DUNMAN SECONDARY SCHOOL

CANDIDATE NAME

CLASS

INDEX NUMBER

PRELIMINARY EXAMINATION 2025
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

MATHEMATICS **4052/01**
Paper 1
20 August 2025
2 hours 15 minutes

Solutions

Question	Answer
1	$x-4=\dfrac{2}{x-3}$
	$(x-4)(x-3)=2$
	$x^2-7x+12=2$
	$x^2-7x+10=0$
	$(x-2)(x-5)=0$
	$x=2$ or 5
	When $x=2, y=-2$
	When $x=5, y=1$
	Length = $\sqrt{(2-5)^2+(-2-1)^2}$ $= 4.24$ unit
2a	$-90° < \tan^{-1} p < 90°$ or $-\dfrac{\pi}{2} < \tan^{-1} p < \dfrac{\pi}{2}$
2b	$\pi - x$
3	$3x^2+15x+20 = 3(x^2+5x)+20$
	$= 3\left[(x+2.5)^2-(2.5)^2\right]+20$
	$= 3(x+2.5)^2+1.25$
	Since $3(x+2.5)^2+1.25 \geq 1.25$, $3x^2+15x+20$ cannot be smaller than 1.

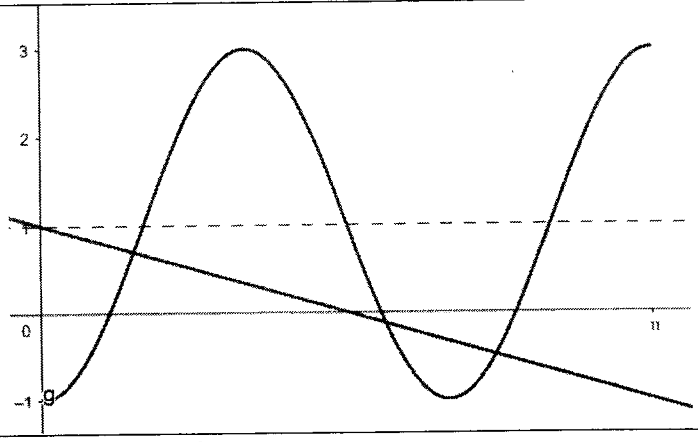
Question	Answer
4a	$\dfrac{x^2+3x+2}{x^2(2x+1)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{2x+1}$
	$x^2+3x+2=Ax(2x+1)+B(2x+1)+Cx^2$
	When $x=-\dfrac{1}{2}$, $\dfrac{1}{4}-\dfrac{3}{2}+2=C\left(\dfrac{1}{4}\right)$ $C=3$
	When $x=0$, $B=2$
	Comparing coefficients of x^2, $A=-1$
	$\dfrac{x^2+3x+2}{x^2(2x+1)}=-\dfrac{1}{x}+\dfrac{2}{x^2}+\dfrac{3}{2x+1}$
4b	$\int\dfrac{x^2+3x+2}{x^2(2x+1)}\,dx=\int-\dfrac{1}{x}+\dfrac{2}{x^2}+\dfrac{3}{2x+1}\,dx$ $=-\ln x-\dfrac{2}{x}+\dfrac{3}{2}\ln(2x+1)+c$
5	$3\log_5 y-\log_y 5=2$
	$3\log_5 y-\dfrac{\log_5 5}{\log_5 y}=2$
	$3\log_5 y-\dfrac{1}{\log_5 y}=2$
	$3(\log_5 y)^2-2\log_5 y-1=0$
	$(3\log_5 y+1)(\log_5 y-1)=0$
	$\log_5 y=1$ or $-\dfrac{1}{3}$
	$y=5$ or $y=5^{-\frac{1}{3}}=0.585$ (3s.f.)
Question	**Answer**
6a	$\sin(105°)=\sin(60°+45°)$
	$=\sin 60°\cos 45°+\cos 60°\sin 45°$
	$=\dfrac{\sqrt{3}}{2}\cdot\dfrac{1}{\sqrt{2}}+\dfrac{1}{2}\cdot\dfrac{1}{\sqrt{2}}$
	$=\dfrac{\sqrt{3}+1}{2\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}$
	$=\dfrac{\sqrt{6}+\sqrt{2}}{4}$
6b	$\dfrac{1}{2}(4)(BC)\sin 105°=10+2\sqrt{3}$

