\left[x^2-3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2\right]+2$$

$$=-4\left(x-\frac{3}{2}\right)^2+9+2$$

$$=-4\left(x-\frac{3}{2}\right)^2+11$$

The turning point is $\left(1\frac{1}{2},11\right)$.

(b) Hence explain why the turning point is a maximum point. [1]

For all real values of x, $\left(x-\frac{3}{2}\right)^2\geq 0$, $-4\left(x-\frac{3}{2}\right)^2\leq 0$, $-4\left(x-\frac{3}{2}\right)^2+11\leq 11$.

Since coefficient of x^2 is negative,

$\left(1\frac{1}{2},11\right)$ is a maximum point.

5 Express $\dfrac{3x^2+5x-2}{(x+1)(x^2+4)}$ in partial fractions. [5]

[Turn over

$$\frac{3x^2+5x-2}{(x+1)(x^2+4)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+4}$$

$$3x^2+5x-2=A(x^2+4)+(Bx+c)(x+1)$$

Sub $x=-1$, $3-5-2=5A$

$$A=-\frac{4}{5}$$

Compare constant: $-2=4A+C$

$$C=-2-4\left(-\frac{4}{5}\right)=\frac{6}{5}$$

Compare coeff of x: $5=B+C$

$$B=5-\frac{6}{5}=\frac{19}{5}$$

$$\frac{3x^2+5x-2}{(x+1)(x^2+4)}=-\frac{4}{5(x+1)}+\frac{19x+6}{5(x^2+4)}$$

6 It is given that $y=\dfrac{2x-1}{\sqrt{x+2}}$ for $x>q$.

(a) State the value of q. [1]

$q = -2$

(b) Find $\frac{dy}{dx}$. [3]

$$y = \frac{2x-1}{\sqrt{x+2}}$$

$$\frac{dy}{dx} = \frac{2\sqrt{x+2} - (2x-1).\frac{1}{2}(x+2)^{-\frac{1}{2}}(1)}{x+2}$$

$$= \frac{2\sqrt{x+2} - \frac{(2x-1)}{2\sqrt{x+2}}}{x+2}$$

$$= \frac{4(x+2)-(2x-1)}{2\sqrt{(x+2)^3}}$$

$$= \frac{2x+9}{2\sqrt{(x+2)^3}}$$

(c) State whether $y = \frac{2x-1}{\sqrt{x+2}}$ is an increasing or decreasing function.
Explain your answer clearly. [2]

For $x > -2$

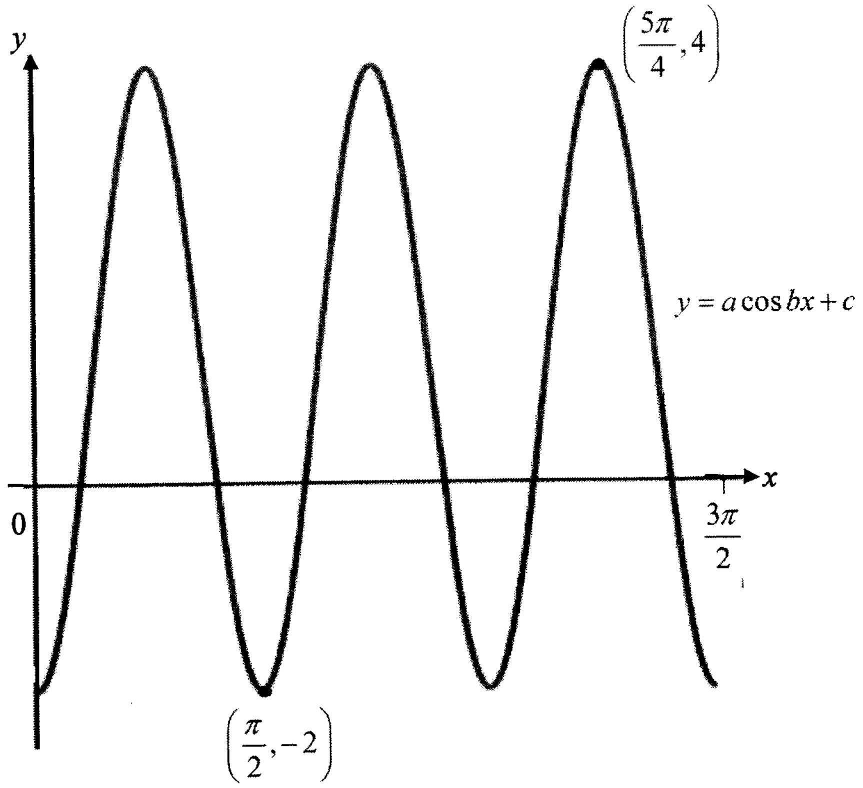
$\sqrt{(x+2)^3} > 0$, $2\sqrt{(x+2)^3} > 0$ and $2x+9 > 0$

$\Rightarrow \frac{dy}{dx} > 0$

$y = \frac{2x-1}{\sqrt{x+2}}$ is an <u>increasing function</u>.

[Turn over

7



The diagram shows the curve $y = a\cos bx + c$ for $0 \le x \le \frac{3\pi}{2}$ radians.

A minimum point $\left(\frac{\pi}{2}, -2\right)$ and a maximum point $\left(\frac{5\pi}{4}, 4\right)$ are indicated on the diagram.

(a) Explain why $b = 4$. [2]

Period $= \frac{\pi}{2}$

Period, $\frac{2\pi}{b} = \frac{\pi}{2}$

$b = 2\pi \div \frac{\pi}{2} = 4$

(b) Explain why $c = 1$. [2]

$y = c$ is the axis of curve

$$c = \frac{\text{maximum} + \text{minimum}}{2}$$

$$= \frac{4+(-2)}{2}$$

$c = 1$

(c) Hence find the equation of the curve. [2]

By observation, $a < 0$.

