| Class | Index Number |
|---|---|
| | |

Name : _____

# METHODIST GIRLS' SCHOOL

Founded in 1887

## PRELIMINARY EXAMINATION 2025
## Secondary 4

Tuesday
26 August 2025

## ADDITIONAL MATHEMATICS
## Paper 1

4049/01
2 h 15 min

Candidates answer on the Question Paper.
No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**
Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

| |
|---|
| / 90 |

*Mathematical Formulae*

## 1. ALGEBRA

***Quadratic Equation***

For the quadratic equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

***Binomial Expansion***

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where $n$ is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$.

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}bc\sin A$$

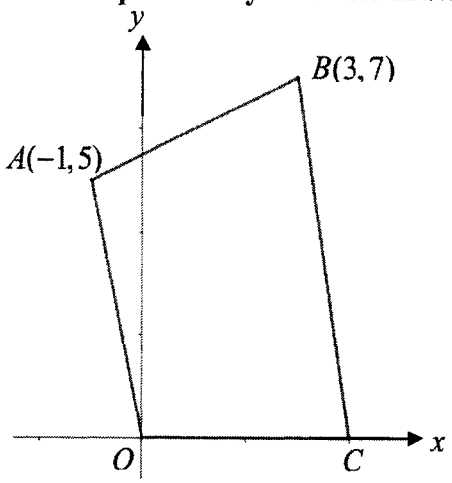
**1** A home-based sticker company that prints stickers has calculated its profit, \$$P$, for each order using the equation $P = 120x - 40 - 30x^2$, where $x$ is the number of stickers produced in **hundreds**.

**(a)** Express $P = 120x - 40 - 30x^2$ in the form $P = a(x-b)^2 + c$, where $a$, $b$ and $c$ are constants. [2]

**(b)** Using your answer in **(a)**, explain clearly if the company should accept an order for printing 400 stickers. [2]

**2** Find the exact coordinates of the stationary point on the curve $y = \dfrac{e^{2x}}{3x^2}$, $x \neq 0$. [6]

**3** **Solutions to this question by accurate drawing will not be accepted.**

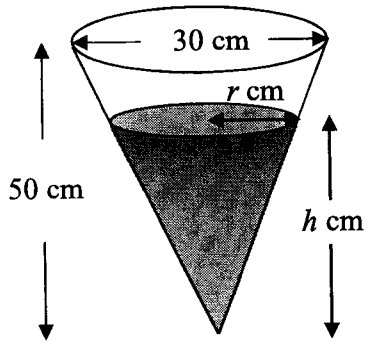



The diagram shows a quadrilateral $OABC$ where $O$ is the origin. The point $A$ is $(-1,5)$ and the point $B$ is $(3, 7)$. The perpendicular bisector of $AB$ meets the $x$-axis at $C$.

**(a)** Find the coordinates of $C$. [4]

**(b)** Find the area of the quadrilateral $OABC$. [2]

4 The diagram shows an inverted right circular cone with a diameter, 30 cm and height, 50 cm.
The cone was initially empty. It is being filled with water at a constant rate of 100 cm³/s. The surface of the water remains horizontal as it fills. At time $t$ seconds, the radius of the horizontal water surface area is $r$ cm and the depth of the water is $h$ cm.

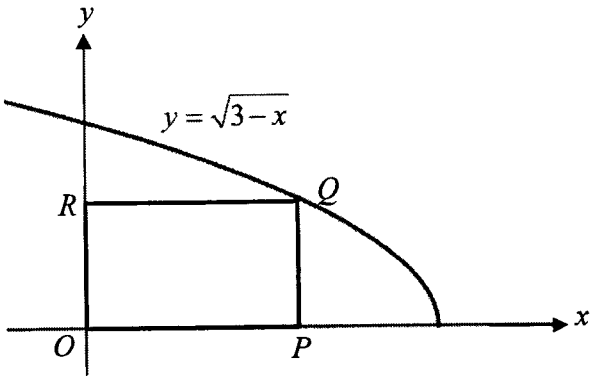



**(a)** Express $r$ in terms of $h$. [2]

**(b)** Show that the volume of water, $V$ cm³, in the cone is $V = \frac{3\pi h^3}{100}$. [1]

**(c)** Find the rate of change of the depth of the water when $h = 35$ cm. [3]

**5** The diagram shows part of the graph of $y=\sqrt{3-x}$ , $x<3$. $O$ is the origin and points $P$ and $R$ lie on the $x$ and $y$ axes respectively. Point $Q$ is a point on the curve such that $OPQR$ forms a rectangle.




