Class/ Index Number	Centre Number/ 'O' Level Index Number	Name
/	/	

新加坡海星中学

MARIS STELLA HIGH SCHOOL
PRELIMINARY EXAMINATION
SECONDARY FOUR

ADDITIONAL MATHEMATICS **4049/1**
Paper 1

21 August 2025

Candidates answer on the Question Paper. **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 90.

For Examiners' Use

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9
/ 5	/ 6	/ 5	/ 6	/ 5	/ 6	/ 7	/ 7	/ 7

Q10	Q11	Q12	Q13	SUBTOTAL		
/ 8	/ 8	/ 10	/ 10	Statement		
				Presentation		
				Units		90
				Rounding Off		

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The curve $(x-1)^2+(y-3)^2=5$ intersects the line $y-3x=5$ at two points. Find the coordinates of these two points. [5]

$(x-1)^2+(y-3)^2=5$ ------(1)

$y-3x=5$

$y=3x+5$ -----------(2)

Subst (2) to (1), $(x-1)^2+(3x+5-3)^2=5$ M1

$x^2-2x+1+9x^2+12x+4=5$

$10x^2+10x=0$ M1

$10x(x+1)=0$ M1

$x=0$ or $x=-1$

$y=5$ or $y=2$

the coordinates are (0, 5) and (–1, 2) A2

2 A stone is thrown vertically upwards such that its height, h metres from the ground at time t seconds after being thrown is given by the formula $h = -9t^2 + 12t + 1$.

(a) Explain the meaning of the constant term in the formula. [1]

The **initial height** of the stone is **1m above the ground.** B1

(b) Express h in the form $a(t+b)^2 + c$, where a, b and c are constants to be determined. [3]

$$h = -9\left(t^2 - \frac{4}{3}\right) + 1$$

$$= -9\left(t^2 - \frac{4}{3}t + \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^2\right) + 1$$

$$= -9\left(t - \frac{2}{3}\right)^2 + 5$$

$a = -9$, $b = -\frac{2}{3}$, $c = 5$ B3 for each of the value

(c) Hence state the maximum height attained by the stone and the time at which this occurs. [2]

Maximum height = 5m when time $= \frac{2}{3}$ s √B2 FT from (b)

3 Find the range of values of the constant p such that $y = px^2 - 4x + p - 3$ is always positive for all real values of x. [5]

for $y = px^2 - 4x + p - 3$ to be always positive,
$p>0$ and discriminant < 0,

$(-4)^2 - 4(p)(p-3) < 0$	M1 for using discriminant < 0
$16 - 4p^2 + 12p < 0$	
$p^2 - 3p - 4 > 0$	M1 for quadratic eqn
$(p+1)(p-4) > 0$	M1 for factorising
$p < -1$ or $p > 4$	M1
since $p > 0$, $p > 4$	A1

4 Express $\dfrac{15x^2-13x+18}{(x-2)(3x^2+1)}$ in partial fractions. [6]

Let $\dfrac{15x^2-13x+18}{(x-2)(3x^2+1)}=\dfrac{A}{x-2}+\dfrac{Bx+C}{3x^2+1}$ M1

$15x^2-13x+18=A(3x^2+1)+(Bx+C)(x-2)$ M1

when $x=2$, $13A=52$

$A=4$ M1

when $x=0$, $-2C+4=18$

$C=-7$ M1

when $x=1$, $16-B+7=20$

$B=3$ M1

$\dfrac{15x^2-13x+18}{(x-2)(3x^2+1)}=\dfrac{4}{x-2}+\dfrac{3x-7}{3x^2+1}$ A1

5 **(a)** Find the first 4 terms in the expansion of $\left(\frac{a}{x}+3x^2\right)^6$ in ascending power of x, simplifying each term. [3]

$$\left(\frac{a}{x}+3x^2\right)^6=\left(\frac{a}{x}\right)^6+\binom{6}{1}\left(\frac{a}{x}\right)^5\left(3x^2\right)+\binom{6}{2}\left(\frac{a}{x}\right)^4\left(3x^2\right)^2+\binom{6}{3}\left(\frac{a}{x}\right)^3\left(3x^2\right)^3+\ldots$$

B1 if at least 2 terms correct and
B2 if all 4 terms correct

$$=\frac{a^6}{x^6}+\frac{18a^5}{x^3}+135a^4+540a^3x^3+\ldots$$ B1

(b) Given that there is no term in x^3 in the expansion of $(2x^3-1)\left(\frac{a}{x}+3x^2\right)^6$, find the value of the positive constant a. [2]

$$\left(2x^3-1\right)\left(\frac{a^6}{x^6}+\frac{18a^5}{x^3}+135a^4+540a^3x^3+\ldots\right)$$

since there is no x^3 term, $2\left(135a^4\right)-540a^3=0$ M1

$$270a^4-540a^3=0$$
$$270a^3\left(a-2\right)=0$$
$a=0$ (rej), $a=2$ A1

6 A particle moves along the curve $y = \frac{5}{(3x-1)^2}$, where $x \neq \frac{1}{3}$, in such a way that the x-coordinate is decreasing at a rate of 0.1 units per second.