$a = -(4-1)$

OR

$a = -[1-(-2)]$

OR

$$a = -\left(\frac{\text{maximum} - \text{minimum}}{2}\right)$$

$$= -\left(\frac{4-(-2)}{2}\right)$$

$= -3$

Equation of curve is $y = -3\cos 4x + 1$.

8 **(a)** Prove the identity $\dfrac{\sin\theta}{1+\cos\theta}=\tan\dfrac{\theta}{2}$. [3]

$$\frac{\sin\theta}{1+\cos\theta}=\tan\frac{\theta}{2}$$

$$\text{LHS}=\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1+(2\cos^2\frac{\theta}{2}-1)}$$

$$=\frac{\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}$$

$$=\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$$

$$=\tan\frac{\theta}{2}$$

(b) Hence solve the equation $\dfrac{1+\cos\theta}{\sin\theta}=-3$ for $0^o \le \theta \le 360^o$. [4]

$\dfrac{1+\cos\theta}{\sin\theta}=-3$

$0^o \le \theta \le 360^o$

$\dfrac{\sin\theta}{1+\cos\theta}=-\dfrac{1}{3}$

$0^o \le \dfrac{\theta}{2} \le 180^o$

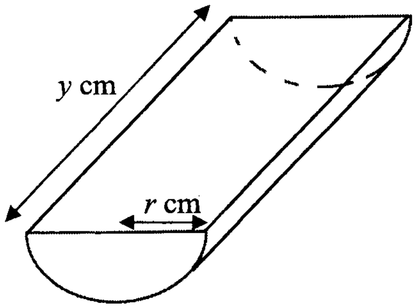
$\tan\dfrac{\theta}{2}=-\dfrac{1}{3}$

Reference angle = 18.435° (to 3 d.p.)

$\dfrac{\theta}{2}=180^o-18.435^o$

$\theta=323.1^o$ (to 1 d.p.)

9 The diagram shows a container, made of a thin sheet of plastic, in the shape of a hollow cylinder. The length of the container is y cm and each of the semicircular ends has radius r cm. The total external area of the plastic used is 8500 cm^2.



(a) Show that the volume, V cm^3, of the container is given by $4250r - \frac{1}{2}\pi r^3$. [3]

External surface area, $\pi r^2 + \frac{1}{2}(2\pi r y) = 8500$

$$\pi r^2 + \pi r y = 8500$$

$$y = \frac{8500 - \pi r^2}{\pi r}$$

Volume of the container, $V = \frac{1}{2}\pi r^2 y$

$$= \frac{1}{2}\pi r^2\left(\frac{8500 - \pi r^2}{\pi r}\right)$$

$$= 4250r - \frac{1}{2}\pi r^3 \text{ (shown)}$$

(b) Given that r can vary, find the stationary value of V and determine its nature. [4]

$V = 4250r - \frac{1}{2}\pi r^3$

$\frac{dV}{dr} = 4250 - \frac{3}{2}\pi r^2$

For stationary value of V, $\frac{dV}{dr} = 0$

$4250 - \frac{3}{2}\pi r^2 = 0$

$\frac{3}{2}\pi r^2 = 4250$

$r = \sqrt{\frac{8500}{3\pi}}$ $(r > 0)$

$= 30.031$ cm (to 5 s.f.)

$\frac{d^2V}{dr^2} = -3\pi r$

When $r = 30.031$, $\frac{d^2V}{dr^2} = -3\pi(30.031) < 0$

When $r = 30.031$, V is maximum.

Maximum $V = 4250(30.031) - \frac{1}{2}\pi(30.031)^3$

$= 85\ 100$ cm^3 (to 3 s.f.)

10 The solution of the equation $3^{2x-1} = 4\left(3^x\right)$ can be expressed as $\dfrac{\ln p}{\ln q}$.

(a) Find the value of p and of q. [4]

$$\frac{3^{2x-1}}{3^x} = 4$$

$$3^{2x-1-x} = 4$$

$$3^{x-1} = 4$$

$$(x-1)\ln 3 = \ln 4$$

$$x - 1 = \frac{\ln 4}{\ln 3}$$

$$x = \frac{\ln 4}{\ln 3} + 1$$

$$= \frac{\ln 4 + \ln 3}{\ln 3}$$

$$= \frac{\ln 12}{\ln 3}$$

Hence $p = 12$, $q = 3$

(b) Express the solution of $\log_{16}(3y-1)+\frac{1}{2}=\log_4(3y)$ as a rational number. [4]

$$\log_{16}(3y-1)-\log_4(3y)=-\frac{1}{2}$$

$$\frac{\log_4(3y-1)}{\log_4 4^2}-\log_4(3y)=-\frac{1}{2}$$

$$\frac{\log_4(3y-1)}{2}-\log_4(3y)=-\frac{1}{2}$$

$$\log_4(3y-1)-2\log_4(3y)=-1$$

$$\log_4\left(\frac{3y-1}{9y^2}\right)=-1$$

$$\frac{3y-1}{9y^2}=4^{-1}$$

$$9y^2-12y+4=0$$

$$(3y-2)^2=0 \Rightarrow y=\frac{2}{3}$$

11(a) Cement is poured into an empty right circular cone mould at a constant rate of 36π cm^3/s. The height of the cone is always twice its radius.