	$\frac{1}{2}(4)(BC)\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)=10+2\sqrt{3}$
	$BC=\frac{2\left(10+2\sqrt{3}\right)}{\sqrt{6}+\sqrt{2}}\times\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$
	$=\frac{4\left(5+\sqrt{3}\right)\left(\sqrt{6}-\sqrt{2}\right)}{6-2}$
	$=5\sqrt{6}+\sqrt{18}-5\sqrt{2}-\sqrt{6}$
	$=4\sqrt{6}+3\sqrt{2}-5\sqrt{2}$
	$=4\sqrt{6}-2\sqrt{2}$
	$=\sqrt{2}\left(4\sqrt{3}-2\right)$

Question	Answer
7a	$9^x + 6^x = 6(4^x)$
	$\frac{9^x}{4^x} + \frac{6^x}{4^x} = 6$
	$\frac{3^{2x}}{2^{2x}} + \left(\frac{6}{4}\right)^x - 6 = 0$
	$\left(\frac{3}{2}\right)^{2x} + \left(\frac{3}{2}\right)^x - 6 = 0$
	When $u = \left(\frac{3}{2}\right)^x$, $u^2 + u - 6 = 0$
7b	$(u+3)(u-2) = 0$
	$u = 2$ or -3 (n.a.)
	$\left(\frac{3}{2}\right)^x = 2$
	$x = \frac{\ln 2}{\ln \frac{3}{2}}$ or $\frac{\lg 2}{\lg \frac{3}{2}} = 1.71$
8	When $x = -2$, $2(-2)^3 + a(-2)^2 + b(-2) + 10 = 0$
	$4a - 2b = 6$
	$2a - b = 3$ - eqn 1
	When $x = 3$, $2(3)^3 + a(3)^2 + b(3) + 10 = 10$
	$9a + 3b = -54$
	$3a + b = -18$ - eqn 2
	eqn 1 + eqn 2: $5a = -15$
	$a = -3$
	$\therefore b = -9$
Question	**Answer**
9a	$12t - \frac{3}{2}t^2 = 0$
	$t\left(12 - \frac{3}{2}t\right) = 0$
	$t = 0$ or 8
	$\therefore 8$ s
9b	distance $= \int_0^8 12t - \frac{3}{2}t^2 \, dx$
	$= \left[6t^2 - \frac{1}{2}t^3\right]_0^8$

	$=[128]-[0]$
	$= 128$ m
9c	$a = 12 - 3t$
	When $t = 5$, $a = -3$ m/s^2

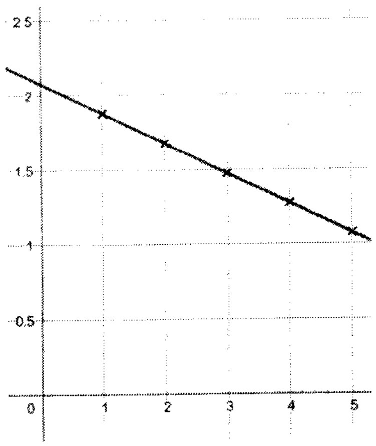
Question	Answer
10	$y = xe^{-3x}$
	$\frac{dy}{dx} = e^{-3x}(1) + x(-3e^{-3x})$
	$= e^{-3x}(1-3x)$
	$e^{-3x}(1-3x) = 0$
	$x = \frac{1}{3}$
	$y = \frac{1}{3e}$ or 0.123
	$\frac{d^2y}{dx^2} = (1-3x)(-3e^{-3x}) + e^{-3x}(-3)$
	When $x = \frac{1}{3}$, $\frac{d^2y}{dx^2} < 0$
	$\left(\frac{1}{3}, -\frac{1}{3e}\right)$ is a maximum point.