(a) Find the rate of change of the y-coordinate when $x = 1$. [4]

$y = 5(3x-1)^{-2}$

$\frac{dy}{dx} = 5(-2)(3x-1)^{-3}(3)$ M1

$= \frac{-30}{(3x-1)^3}$ M1

when $x = 1$, $\frac{dy}{dx} = \frac{-30}{8}$

$\frac{dy}{dt} = \frac{-30}{8}(-0.1)$ M1

$= \frac{3}{8}$ units/s A1

(b) Find the value of x when y increases at the rate of $\frac{3}{125}$ units per second. [2]

$\frac{3}{125} = \frac{3}{(3x-1)^3}$

$(3x-1)^3 = 125$ M1

$3x - 1 = 5$

$x = 2$ A1

7 **A calculator must not be used in this question.**

(a) Show that $\tan 15° = 2-\sqrt{3}$. [4]

$$\tan(60°-45°) = \frac{\tan 60° - \tan 45°}{1+\tan 60° \tan 45°}$$ M1

$$= \frac{\sqrt{3}-1}{1+\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$ M1 for ratios, M1 for rationalization

$$= \frac{3-2\sqrt{3}+1}{3-1}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= 2-\sqrt{3} \text{ (shown)}$$ A1

(b) Use the result from **part (a)** to find an expression for $\text{cosec}^2 15°$, in the form $a+b\sqrt{3}$ where a and b are integers. [3]

$$\text{cosec}^2 15° = 1+\cot^2 15°$$

$$= 1+\frac{1}{\left(2-\sqrt{3}\right)^2}$$ M1

$$= 1+\frac{1}{4-4\sqrt{3}+3}$$

$$= 1+\frac{1}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}}$$ M1

$$= 1+\frac{7+4\sqrt{3}}{49-48}$$

$$= 8+4\sqrt{3}$$ A1

8 **(a)** Prove the identity $\frac{\sin x}{1-\cos x}+\frac{1-\cos x}{\sin x}=2\operatorname{cosec} x$. [4]

$$\text{LHS}=\frac{\sin x}{1-\cos x}+\frac{1-\cos x}{\sin x}$$

$$=\frac{\sin^2 x+(1-\cos x)^2}{(1-\cos x)\sin x} \quad \text{M1}$$

$$=\frac{\sin^2 x+1-2\cos x+\cos^2 x}{(1-\cos x)\sin x}$$

$$=\frac{1+1-2\cos x}{(1-\cos x)\sin x} \quad \text{M1}$$

$$=\frac{2(1-\cos x)}{(1-\cos x)\sin x} \quad \text{M1}$$

$$=2\operatorname{cosec} x \quad \text{A1}$$

$$=\text{RHS}$$

(b) Hence solve the equation $\frac{\sin x}{1-\cos x}+\frac{1-\cos x}{\sin x}=5$ for $0 \le x \le 2\pi$. [3]

$2\operatorname{cosec} x = 5$

$\operatorname{cosec} x = \frac{5}{2}$

$\sin x = \frac{2}{5}$ M1

Basic angle $= 0.41151$ M1

$x = 0.412, 2.73$ (3s.f) A1

9 A curve is such that $\frac{d^2y}{dx^2} = 3e^{2x} - e^{-x}$. The curve cuts the y-axis at P (0, 1) and has a gradient of 4 at P.

(a) Show that the curve has no stationary points. [3]

$\frac{dy}{dx} = \int (3e^{2x} - e^{-x})dx$

$= \frac{3}{2}e^{2x} + e^{-x} + c$ M1

Gradient = 4 at (0, 1),

$4 = \frac{3}{2} + 1 + c$

$c = 1.5$ M1

$\frac{dy}{dx} = \frac{3}{2}e^{2x} + e^{-x} + \frac{3}{2}$

since $\frac{3}{2}e^{2x} > 0,\ e^{-x} > 0,$

$\frac{dy}{dx} = \frac{3}{2}e^{2x} + e^{-x} + \frac{3}{2} > 0,$ A1

the curve has no stationary points.

(b) Find the equation of the curve. [4]

$y = \int \left(\frac{3}{2}e^{2x} + e^{-x} + \frac{3}{2}\right)dx$

$= \frac{3}{4}e^{2x} - e^{-x} + \frac{3}{2}x + d$ M2

at (0, 1), $1 = \frac{3}{4} - 1 + 0 + d$

$d = \frac{5}{4}$ M1

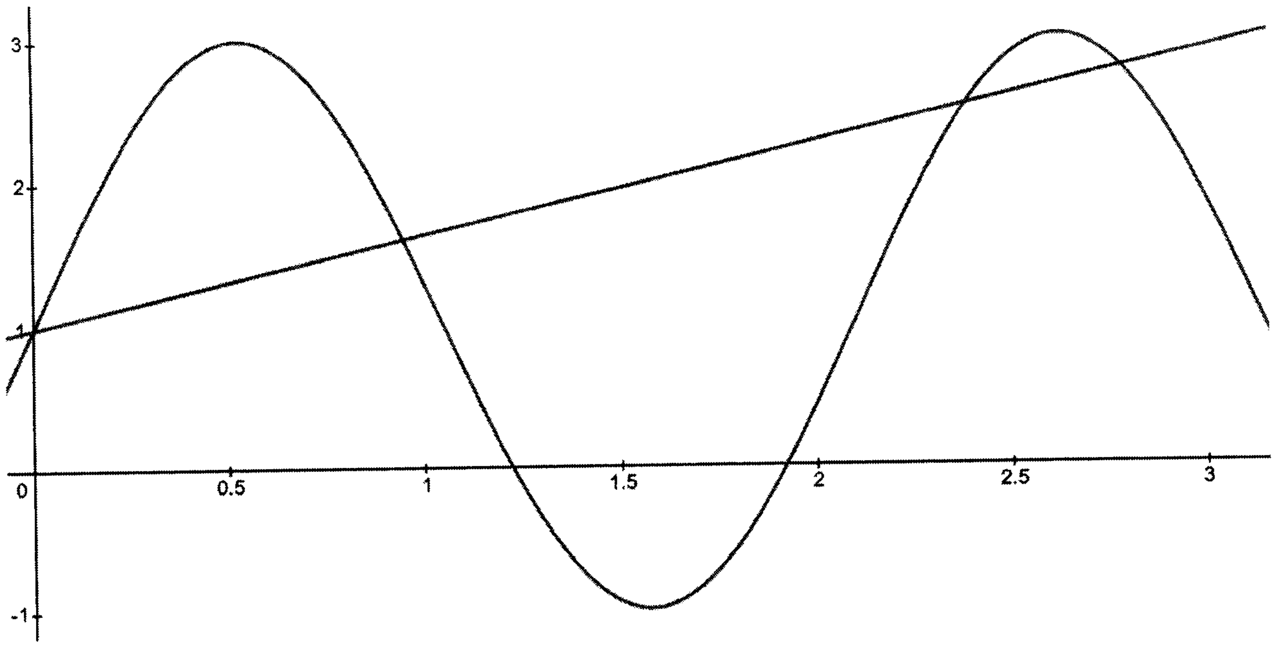
$y = \frac{3}{4}e^{2x} - e^{-x} + \frac{3}{2}x + \frac{5}{4}$ A1