(i) Show that the radius of the cone after 1.5 minutes is approximately 16.94 cm. [2]

[The volume of a cone with radius r and height h is $\frac{1}{3}\pi r^2 h$.]

$$\frac{dV}{dt} = 36\pi \text{ cm}^3/\text{s}$$

$$36\pi(90) = \frac{1}{3}\pi r^2(2r)$$

$$r = \sqrt[3]{\frac{3240(3)}{2}} \approx 16.94 \text{ cm (shown)}$$

(ii) Find the rate of change of the radius of the cone after 1.5 minutes. [3]

$$V = \frac{1}{3}\pi r^2(2r)$$

$$= \frac{2}{3}\pi r^3$$

$$\frac{dV}{dr} = 2\pi r^2$$

When $r = 16.94$, $\frac{dV}{dr} = 2\pi(16.94)^2$

$= 1803.0$ (to 5 s.f.)

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{36\pi}{1803.0}$$

$= 0.0627$ cm/s (to 3 s.f.)

The radius is increasing at a rate of 0.0627 cm/s after 1.5 minutes.

(b) The intensity of light, I candelas, at a certain distance, x metres, from a light source is given by the formula $I = \frac{k}{x^2}$, where k is a constant.
Given that the intensity of light is 12 candelas at a distance of 4 metres from the source, find the rate at which the intensity of light is changing with respect to the distance when $x = 8$. [3]

$I = \frac{k}{x^2}$, k is a constant

When $I = 12$, $x = 4$,

$k = 12(4)^2$

$= 192$

$I = 192x^{-2}$

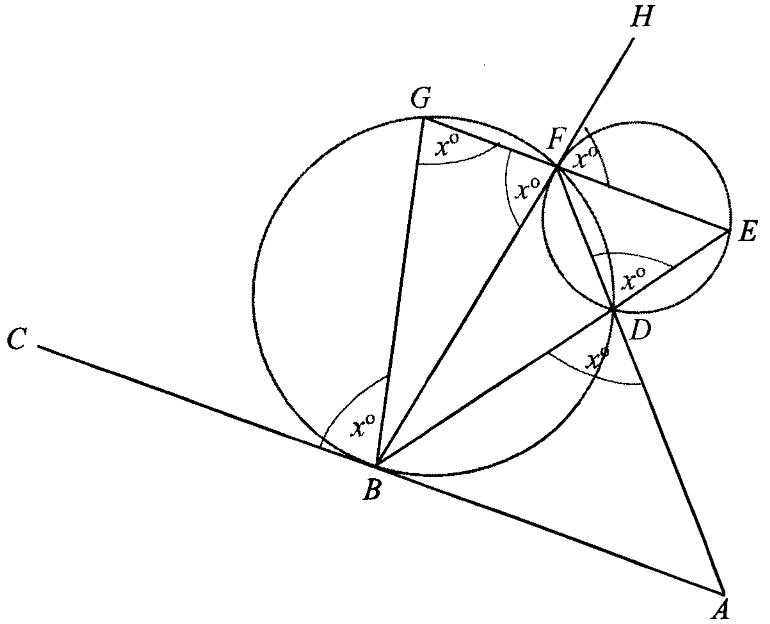
$\frac{dI}{dx} = 192(-2)x^{-3}$

$= -\frac{384}{x^3}$

When $x = 8$, the rate at which the intensity of light is changing with respect to the distance is $-\frac{384}{8^3} = -0.75$ candelas per metre.

12 The diagram shows two circles intersecting at D and F. **[Turn over**

ABC is a tangent to the bigger circle at B and BFH is a tangent to the smaller circle at F.
ADF, BDE and GFE are straight lines.
It is given that angle $BFG = x°$.



(a) Prove that triangle BGF is an isosceles triangle. [4]

$\angle BFG = \angle EFH = x^o$ (vert. opp. angles)

$\angle EFH = \angle EDF = x^o$ (Alternate Segment Theorem)

$\angle BDF = 180^o - \angle EDF$

$= 180^o - x^o$ (adj. angles on a straight line)

$\angle BGF = 180^o - \angle BDF$

$= 180^o - (180^o - x^o)$

$= x^o$ (angles in opp. segment)

<u>Alternatively,</u>

$\angle EDF = \angle ADB = x^o$ (vert. opp. angles)

$\angle BGF = \angle EDF = x^o$ (ext. angle of cyclic quad.)

Since $\angle BFG = \angle BGF = x^o$, $BG = BF$.

Triangle BGF is an isosceles triangle.

(b) Show that *CBA* is parallel to *GFE*. [2]

$\angle BGF = \angle ABF$

$= x^o$ (Alternate Segment Theorem)

$\angle ABF = \angle BFG = x^o$

$\therefore$ *CBA* is parallel to *GFE* (Converse of alternate angles)

(c) Show that triangle *ABD* is similar to triangle *BEG*. [2]

In ΔABD and ΔBEG,

$\angle ABD = \angle BEG$ (alternate angles, *CBA* // *GFE*)

$\angle ADB = \angle EDF$ (vert. opp. angles)

$= \angle BGE$ (ext. angle of cyclic quad)

$= x^o$

<u>Alternatively,</u>

$\angle ADB = 180^o - \angle BDF$ (adj. angles on a str. line)