Question	Answer
11a	
11b	The $2\cos 3x = \frac{2}{\pi}x$ is equivalent to $-2\cos 3x + 1 = -\frac{2}{\pi}x + 1$ and so its solutions can be found from the interception points of the two graphs.

Question	Answer
12a	Midpoint of $PQ = \left(\frac{1+9}{2}, \frac{-2+10}{2}\right) = (5,4)$
	Gradient of $PQ = \frac{-2-10}{1-9} = \frac{3}{2}$
	Gradient of perpendicular bisector $= -\frac{2}{3}$
	$y-4=-\frac{2}{3}(x-5)$
	$y=-\frac{2}{3}x+\frac{22}{3}$
12b	$-\frac{2}{3}x+\frac{22}{3}=8x-10$
	$-\frac{26}{3}x=-\frac{52}{3}$
	$x=2$
	$R(2,6)$
12c	Area $= \frac{1}{2}\begin{vmatrix} 9 & 2 & 1 & 9 \\ 10 & 6 & -2 & 10 \end{vmatrix}$
	$= \frac{1}{2}\left[(54-4+10)-(20+6-18)\right]$
	$= 26$ unit^2

Question	Answer
13a	see table and graph below

t	1	2	3	4	5
m	6.55	5.36	4.40	3.60	2.94
$\ln m$	1.88	1.68	1.48	1.28	1.08



Question	Answer
13b	$\ln M = 2.08$
	$M = 8.00$
13c	Gradient = -0.2
	$k = 0.2$

Question	Answer
14a	$\frac{d^2y}{dx^2}=-4\sin(2x+\pi)$
	$\frac{dy}{dx}=2\cos(2x+\pi)+c_1$
	$3=2\cos\left[2\left(\frac{\pi}{2}\right)+\pi\right]+c_1$
	$3=2(1)+c_1$
	$c_1=1$
	$\frac{dy}{dx}=2\cos(2x+\pi)+1$
	$y=\sin(2x+\pi)+x+c_2$
	$\pi=\sin\left[2\left(\frac{\pi}{2}\right)+\pi\right]+\frac{\pi}{2}+c_2$
	$c_2=\frac{\pi}{2}$
	$y=\sin(2x+\pi)+x+\frac{\pi}{2}$
14b	When $x=3$, $\frac{dy}{dx}=2\cos[2(3)+\pi]+1=-0.920$
	y is decreasing when $x=3$.

Question	Answer
15	$y=\dfrac{16}{(x+1)^2}-1=16(x+1)^{-2}-1$
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-32(x+1)^{-3}$
	$-4=-32(x+1)^{-3}$
	$(x+1)^3=8$
	$x=1$
	At L, $y=\dfrac{16}{(1+1)^2}-1=3$
	$y-3=-4(x-1)$
	$y=-4x+7$
	At N, $0=-4x+7$
	$x=1.75$
	At M, $0=\dfrac{16}{(x+1)^2}-1$
	$x=3$ or -5 (n.a.)
	Area $=\left[\int_1^3 16(x+1)^{-2}-1\ \mathrm{d}x\right]-\int_1^{1.75}-4x+7\ \mathrm{d}x$
	$=\left[-16(x+1)^{-1}-x\right]_1^3-\left[-2x^2+7x\right]_1^{1.75}$
	$=2-1.125$
	$=0.875$ unit^2