10 **(a)** State the amplitude and period of $2\sin 3x+1$. [2]

Amplitude = 2 B1

Period = $\dfrac{2\pi}{3}$ (accept 120°) B1

(b) Sketch the graph of $y=2\sin 3x+1$ for $0\le x\le \pi$. [3]



Correct number of cycles: B1
Correct shape of curve: B1
Curve pass through critical pts (start, end and the max, min) : B1

(c) By drawing a suitable straight line on your sketch, determine the number of solutions of the equation $\pi\sin 3x-x=0$. [3]

$\pi\sin 3x-x=0$

$2\sin 3x-\dfrac{2x}{\pi}=0$

$2\sin 3x+1=\dfrac{2x}{\pi}+1$ M1

Draw $y=\dfrac{2x}{\pi}+1$ M1

Number of solutions = 4 A1

11 **Solutions to this equation by accurate drawing will not be accepted.**

ABC is a triangle with vertices *A* (0, *h*), *B* (2*h*, – 2) and *C* (7, 5), where *h* is a positive integer. The area of the triangle is 20 units².

(a) Find the value of *h*. [3]

Area of triangle $\frac{1}{2}\begin{vmatrix} 0 & 2h & 7 & 0 \\ h & -2 & 5 & h \end{vmatrix}$

$10h + 7h - 2h^2 + 14 = 40$ M1

$2h^2 - 17h + 26 = 0$

$(h-2)(2h-13) = 0$ M1

$h = 2,\ h = \frac{13}{2}$ (rejected) A1

(b) Find the equation of the perpendicular bisector of *AB*. [3]

Midpoint of $AB = \left(\frac{0+4}{2}, \frac{2-2}{2}\right)$

$= (2, 0)$ M1

gradient of $AB = \frac{-2-2}{4-0}$

$= -1$ M1

Equation of perpendicular bisector of *AB*: $y = x - 2$ A1

(c) Explain why triangle *ABC* is isosceles. [2]

$AC = \sqrt{(0-7)^2 + (2-5)^2} = \sqrt{58}$

$BC = \sqrt{(4-7)^2 + (-2-5)^2} = \sqrt{58}$ M1

since *AC* = *BC*, triangle *ABC* is isosceles (shown) A1

Or

from part (b), when $x = 7, y = 5$ M1

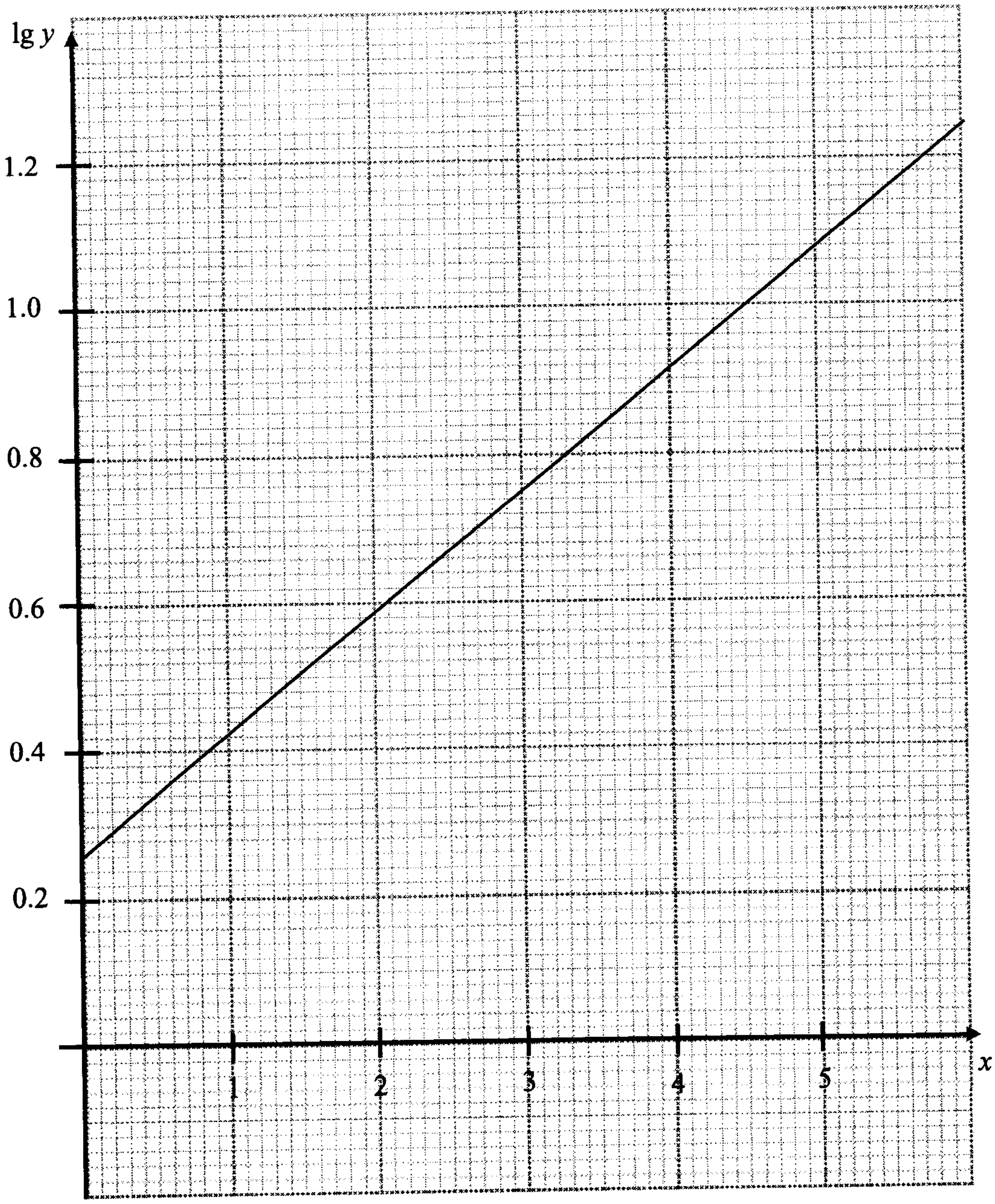
vertex *C* lies on the perpendicular bisector of *AB*, *AC* = *BC* A1