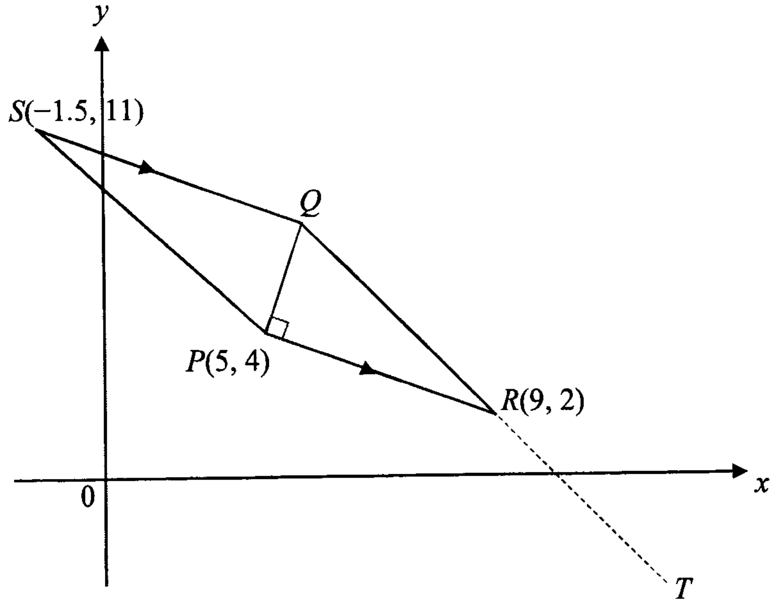
$= 180^o - (180^o - \angle BGF)$ (angles in opp. segment)

$= x^o$

$= \angle BGE$

Triangle *ABD* is similar to triangle *BEG* (AA Similarity Test).

13 The diagram (not drawn to scale) shows a right-angled triangle PQR in which point P is (5, 4) and point R is (9, 2).
Point S is (−1.5, 11) and PR is parallel to SQ.



(a) Find the equation of PQ. [2]

Gradient of PR is $\frac{4-2}{5-9} = -\frac{1}{2}$

Gradient of $PQ = 2$

Equation of PQ is $y - 4 = 2(x - 5)$

$y = 2x - 6$ ----- (1)

(b) Find the coordinates of Q. [3]

Gradient of SQ is $-\frac{1}{2}$

Equation of SQ is $y - 11 = -\frac{1}{2}(x + 1.5)$

$y = -\frac{1}{2}x + \frac{41}{4}$ ----- (2)

(1) = (2): $2x - 6 = -\frac{1}{2}x + \frac{41}{4}$

$\frac{5}{2}x = \frac{65}{4} \Rightarrow x = 6.5$

Sub $x = 6.5$ into (1): $y = 2(6.5) - 6 = 7$

Coordinates of Q are (6.5, 7).

(c) Given that T is a point on QR extended such that $QT = 10\sqrt{5}$ units, find the coordinates of T. [5]

Gradient of QR is $\dfrac{7-2}{6.5-9} = -2$

Equation of QR is $y - 2 = -2(x-9)$

$$y = -2x + 20$$

Let the coordinates of T be $(a, -2a+20)$.

$QT = \sqrt{500}$ units

$(a-6.5)^2 + (-2a+20-7)^2 = 500$

$(a-6.5)^2 + (13-2a)^2 = 500$

$a^2 - 13a + \dfrac{169}{4} + 169 - 52a + 4a^2 - 500 = 0$

$20a^2 - 260a - 1155 = 0$

$4a^2 - 52a - 231 = 0$

$(2a-33)(2a+7) = 0$

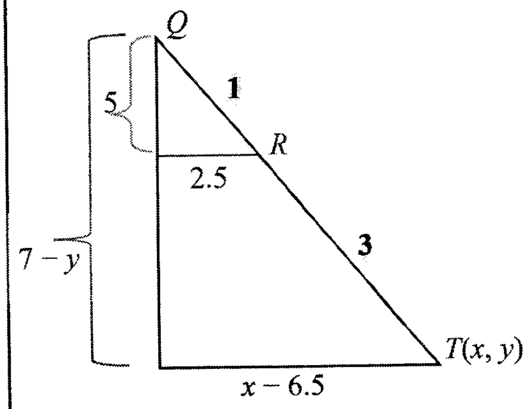
$a = 16.5$ or $a = -3.5$ (rejected as $a > 0$)

y-coordinate of T is $-2(16.5) + 20 = -13$.

Coordinates of T are $(16.5, -13)$.

[Turn over

Alternative method



$QR=\sqrt{(6.5-9)^2+(7-2)^2}$

$=\sqrt{\frac{125}{4}}$ units

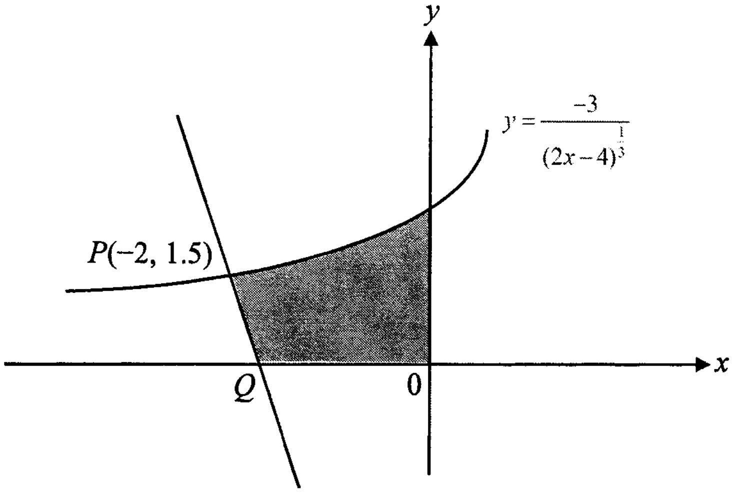
$\frac{QR}{QT}=\frac{\frac{1}{2}\sqrt{125}}{2\sqrt{125}}=\frac{1}{4}$

$\frac{x-6.5}{2.5}=4 \quad \Rightarrow x=16.5$

$\frac{7-y}{5}=4 \quad \Rightarrow y=-13$

Therefore, $T(16.5,-13)$.