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MATHEMATICS **4052/02**

Paper 2

25 August 2025

2 hours 15 minutes

Marking Scheme

Question	Answer
1a	$h=7+20t-5t^2$
	$=-5(t^2-4t)+7$
	$=-5\left[(t-2)^2-4\right]+7$
	$=-5(t-2)^2+27$
	maximum height $=27$
	OR
	$\frac{dh}{dt}=-10t+20$
	$-10t+20=0$
	$t=2$
	When $t=2,\ h=7+20(2)-5(2)^2=27\,\text{m}$
1b	$x^2+6x+k=k(x-1)$
	$x^2+(6-k)x+2k=0$
	$(6-k)^2-4(1)(2k)<0$
	$36-12k+k^2-8k<0$
	$k^2-20k+32<0$
	$(k-2)(k-18)<0$
	$\therefore 2<x<18$
2a	$\tan A=\frac{4}{3}$
	$\tan 2A=\dfrac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2}$
	$=-\frac{24}{7}$
Question	**Answer**
2b	$\sin B=\frac{12}{13}$, $\cos B=\frac{5}{13}$
	$\sin\left(B+\frac{3\pi}{2}\right)=\sin B\cos\frac{3\pi}{2}+\sin\frac{3\pi}{2}\cos B$
	$=\frac{12}{13}\times 0+(-1)\times\frac{5}{13}$
	$=-\frac{5}{13}$
	OR
	$\sin\left(B+\frac{3\pi}{2}\right)=\sin\left(2\pi-\frac{\pi}{2}+B\right)$

	$= \sin\left(-\frac{\pi}{2}+B\right) = -\sin\left(\frac{\pi}{2}-B\right)$
	$= -\cos B = -\frac{5}{13}$
2c	$\cos B = 2\cos^2\frac{B}{2} - 1$
	$\frac{5}{13} = 2\cos^2\frac{B}{2} - 1$
	$2\cos^2\frac{B}{2} = \frac{18}{13}$
	$\cos^2\frac{B}{2} = \frac{9}{13}$
	$\cos\frac{B}{2} = \frac{3}{\sqrt{13}}$ or $-\frac{3}{\sqrt{13}}$ (rej)

Question	Answer
3a	$\angle PQC = \angle PSQ$ (Alternate segment theorem)
	$\angle PQC = \angle PBC$ (angles in same segment)
	Hence $\angle PBC = \angle PSQ$
	By alternate angles of parallel lines, BC is parallel to QS
3b	$\angle PQC = \angle PSQ$ (Alternate segment theorem)
	$\angle PMQ = \angle QMS$ (Common angle)
	ΔPQM is similar to ΔQSM (AA Similarity Test)
3c	Since ΔPQM is similar to ΔQSM, $\frac{QM}{SM} = \frac{PM}{QM}$
	$(QM)^2 = PM \times SM$

4a	$\frac{d}{dx}(\cot x)=\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$
	$=\frac{\sin x(-\sin x)-\cos x(\cos x)}{\sin^2 x}$
	$=\frac{-\sin^2 x-\cos^2 x}{\sin^2 x}$
	$=-\frac{1}{\sin^2 x}$
	$=-\text{cosec}^2\, x$ (Shown)
	OR
	$\frac{d}{dx}(\cot x)=\frac{d}{dx}(\tan x)^{-1}$
	$=-(\tan x)^{-2}(\sec^2 x)$
	$=-\left(\frac{\sin x}{\cos x}\right)^{-2}\left(\frac{1}{\cos^2 x}\right)$
	$=-\left(\frac{\cos^2 x}{\sin^2 x}\right)\left(\frac{1}{\cos^2 x}\right)$
	$=-\frac{1}{\sin^2 x}$
	$=-\text{cosec}^2\, x$ (Shown)
4b	$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} -\cot^2 x\, dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 1-\text{cosec}^2 x\, dx$
	$=\left[x+\cot x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$
	$=\left[\frac{\pi}{3}+\cot\frac{\pi}{3}\right]-\left[\frac{\pi}{4}+\cot\frac{\pi}{4}\right]$
	$=\frac{\pi}{3}+\frac{1}{\sqrt{3}}-\frac{\pi}{4}-1$
	$=\frac{\pi}{12}+\frac{1}{\sqrt{3}}-1$ or -0.161