triangle *ABC* is isosceles (shown)

12 **(a)** The table shows experimental values of two variables x and y. It is known that x and y are related by the equation $y = Ak^x$, where A and k are constants.

x	1	2	3	4	5
y	2.63	3.89	5.62	8.32	12.02
lg$y$	0.420	0.590	0.750	0.920	1.08

(i) On the grid below plot lg y against x and draw a straight line graph to illustrate the information. [2]



x	1	2	3	4	5
y	2.63	3.89	5.62	8.32	12.02
lg$y$	0.420	0.590	0.750	0.920	1.08

P1 for correct points plot. L1 for straight line drawn.

(ii) Use your graph to estimate the value of A and of k. [4]

$y = Ak^x$

$\lg y = \lg Ak^x$

$\lg y = x\lg k + \lg A$ M1

From the graph, gradient $= \dfrac{0.92 - 0.59}{4-2}$

$= 0.165$ M1

$\lg y$-intercept $= 0.26$,

$\lg A = 0.26$

$A = 1.82$ (accept 1.78 to 1.86) A1

$\lg k = 0.165$

$k = 1.46$ (accept 1.43 to 1.50) A1

(b) Variables x and y are related by the formula $y = ax^3 + bx^2$, where a and b are constants. Explain clearly how a straight line graph can be drawn to represent this formula. You should state which variables should be plotted on each axis and explain how the values of a and b can be calculated. [4]

$y = x^2(ax + b)$

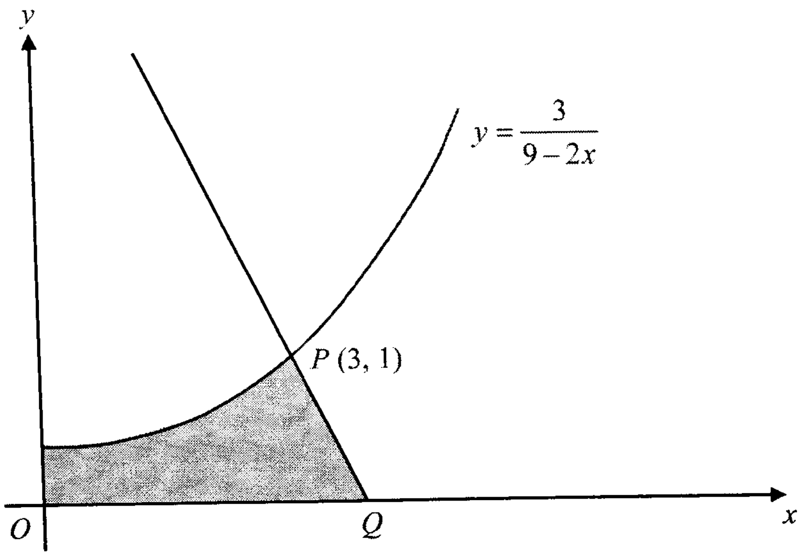
$\dfrac{y}{x^2} = ax + b$ or alternative $\dfrac{y}{x^3} = a + \dfrac{b}{x}$ M1

Plot $\dfrac{y}{x^2}$ (or $\dfrac{y}{x^3}$) on the vertical axis against x (or $\dfrac{1}{x^2}$) on the horizonal axis (M1)

Gradient of line $= a$ (or b) A1

Vertical intercept ($\dfrac{y}{x^2}$ intercept) of line $= b$ (or a) A1

13



The diagram shows part of the curve $y = \frac{3}{9-2x}$. The point P (3, 1) lies on the curve and the normal to the curve at P meets the x-axis at Q.

(a) Find the coordinates of Q. [5]

$\frac{dy}{dx} = \frac{6}{(9-2x)^2}$ M1

when $x = 3$, $\frac{dy}{dx} = \frac{6}{(9-6)^2}$

$= \frac{2}{3}$ M1

Gradient of normal $= -\frac{3}{2}$

Equation of normal: $y - 1 = -\frac{3}{2}(x-3)$ M1

when $y = 0$, $-1 = -\frac{3}{2}x + \frac{9}{2}$

$x = \frac{11}{3}$ M1

Coordinates of Q $\left(\frac{11}{3}, 0\right)$ A1

(b) Find the area of the shaded region bounded by the curve, the normal PQ and the coordinate axes. [5]

Area of shaded region $= \int_0^3 \frac{3}{9-2x}\,dx + \frac{1}{2}(1)\left(3\frac{2}{3}-3\right)$ M2

$= 3\left[\frac{\ln(9-2x)}{-2}\right]_0^3 + \frac{1}{3}$ M1

$= 3(-0.54931+1.0986)+\frac{1}{3}$ M1

$= 1.98$ units^2 (3s.f) A1

End of Paper

BLANK PAGE

Class/ Index Number	Centre Number/ 'O' Level Index Number	Name
/	/	

新加坡海星中学

MARIS STELLA HIGH SCHOOL
PRELIMINARY EXAMINATION
SECONDARY FOUR

ADDITIONAL MATHEMATICS **4049/2**
Paper 2
Solution **26 August 2025**
Candidates answer on the Question Paper. **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 90.