14 The graph shows part of the curve $y = \dfrac{-3}{(2x-4)^{\frac{1}{3}}}$. The point $P(-2, 1.5)$ lies on the curve and the normal to the curve at P meets the x-axis at Q.



(a) Find the coordinates of Q. [5]

$y = -3(2x-4)^{-\frac{1}{3}}$

$\dfrac{dy}{dx} = -3\left(-\dfrac{1}{3}\right)(2x-4)^{-\frac{4}{3}}(2)$

$= \dfrac{2}{(2x-4)^{\frac{4}{3}}}$

When $x = -2$, $\dfrac{dy}{dx} = \dfrac{2}{[2(-2)-4]^{\frac{4}{3}}} = \dfrac{1}{8}$

Gradient of normal at $P = -8$.

Equation of normal at P is $y - 1.5 = -8(x+2)$

$$y = -8x - \frac{29}{2}$$

At Q, $y = 0$.

$0 = -8x - \dfrac{29}{2} \Rightarrow x = -\dfrac{29}{16}$

$Q\left(-\dfrac{29}{16}, 0\right)$

(b) Find the area of the shaded region bounded by the curve, the normal PQ and the coordinate axes.
Give your answer correct to 3 decimal places. [5]

Area of shaded region

$$= \int_{-2}^{0} -3(2x-4)^{-\frac{1}{3}}\,dx - \frac{1}{2}(2-1.8125)(1.5)$$

$$= \left[\frac{-3(2x-4)^{\frac{2}{3}}}{\frac{2}{3}(2)}\right]_{-2}^{0} - \frac{9}{64}$$

$$= \left[-\frac{9}{4}(2x-4)^{\frac{2}{3}}\right]_{-2}^{0} - \frac{9}{64}$$

$$= -\frac{9}{4}\left[(-4)^{\frac{2}{3}} - (-8)^{\frac{2}{3}}\right] - \frac{9}{64}$$

$= 3.190$ units^2 (to 3 d.p.)

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Name: Solution	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR 2025
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS — **4049/02**

Paper 2 — **3 September 2025**

Candidates answer on the Question Paper. **2 hours 15 mins**

No Additional Materials are required.

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Marks										

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This document consists of **16** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-r)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Show that the equation $3e^{2x}+7e^{x}=6$ has only one solution and find its value correct to 2 significant figures. [4]

$3e^{2x}+7e^{x}=6$

Let $u=e^{x}$

$3u^2+7u-6=0$

$(3u-2)(u+3)=0$

$u=\frac{2}{3}$ or $u=-3$

$e^{x}=\frac{2}{3}$ or $e^{x}=-3$ (no solution)

$x=\ln\frac{2}{3}$

$=-0.41$ (to 2 s.f.)

(b) Factorise $(3x+1)^3-8$ completely. [2]

$(3x+1)^3-2^3$

$=\left[(3x+1)-2\right]\left[(3x+1)^2+2(3x+1)+2^2\right]$

$=(3x-1)(9x^2+6x+1+6x+2+4)$

$=(3x-1)(9x^2+12x+7)$

2 **(a)** Show that $x-1$ is a factor of $3x^3+8x^2-5x-6$. [1]

Let $f(x)=3x^3+8x^2-5x-6$

$f(1)=3+8-5-6=0$

$\therefore$ By factor theorem, $x-1$ is a factor of $f(x)$

(b) Hence solve the equation $3x^3+8x^2-5x=6$ completely. [4]

$f(x)=3x^3+8x^2-5x-6=(x-1)(Ax^2+Bx+C)$

Comparing coefficients of x^3 : $A=3$

Comparing coefficients of x^2 : $8=B-3$

$B=11$

Compare constant : $C=6$

$f(x)=(x-1)(3x^2+11x+6)=0$

$(x-1)(3x+2)(x+3)=0$

$x=1, x=-\dfrac{2}{3}, x=-3$

OR

$$
\begin{array}{r|l}
 & 3x^2+11x+6 \\
\hline
x-1 & 3x^3+8x^2-5x-6 \\
 & -\underline{(3x^3-3x^2)} \\
 & 11x^2-5x \\
 & -\underline{(11x^2-11x)} \\
 & 6x-6 \\
 & -\underline{(6x-6)} \\
 & \underline{0}
\end{array}
$$

$f(x)=(x-1)(3x^2+11x+6)=0$

$(x-1)(3x+2)(x+3)=0$

$x=1, x=-\dfrac{2}{3}, x=-3$

(c) Hence solve the equation $3(y-2)^3+8(y-2)^2-5y+4=0$. [2]

$3(y-2)^3+8(y-2)^2-5(y-2)-6=0$

$y-2=1,\quad y-2=-\dfrac{2}{3},\quad y-2=-3$

$y=3,\quad y=\dfrac{4}{3},\quad y=-1$

3 **(a)** Show that $\frac{d}{dx}(\tan^3 2x) = 6\sec^4 2x - 6\sec^2 2x$. [3]

$$\frac{d}{dx}(\tan^3 2x) = 6\tan^2 2x \sec^2 2x$$

$$= 6\sec^2 2x(\sec^2 2x - 1)$$

$$= 6\sec^4 2x - 6\sec^2 2x$$

(b) Hence, find the exact value of $\int_0^{\frac{\pi}{8}} \sec^4 2x \ dx$. [5]

$$\int_0^{\frac{\pi}{8}} (6\sec^4 2x - 6\sec^2 2x)\, dx = \left[\tan^3 2x\right]_0^{\frac{\pi}{8}}$$

$$\int_0^{\frac{\pi}{8}} 6\sec^4 2x \ dx = \left[\tan^3 2x\right]_0^{\frac{\pi}{8}} + \int_0^{\frac{\pi}{8}} 6\sec^2 2x\, dx$$