Question	Answer
5a	General term $=\binom{8}{r}x^{8-r}\left(-\frac{k}{x^3}\right)^r$
	$=\binom{8}{r}x^{8-r}(-1)^r k^r (x^{-3})^r$
	$=\binom{8}{r}(-1)^r k^r x^{8-4r}$
	When $8-4r=0$, $r=2$
	$\binom{8}{2}(-1)^2 k^2 x^{8-4(2)}=7$
	$28k^2=7$
	$k^2=\frac{1}{4}$
	$k=\frac{1}{2}$
5b	When $8-4r=-4$, $r=3$
	$T_4=\binom{8}{3}(-1)^3\left(\frac{1}{2}\right)^3 x^{-4}$
	$=-7x^{-4}$
	$(1+x^4)\left(x-\frac{k}{x^3}\right)^8=(1+x^4)(...+7-7x^{-4}+...)$
	$=...+7-7+...=...+0+...$
	There is no constant term in the expansion. (Shown)

Question	Answer
6a	$AM = 2\cos\theta$
	$BM = 2\sin\theta$
	$BC = 4\sin\theta$
	Total area $= (4\sin\theta)^2 + \frac{1}{2}(2\cos\theta)(4\sin\theta)$
	$= 16\sin^2\theta + 4\sin\theta\cos\theta$
6b	$16\sin^2\theta + 4\sin\theta\cos\theta$
	$= 8(2\sin^2\theta) + 2(2\sin\theta\cos\theta)$
	$= 8(1 - \cos 2\theta) + 2(\sin 2\theta)$
	$= 2\sin 2\theta - 8\cos 2\theta + 8$
6c	$R = \sqrt{2^2 + 8^2} = \sqrt{68}$
	$\tan\alpha = \frac{8}{2}$
	$\alpha = 76.0°$
	$2\sin 2\theta - 8\cos 2\theta = \sqrt{68}\sin(2\theta - 76.0°)$
7a	$LHS = \dfrac{\cos^2 x - \sin^2 x}{1 + 2\sin x\cos x}$
	$= \dfrac{\cos^2 x - \sin^2 x}{\sin^2 x + \cos^2 x + 2\sin x\cos x}$
	$= \dfrac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^2}$
	$= \dfrac{\cos x - \sin x}{\cos x + \sin x}$
	$= \dfrac{1 - \tan x}{1 + \tan x} = RHS$

Question	Answer
7b	$\dfrac{1-\tan x}{1+\tan x}=\dfrac{2}{3}\tan x$
	$3(1-\tan x)=2\tan x+\tan^2 x$
	$2\tan^2 x+5\tan x-3=0$
	$(2\tan x-1)(\tan x+3)=0$
	$\tan x=\dfrac{1}{2}$ or -3
	$x=0.464$ or 3.61
	$x=1.89$ or 5.03
8a	Quotient: $x^2-\frac{3x}{2}-2$ Divisor: $2x-4$, Dividend: $2x^3-7x^2+2x+3$ $-$ $2x^3-4x^2$ $-3x^2+2x+3$ $-$ $-3x^2+6x$ $3-4x$ $-$ $8-4x$ -5