For Examiners' Use

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
/ 7	/ 7	/ 9	/ 7	/ 12	/ 9	/ 8	/ 11

Q9	Q10
/ 9	/ 11

SUBTOTAL		90
Statement		
Presentation		
Units		
Rounding Off		

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 **(a)** A polynomial $f(x)$ has a remainder of -1 when divided by $(2x + 3)$.

(i) Find the remainder when $f(x) - 1$ is divided by $(2x+3)$ [1]

$$f(x) = (2x+3)h(x) - 1$$
$$f(x) - 1 = (2x+3)h(x) - 1 - 1$$
$$= (2x+3)h(x) - 2$$

Remainder $= -2$

(ii) Find in terms of $f(x)$, a polynomial which is divisible by $(2x + 3)$. [1]

$$Polynomial = f(x) + 1$$

(b) The cubic polynomial $g(x)$ is such that the coefficient of x^3 is 2 and the roots of $g(x) = 0$ are -1, m and $2m$, where m is an integer. It is given that $g(x)$ has a remainder of 12 when divided by $x - 1$.

Find an expression of $g(x)$ in descending powers of x. [5]

$$g(x) = 2(x+1)(x-m)(x-2m)$$
$$g(1) = 2(2)(1-m)(1-2m) = 12$$
$$4(1 - 3m + 2m^2) = 12$$
$$2m^2 - 3m + 1 = 3$$
$$2m^2 - 3m - 2 = 0$$
$$(2m+1)(m-2) = 0$$
$$m = 2 \text{ or } -\frac{1}{2} (reject)$$
$$g(x) = 2(x+1)(x-2)(x-4)$$
$$= 2(x^2 - x - 2)(x-4)$$
$$= 2(x^3 - 4x^2 - x^2 + 4x - 2x + 8)$$
$$= 2(x^3 - 5x^2 + 2x + 8)$$
$$= 2x^3 - 10x^2 + 4x + 16$$

2 The equation of a curve is $y = \frac{\ln x}{x}$.

(i) Find the coordinates of the stationary point of this curve. Leave your answer in terms of e. [4]

$$y = \frac{\ln x}{x}$$

$$\frac{dy}{dx} = \frac{x\left(\frac{1}{x}\right) - \ln x}{x^2}$$

$$= \frac{1 - \ln x}{x^2}$$

$$\frac{dy}{dx} = 0$$

$$1 - \ln x = 0$$

$$1 = \ln x$$

$$x = e$$

$$y = \frac{1}{e}$$

Coordinates of stationary point are $\left(e, \frac{1}{e}\right)$

(ii) Determine the nature of the stationary point. [3]

$$\frac{dy}{dx} = \frac{1 - \ln x}{x^2}$$

$$\frac{d^2y}{dx^2} = \frac{x^2\left(-\frac{1}{x}\right) - 2x(1 - \ln x)}{x^4}$$

$$= \frac{-x - 2x(1 - \ln x)}{x^4}$$

$$x = e$$

$$\frac{d^2y}{dx^2} = -\frac{1}{e^3} < 0$$

Therefore $\left(e, \frac{1}{e}\right)$ is a maximum point.

Alternative method : First derivative test.

x	2.6	e	2.8
dy/dx	>0	0	<0
sign	+ve	0	-ve

Therefore it is a maximum point.

3 The decay of a certain radioactive isotope can be modelled by the exponential equation $N = N_0 e^{-at}$ after t weeks, where N represents the amount of radioactive isotope, N_0 and a are constants. A sample of this radioactive isotope has a mass of 100.9 g initially.

(i) After 2 weeks, it is found that the amount of this sample left is 84.6 g. Calculate the value of a. [3]

$$84.6 = 100.9e^{-2a}$$

$$e^{-2a} = \frac{84.6}{100.9}$$

$$-2a = \ln\left(\frac{84.6}{100.9}\right)$$

$$a = -\frac{1}{2}\ln\left(\frac{84.6}{100.9}\right)$$

$$\approx 0.088098$$

$$= 0.0881$$

(ii) What percentage of this sample has decayed after 5 weeks? [3]

$$t = 5$$

$$N = 100.9e^{-5(0.088098)}$$

$$= 64.951$$

$$\text{Percentage decayed} = \frac{100.9 - 64.951}{100.9} \times 100\%$$

$$= 35.6\%$$

(iii) Find the number of weeks when the amount of radioactive isotope decayed first exceeds 60 g. Give your answer correct to the nearest week. [3]

$$N < 100.9 - 60$$

$$100.9e^{-(0.088098)t} < 40.9$$

$$e^{-(0.088098)t} < \frac{40.9}{100.9}$$

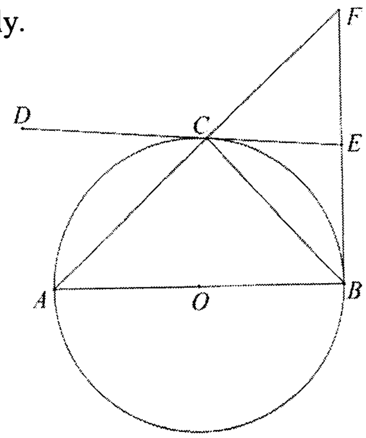
$$-(0.088098)t < \ln\left(\frac{40.9}{100.9}\right)$$

$$t > -\frac{1}{0.088098}\ln\left(\frac{40.9}{100.9}\right)$$

$$> 10.24$$

The nearest week that the amount of radioactive isotope decayed exceeds 60 g = 11

4 In the diagram, AB is the diameter of the circle with centre O.
DE and BF are tangents to the circle at C and B respectively.
DCE and BEF are straight lines.