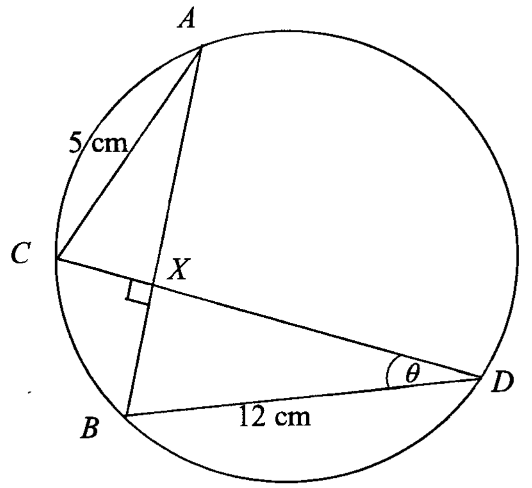
$$= \tan^3\frac{\pi}{4} - \tan^3 0 + \frac{6}{2}\left[\tan 2x\right]_0^{\frac{\pi}{8}}$$

$$= 1 + 3\left(\tan\frac{\pi}{4} - \tan 0\right)$$

$$= 4$$

$$\int_0^{\frac{\pi}{8}} \sec^4 2x \ dx = \frac{2}{3}$$

4 The diagram shows two chords AB and CD intersect at X such that angle $CXB = 90°$.



Given that $AC = 5$ cm, $BD = 12$ cm and angle $BDC = \theta$.

(a) Prove that $AB = 5\cos\theta + 12\sin\theta$. [3]

$\angle CAX = \theta$ (can shown in diagram)

$\cos\theta = \dfrac{AX}{5}$ $\sin\theta = \dfrac{BX}{12}$

$AX = 5\cos\theta$ $BX = 12\sin\theta$

$AB = AX + BX = 5\cos\theta + 12\sin\theta$

(b) Express AB in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [4]

$5\cos\theta + 12\sin\theta = R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$\tan\alpha = \dfrac{12}{5}$ $R = 13$

$\alpha = 67.38^0$

$AB = 13\cos(\theta - 67.4^0)$

(c) Find the maximum length of AB and the corresponding value of θ.

Hence state the radius of the circle. [3]

Maximum $AB = 13$ cm
$\theta = 67.4^0$
Radius = 6.5 cm

5 **(a)** Find the range of values of k for which $kx^2 + 8x + k < 6$ for all real values of x. [4]

$kx^2 + 8x + k - 6 < 0$
$b^2 - 4ac < 0$
$8^2 - 4k(k - 6) < 0$
$64 - 4k^2 + 24k < 0$
$k^2 - 6k - 16 > 0$
$(k - 8)(k + 2) > 0$
$k < -2$ or $k > 8$
$\therefore k < -2$

(b) Given the curve $y = 8x^2 + mx + m - 6$, where m is a constant, find

(i) the possible values of m for which the x-axis is a tangent to the curve. [3]

$8x^2 + mx + m - 6 = 0$
$b^2 - 4ac = 0$
$m^2 - 4(8)(m - 6) = 0$
$m^2 - 32m + 192 = 0$
$(m - 8)(m - 24) = 0$
$m = 8$ or $m = 24$

(ii) the range of values of m for which the curve has a positive y-intercept. [1]

$m - 6 > 0$
$m > 6$

(c) Show that the equation $2x^2 - cx + c^2 + 6 = 0$ has no real roots for all real values of c. [2]

$b^2 - 4ac = c^2 - 4(2)(c^2 + 6)$
$= -7c^2 - 48$

Since $c^2 \geq 0$ for all real values of c, $-7c^2 \leq 0$, $b^2 - 4ac < 0$
$\therefore$ the equation has no real roots for all real values of c.

6 It is given that $\left(x+\frac{k}{x^2}\right)^n$ is a binomial expansion where k and n are positive constants.

(a) Write down the first 4 terms in the expansion $\left(x+\frac{k}{x^2}\right)^5$ in terms of k. [2]

$$\left(x+\frac{k}{x^2}\right)^5 = x^5 + \binom{5}{1}x^4\left(\frac{k}{x^2}\right)+\binom{5}{2}x^3\left(\frac{k}{x^2}\right)^2+\binom{5}{3}x^2\left(\frac{k}{x^2}\right)^3+...$$

$$= x^5+5kx^2+\frac{10k^2}{x}+\frac{10k^3}{x^4}+...$$

(b) Hence, find the value of k if the term independent of x in the expansion $\left(5x+\frac{3}{x^2}\right)\left(x+\frac{k}{x^2}\right)^5$ is 5. [3]

$$\left(5x+\frac{3}{x^2}\right)\left(x+\frac{k}{x^2}\right)^5 = \left(5x+\frac{3}{x^2}\right)\left(x^5+5kx^2+\frac{10k^2}{x}+\frac{10k^3}{x^4}+...\right)$$

constant term = 5

$5\times10k^2+3\times5k=5$

$10k^2+3k-1=0$

$(5k-1)(2k+1)=0$

$k=\frac{1}{5}$ or $k=-\frac{1}{2}$ (reject)

(c) By using the formula for the general term, suggest two possible powers of n such that $\left(x+\frac{k}{x^2}\right)^n$ contains a term independent of x. [3]

$$T_{r+1}=\binom{n}{r}x^{n-r}\left(\frac{k}{x^2}\right)^r$$

$$=\binom{n}{r}k^r x^{n-3r}$$

For term independent of x, $n=3r$.