Question	Answer
8b	$2x^3-7x^2+2x+3=0$
	Let $\mathrm{f}(x)=2x^3-7x^2+2x+3$
	When $x=3$, $\mathrm{f}(3)=2(3)^3-7(3)^2+2(3)+3=0$
	By factor theorem, $(x-3)$ is a factor of f(x).
	$(x-3)(2x^2-x-1)=0$
	$(x-3)(2x+1)(x-1)=0$
	$x=-\frac{1}{2}$, 1 or 3
8c	$3y^6+2y^4-7y^2+2=0$
	$3+\frac{2}{y^2}-\frac{7}{y^4}+\frac{2}{y^6}=0$
	$3+2\left(\frac{1}{y^2}\right)-7\left(\frac{1}{y^4}\right)+2\left(\frac{1}{y^6}\right)=0$
	$3+2\left(\frac{1}{y^2}\right)-7\left(\frac{1}{y^2}\right)^2+2\left(\frac{1}{y^2}\right)^3=0$
	$\therefore x=\frac{1}{y^2}$, Kun Ye is correct

Question	Answer
9a	Height = $3x^2$
	Base = $2x$
	Area = $\frac{1}{2}(2x)(3x^2)=3x^3$
9b	$\frac{dy}{dx}=6x$
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$
	$3=6x\times\frac{dx}{dt}$
	$3=6x\times\frac{dx}{dt}$
	$\frac{dx}{dt}=\frac{1}{2x}$
	When $x=4$, $\frac{dx}{dt}=\frac{1}{8}$
	Rate of change of $PQ=\frac{1}{4}$ unit/s
9c	$\frac{dA}{dx}=9x^2$
	$\frac{dA}{dt}=\frac{dA}{dx}\times\frac{dx}{dt}$
	$\frac{dA}{dt}=9x^2\times\frac{1}{8}$
	When $x=4$, $\frac{dA}{dt}=18$ unit²/s

Question	Answer
10a	y-coordinate of centre = $\frac{-1+9}{2}=4$
10b	Let the x-coordinate of the centre be a,
	$(x-a)^2+(y-4)^2=25$
	From the line, $y=\frac{4x+9}{3}$
	$(x-a)^2+\left(\frac{4x+9}{3}-4\right)^2=25$
	$(x-a)^2+\left(\frac{4}{3}x-1\right)^2=25$
	$x^2-2ax+a^2+\frac{16}{9}x^2-\frac{8}{3}x+1=25$
	$\frac{25}{9}x^2+\left(-2a-\frac{8}{3}\right)x+a^2-24=0$
	Since the line is a tangent, $b^2-4ac=0$
	$\left(-2a-\frac{8}{3}\right)^2-4\left(\frac{25}{9}\right)(a^2-24)=0$
	$4a^2+\frac{32}{3}a+\frac{64}{9}-\frac{100}{9}a^2+\frac{2400}{9}=0$
	$36a^2+96a+64-100a^2+2400=0$
	$-64a^2+96a+2464=0$
	$2a^2-3a-77=0$
	$(2a+11)(a-7)=0$
	$a=7$ or -5.5 (rej)

Question	Answer
10c	Distance between point and centre $=\sqrt{(7-3)^2+(4-0)^2}$
	$=5.66$ unit
	Since the distance is longer than the radius, the point lies outside.
11a	$CX=\sqrt{(30-x)^2+20^2}$
	$=\sqrt{x^2-60x+1300}$
	$T=\frac{x}{4}+\frac{1}{2}\sqrt{x^2-60x+1300}$
11b	$\frac{dT}{dx}=\frac{1}{4}+\frac{1}{2}\left[\frac{1}{2}\left(x^2-60x+1300\right)^{-\frac{1}{2}}(2x-60)\right]$
	$=\frac{1}{4}+\frac{2x-60}{4\sqrt{x^2-60x+1300}}$
11c	$\frac{1}{4}+\frac{2x-60}{4\sqrt{x^2-60x+1300}}=0$
	$\sqrt{x^2-60x+1300}+2x-60=0$
	$\sqrt{x^2-60x+1300}=60-2x$
	$x^2-60x+1300=3600-240x+4x^2$
	$3x^2-180x+2300=0$
	$x=\frac{180\pm\sqrt{180^2-4(3)(2300)}}{2(3)}$
	$x=41.5$ (rej) or 18.5