Prove that

(i) Triangle ABC and triangle AFB are similar. [3]

$\angle ACB = 90°$ (Angles in semicircle)
$\angle ABF = 90°$ (Radius perpendicular to tangent)
$\angle ACB = \angle ABF$
$\angle CAB = \angle BAF$ (common angle)
Triangle ABC and triangle AFB are similar (AA similarity)

(ii) Show that $EC = EF$. [4]

Triangle ABC and triangle AFB are similar.
$\angle ABC = \angle AFB$.
$\angle CFE = \angle AFB$ (same angle)
$\angle ABC = \angle DCA$ (tangent chord theorem)
$\angle DCA = \angle FCE$ (vertically opposite angle)
$\angle FCE \ = \ \angle ABC \ = \ \angle AFB$
Therefore $\angle CFE = \angle$FCE, triangle CFE is isosceles, EC = EF

Alternative solution:
EB = EC (common tangent)
Triangle ECB is isosceles.
Let $\angle EBC = x$.
$\angle ECB = x$.
$\angle FCE = 180 - 90 - x$ (Adjacent angles in a straight line)
$= 90 - x$
$\angle CAB = \angle ECB$ (tangent chord theorem)
$= x$
$\angle CFE = 180 - 90 - x$ (angle sum of triangle)
$= 90 - x$
$= \angle FCE$
Therefore triangle CFE is isosceles, EC = EF

5 **(a)** Find the integer a and b which satisfy the equation

$$\left(\frac{1}{8}y^3\right)^{-1} \div \left(4y^2\right)^{\frac{1}{2}} = 2^a y^b$$ [3]

$$\left(\frac{1}{8}y^3\right)^{-1} \div \left(4y^2\right)^{\frac{1}{2}} = \left(2^{-3}y^3\right)^{-1} \div \left(2^2y^2\right)^{\frac{1}{2}}$$

$$= \left(2^3y^{-3}\right) \div \left(2y\right)$$

$$= 2^2y^{-4}$$

$a = 2, b = -4$

(b) Solve the equation $\log_3(x-8) = 2 - \frac{1}{\log_x 3}$. [5]

$\log_3(x-8) = 2 - \frac{1}{\log_x 3}$

$\log_3(x-8) = 2 - \log_3 x$

$\log_3(x-8) + \log_3 x = 2$

$\log_3 x(x-8) = 2$

$x(x-8) = 3^2$

$x^2 - 8x - 9 = 0$

$(x-9)(x+1) = 0$

$x = 9, -1(rej, x-8 > 0)$

(c) Solve the equation $4^{x+1} + 7(2^x) = 2$. [4]

$4^{x+1} + 7(2^x) = 2$

$4(4^x) + 7(2^x) - 2 = 0$

$4(2^x)^2 + 7(2^x) - 2 = 0$

Let $y = 2^x$

$4y^2 + 7y - 2 = 0$

$(4y-1)(y+2) = 0$

$y = \frac{1}{4}, -2(rej)$

$2^x = \frac{1}{4} = 2^{-2}$

$x = -2$

6 **(a)** Given that $y = 2\cos x \sin 2x - \sin x \cos 2x$, show that $\frac{dy}{dx} = k\cos x \cos 2x$, stating the value of k. [4]

$y = 2\cos x \sin 2x - \sin x \cos 2x$

$\frac{dy}{dx} = 2\left(\cos x\left(2\cos 2x\right) + (-\sin x)(\sin 2x)\right) - \left(\sin x\left(-2\sin 2x\right) + \cos x \cos 2x\right)$

$= 4\cos x \cos 2x - 2\sin x \sin 2x + 2\sin x \sin 2x - \cos x \cos 2x$

$= 3\cos x \cos 2x$

$k = 3$

(b) Hence, show that $\int_0^{\frac{\pi}{4}} \left(7\cos x \cos 2x + \sec^2 x\right) dx = \frac{7\sqrt{2}}{3} + 1$ [5]

$$\int_0^{\frac{\pi}{4}} \left(7\cos x \cos 2x + \sec^2 x\right) dx = \int_0^{\frac{\pi}{4}} 7\cos x \cos 2x \, dx + \int_0^{\frac{\pi}{4}} \sec^2 x \, dx$$

$$= \frac{7}{3}\int_0^{\frac{\pi}{4}} 3\cos x \cos 2x \, dx + \left[\tan x\right]_0^{\frac{\pi}{4}}$$

$$= \frac{7}{3}\left[2\cos x \sin 2x - \sin x \cos 2x\right]_0^{\frac{\pi}{4}} + \left[\tan\frac{\pi}{4} - 0\right]$$

$$= \frac{7}{3}\left[2\left(\frac{\sqrt{2}}{2}\right)(1) - 0\right] + 1$$

$$= \frac{7\sqrt{2}}{3} + 1$$

7 Two points $A(-3, -6)$ and $B(7, -6)$ lie on a circle centre G. The equation of the tangent to the circle at the point $Q(1, -12)$ is $x + 5y + 59 = 0$.

(a) Show that the coordinates of G is $(2, -7)$. [5]

Mid-point of AB $= \left(\frac{-3+7}{2}, -6\right)$

$= (2, -6)$

The x- coordinate of G = 2

Gradient of tangent at Q $= -\frac{1}{5}$

Gradient of normal at Q = 5

Equation of normal at Q:

$y - (-12) = 5(x - 1)$

$y = 5x - 5 - 12$

$y = 5x - 17$

The normal at Q passes through the center of the circle.