Since n must be a multiple of 3,
possible values of n are 6 and 9.

7 The equation of a circle with centre C is $x^2 + y^2 - 8x - 10y + 16 = 0$.

A point B (7, 9) lies on the circumference of the circle. P is the lowest point on the circle.

(a) Find the coordinates of C and the radius of the circle. [3]

$x^2 + y^2 - 8x - 10y + 16 = 0$
$(x-4)^2 - 4^2 + (y-5)^2 - 5^2 + 16 = 0$
$(x-4)^2 + (y-5)^2 = 25$
$C(4, 5)$ A1 radius = 5 units

(b) Find the equation of the tangent to the circle at B. [3]

Gradient of $BC = \frac{9-5}{7-4} = \frac{4}{3}$

Gradient of tangent at $B = -\frac{3}{4}$

Equation of tangent at B: $y - 9 = -\frac{3}{4}(x-7)$

$y = -\frac{3}{4}x + \frac{57}{4}$

(c) Explain why P lies on the x-axis. [1]

P is vertically below centre C.
x-coordinate of P = x-coordinate of $C = 4$
y-coordinate of P – radius = $5 - 5 = 0$
$\therefore P(4, 0)$ lies on the x-axis.

8 A particle moves in a straight line so that t seconds after leaving a fixed point O, its velocity v m/s, is given by $v = 16(2e^{-2t} - e^{-t})$. Find

(a) the time when the particle reaches its greatest distance from O, [3]

When $v = 0$, $2e^{-2t} - e^{-t} = 0$

$e^{-t}(2e^{-t} - 1) = 0$

$e^{-t} = 0$ (NA) or $e^{-t} = \frac{1}{2}$

$t = \ln 2$ or 0.693

(b) the value of the greatest distance from O. [3]

$s = \int 16(2e^{-2t} - e^{-t})\, dt$

$= 16(-e^{-2t} + e^{-t}) + c$

When $t = 0$, $s = 0$, $c = 0$

$s = 16(-e^{-2t} + e^{-t})$

when $t = \ln 2$, $s = 4$ m

(c) the minimum velocity of the particle. [4]

$a = 16(-4e^{-2t} + e^{-t})$

when $a = 0$, $-4e^{-2t} + e^{-t} = 0$

$e^{-t}(-4e^{-t} + 1) = 0$

$e^{-t} = 0$ (NA) or $4e^{-t} = 1$

$t = \ln 4$

When $t = \ln 4$, $v = -2$ m/s

9 **(a)** For the equation $py = xy + 1$ where p is an unknown constant, may be represented

by a straight line, expressed in the form $Y = mX + c$, where X and Y are functions of x and/or y and m and c are constants. Using the following table, insert in it an expression for Y, X, m and c. [2]

	Y	X	m	c
$py = xy + 1$	y	xy	$\frac{1}{p}$	$\frac{1}{p}$ B1
$xy = py - 1$	OR xy	Y	P	-1
$py = xy + 1$	OR $\frac{1}{y}$	x	-1	P

(b) The variables x and y are related by the equation $x^a y = b$, where a and b are constants. The table below shows some corresponding values of x and y.

x	1.31	1.63	1.97	2.54	4.03
y	1270	398	131	35.1	2.63

(i) Plot $\lg y$ against $\lg x$ and draw a straight line graph. [3]

$\lg x$	0.117	0.212	0.294	0.405	0.605
$\lg y$	3.10	2.60	2.12	1.55	0.420

$a\lg x + \lg y = \lg b$

$\lg y = -a\lg x + \lg b$

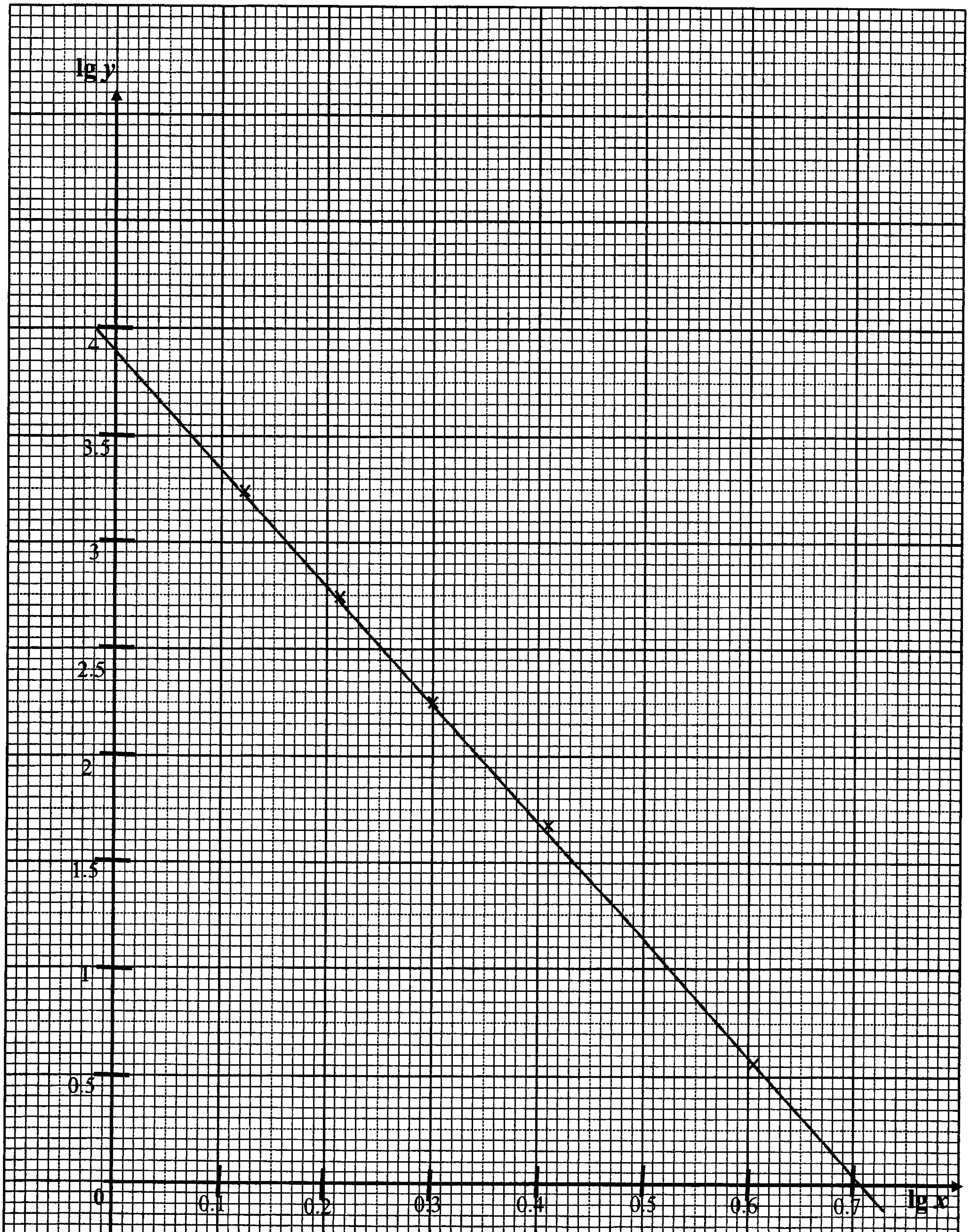
(ii) Use your graph to estimate the value of a and of b. [4]

Gradient = -5.51

$a \approx 5.51 \quad (\pm 0.1)$

Y-intercept = $\lg b = 3.8\ (\pm 0.1)$

$b = 6310\ (5012 - 7943)$


lg y
4
3.5
3
2.5
2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
lg x

10 **(a)** **(i)** Show that $\dfrac{(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)}{\cos^3\theta}=\tan^3\theta-1$. [4]

From LHS $\dfrac{(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)}{\cos^3\theta}$

$$=\frac{\sin\theta+\sin^2\theta\cos\theta-\cos\theta-\sin\theta\cos^2\theta}{\cos^3\theta}$$

$$=\frac{\sin\theta+\cos\theta(1-\cos^2\theta)-\cos\theta-\sin\theta(1-\sin^2\theta)}{\cos^3\theta}$$

$$=\frac{\sin\theta+\cos\theta-\cos^3\theta-\cos\theta-\sin\theta+\sin^3\theta}{\cos^3\theta}$$

$$=\frac{\sin^3\theta-\cos^3\theta}{\cos^3\theta}$$

$$=\tan^3\theta-1 \text{ (shown)}$$

(ii) Hence, for $0<\theta<2\pi$, find the reflex angle θ such that

$(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)=\cos^3\theta$. [3]

$$(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)=\cos^3\theta$$

$$\frac{(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)}{\cos^3\theta}=1$$

$$\tan^3\theta-1=1$$

$$\tan\theta=\sqrt[3]{2}$$

$$\theta=\tan^{-1}\left(\sqrt[3]{2}\right)$$

$$=4.04$$

(b) Solve the equation $\sin 3A \cos 2A = \cos 3A \sin 2A$ for $0 \le A \le 180°$. [2]

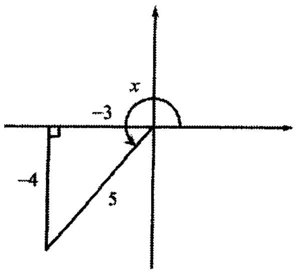
$\sin 3A \cos 2A - \cos 3A \sin 2A = 0$

$\sin(3A - 2A) = 0$

$\sin A = 0$

$A = 0^0, 180°$

(c) Given that $\sin x = -\frac{4}{5}$ and $\tan x > 0$, find, without using a calculator, the value of $\cos 4x$. [2]



$$\cos 4x = 1 - 2\sin^2 2x$$
$$= 1 - 2(2\sin x \cos x)^2$$
$$= 1 - 2\left(2 \times \left(-\frac{4}{5}\right) \times \left(-\frac{3}{5}\right)\right)^2$$
$$= -\frac{527}{625}$$

OR

$$\cos 2x = 1 - 2\sin^2 x$$
$$= 1 - 2\left(-\frac{4}{5}\right)^2$$
$$= -\frac{7}{25}$$
$$\cos 4x = 2\cos^2 2x - 1$$
$$= 2\left(-\frac{7}{25}\right)^2 - 1$$
$$= -\frac{527}{625}$$

(d) **Without using a calculator,** show that

$\tan\frac{\pi}{12} = 2-\sqrt{3}$. [4]

$$\tan\frac{\pi}{12} = \tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$$

$$= \frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$$

$$= \frac{\sqrt{3}-1}{1+\sqrt{3}}$$

$$= \frac{\sqrt{3}-1}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$$

$$= \frac{2\sqrt{3}-4}{-2}$$

$$= 2-\sqrt{3}$$