When $x = 2$, $y = 5(2) - 17 = -7$

Therefore G = $(2, -7)$

Alternative solution:

Mid-point of AB $= \left(\frac{-3+7}{2}, -6\right)$

$= (2, -6)$

The x- coordinate of G = 2

Mid-point of AQ $= \left(\frac{-3+1}{2}, \frac{-6-12}{2}\right)$

$= (-1, -9)$

Gradient of AQ $= \frac{-6-(-12)}{-3-1}$

$= -\frac{6}{4} = -\frac{3}{2}$

Equation of perpendicular bisector of AQ:

$y+9 = \frac{2}{3}(x+1)$

$y = \frac{2}{3}x - 8\frac{1}{3}$

$x = 2$

$y = \frac{4}{3} - 8\frac{1}{3} = -7$

$G = (2, -7)$

(b) Find the equation of the circle in the form of $x^2 + y^2 + 2gx + 2fy + c = 0$, stating the value of f, g and c.. [3]

Distance of $GA = \sqrt{(2-(-3))^2 + (-7-(-6))^2}$

$= \sqrt{26}$

Equation of circle:

$(x-2)^2 + (y+7)^2 = (\sqrt{26})^2$

$x^2 - 4x + 4 + y^2 + 14y + 49 = 26$

$x^2 - 4x + y^2 + 14y + 27 = 0$

$g = -2, f = 7, c = 27$

8 A particle P moves in a straight line and passes through a fixed point O so that its velocity, v m/s is given by $v = t^2 - 5t + 4$ where t is the time in seconds after passing O.

(a) Find the values of t when the particle is instantaneously at rest. [2]

$$v = t^2 - 5t + 4$$
$$v = 0$$
$$t^2 - 5t + 4 = 0$$
$$(t-1)(t-4) = 0$$
$$t = 1, 4$$

(b) Find the values of t, when the particle returns to O. [4]

$$v = t^2 - 5t + 4$$
$$s = \int v\,dt$$
$$= \frac{1}{3}t^3 - \frac{5}{2}t^2 + 4t + c$$
$$At\ O, s = 0, t = 0, c = 0$$
$$s = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 4t$$
$$s = 0,$$
$$\frac{1}{3}t^3 - \frac{5}{2}t^2 + 4t = 0$$
$$\frac{1}{6}t\left(2t^2 - 15t + 24\right) = 0$$
$$2t^2 - 15t + 24 = 0$$
$$t = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(2)(24)}}{2(2)}$$
$$\approx 2.3139, 5.1861$$
$$= 2.31, 5.19$$

(c) Find the acceleration of the particle when it first returns to O. [2]

$$v = t^2 - 5t + 4$$
$$a = \frac{dv}{dt} = 2t - 5$$
$$t = 2.3139$$
$$a = 2(2.3139) - 5 = -0.372\ m/s^2$$

(d) Find the average speed during the first 5 seconds. [3]

$$s=\frac{1}{3}t^3-\frac{5}{2}t^2+4t$$

$$t=1,\quad s=\frac{1}{3}-\frac{5}{2}+4=\frac{11}{6}$$

$$t=4,\quad s=\frac{1}{3}(64)-\frac{5}{2}(16)+16=-\frac{8}{3}$$

$$t=5,\quad s=\frac{1}{3}(125)-\frac{5}{2}(25)+20=-\frac{5}{6}$$

$$\text{Total distance travelled in first } 5s=2\left(\frac{11}{6}\right)+2\left(\frac{8}{3}\right)-\frac{5}{6}$$

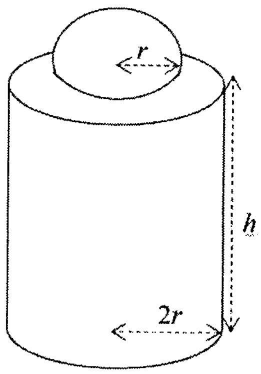
$$=\frac{49}{6}m$$

$$Average\ speed=\frac{\frac{49}{6}}{5}=1.63\ m/s$$

---

9 The diagram shows a paper weight which is made up of a solid hemisphere of radius r cm, resting on top of a solid cylinder of radius $2r$ cm and height h cm.

(i) Given that the total volume of the solid is 216π cm^3, express h in terms of r. [2]



$$216\pi=\frac{2}{3}\pi r^3+\pi(2r)^2h$$

$$216=\frac{2}{3}r^3+4r^2h$$

$$h=\frac{54}{r^2}-\frac{r}{6}$$

(ii) Show that the total surface area, A cm^2, of the solid is given by , $A=\frac{25\pi r^2}{3}+\frac{216\pi}{r}$. [2]

$$A=2\pi r^2+\pi(2r)^2-\pi r^2+2\pi(2r)h+\pi(2r)^2$$

$$A=9\pi r^2+4\pi rh$$

$$=9\pi r^2+4\pi r\left(\frac{54}{r^2}-\frac{r}{6}\right)$$

$$=9\pi r^2+\frac{216\pi}{r}-\frac{2}{3}\pi r^2$$

$$=\frac{25\pi r^2}{3}+\frac{216\pi}{r}$$

(iii) Given that r can vary, find the value of r for which A has a stationary value. Find this value of A and determine whether it is a maximum or a minimum. [5]

$$A=\frac{25\pi r^2}{3}+\frac{216\pi}{r}$$

$$\frac{dA}{dr}=\frac{50\pi r}{3}-\frac{216\pi}{r^2}=0$$

$$\frac{50\pi r}{3}=\frac{216\pi}{r^2}$$

$$r^3=\frac{324}{25}$$

$$r\approx 2.349$$

$$r=2.35$$

$$A=\frac{25\pi(2.349)^2}{3}+\frac{216\pi}{2.349}$$

$$\approx 433.38$$

$$=433$$

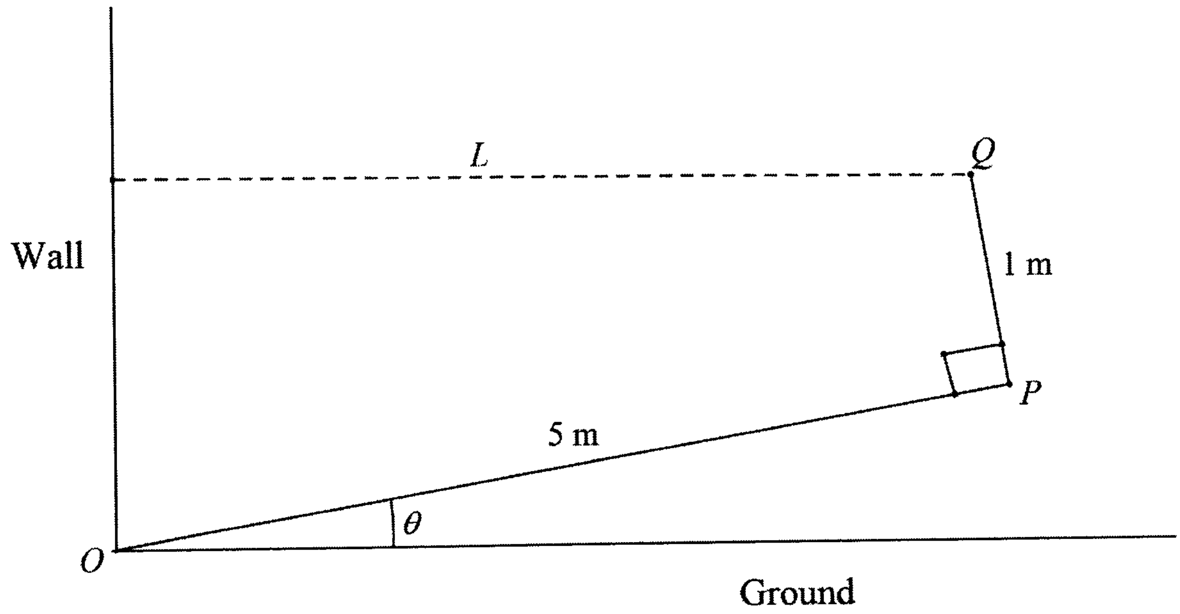
$$\frac{d^2A}{dr^2}=\frac{50\pi}{3}+\frac{432\pi}{r^3}$$

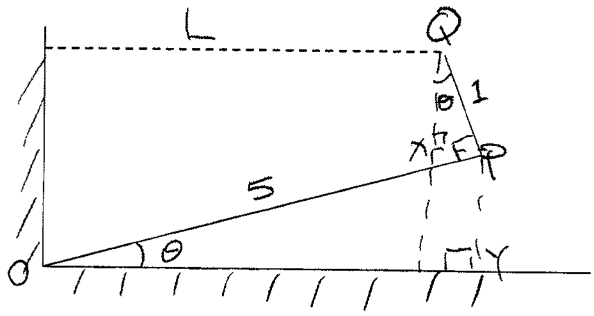
$$r=2.35,\frac{d^2A}{dr^2}>0$$

A is a minimum.

---

10 A L-shaped structure, OPQ can be rotated about O. OP and PQ measures 5 m and 1 m respectively. OP makes an acute angle, θ, with the ground. The shortest distance from Q to the wall is is L m.





(i) Show that $L = 5\cos\theta - \sin\theta$. [2]

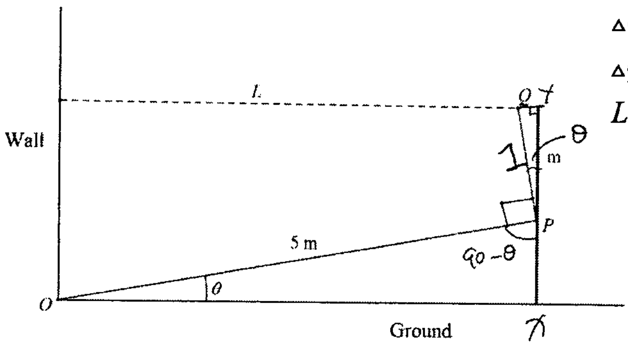
$OY = 5\cos\theta$

$XP = \sin\theta$

$L = OY - XP$

$= 5\cos\theta - \sin\theta$

Alternative solution



$\triangle OPX,\ OX = 5\cos\theta$

$\triangle QPY,\ QY = 1\sin\theta = \sin\theta$

$L = OX - QY = 5\cos\theta - \sin\theta$

(ii) Express L in the form $R\cos(\theta+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

$L = 5\cos\theta - \sin\theta$

$R = \sqrt{5^2 + 1^2} = \sqrt{26}$

$\tan\alpha = \dfrac{1}{5}$

$\alpha \approx 11.31°$

$= 11.3°$

$L = \sqrt{26}\cos(\theta + 11.3°)$

(iii) State the minimum value of L and the corresponding value of θ. [3]

Minimum value of $L = 0$

$\cos(\theta + 11.3°) = 0$

When $\theta + 11.3° = 90°$

$\theta = 78.7°$

(iv) find the value of θ when $L = 3$ [2]

$$3 = \sqrt{26}\cos(\theta + 11.3°)$$

$$\cos(\theta + 11.3°) = \frac{3}{\sqrt{26}}$$

$$\theta + 11.3° = 53.96°$$

$$\theta \approx 42.65° = 42.7°$$

(iv) Explain why the maximum value of L is not R. [1]

Given that θ > 0, when L = R

$$\cos(\theta + 11.3°) = 1$$

$$\theta + 11.3° = 0$$

$$\theta = -11.3° (rej)$$

Therefore the maximum value of L is not R.

End of Paper