**(a)** Given that the $x$-coordinate of $P$ is $h$, write down an expression for the area of the rectangle, $A$ units$^2$, in terms of $h$ and show that $\frac{dA}{dh}=\frac{6-3h}{2\sqrt{3-h}}$. [3]

**(b)** Find the stationary value of $A$ and determine the nature of this stationary value. [4]

**6** **(a)** The expansion of $(1+kx)^n$ in ascending powers of $x$ is $1+\frac{7}{3}x+\frac{7}{3}x^2+....$

Show that $k=\frac{1}{3}$ and calculate the value of $n$. [4]

**(b)** Explain why all terms in the expansion of $\left(px^2 - \frac{1}{x^4}\right)^{12}$ contain only even powers of $x$. [4]

7 **(a)** Find all the angles between $-\pi$ and $\pi$ that satisfy the equation $2\cos 2x = 4\sin x + 3$. [3]

**(b)** Solve the equation $3\text{cosec}^2 x - \text{cosec}\, x = 2 - \cot^2 x$ for $0° \leq x \leq 360°$. [5]

**8** **(a)** Express $\dfrac{1-6x}{(x+2)(3x^2+1)}$ in the form of $\dfrac{a}{x+2}+\dfrac{bx+c}{3x^2+1}$, where $a$, $b$ and $c$ are integers. [4]

**(b)** Differentiate $\ln(3x^2+1)$ with respect to $x$. [2]

**(c)** Using your answer to **parts (a)** and **(b)**, show that

$$\int_0^2 \frac{1-6x}{(x+2)(3x^2+1)}\,dx = \ln 2 - \frac{1}{2}\ln 13.$$ [4]

**9** **(a)** **(i)** Sketch the graph of $y = 3e^x$. [1]

**(ii)** Find the equation of a suitable straight line that can be inserted in **part (a)(i)** to solve the equation $6e^x - x = 6$. [2]

**(b)** **(i)** Without using a calculator, show that $\tan 75° = a + b\sqrt{3}$, where $a$ and $b$ are integers. [3]

**(ii)** Hence, find the exact value of $\cot^2 75°$. [3]

**10** The maximum height that a roller coaster can reach is 90 meters above sea level, and the lowest possible height is 10 meters above sea level. The roller coaster takes 10 seconds to move from its highest position to its lowest position.
The height, $h$ m, of the roller coaster above sea level can be modelled by the function $h = a\cos\left(\frac{\pi}{b}t\right) + c$, where $t$ is the time in seconds.

**(a)** Show that $h = 40\cos\left(\frac{\pi}{10}t\right) + 50$. [3]

**(b)** Sketch the graph of $h = 40\cos\left(\frac{\pi}{10}t\right) + 50$ for $0 \le t \le 30$. [2]

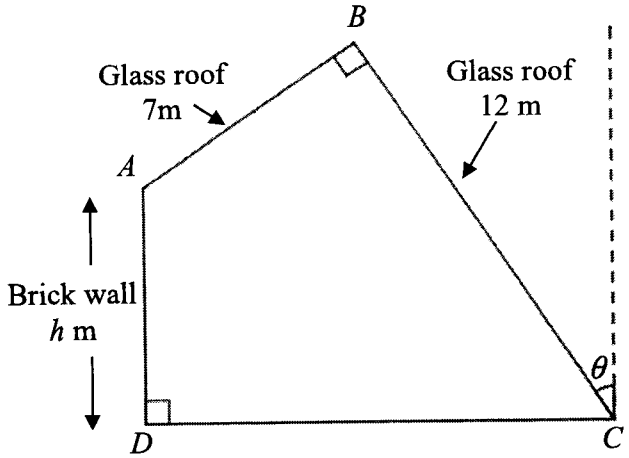
**(c)** Find the time when the roller coaster first reaches a height of 62 m. [2]

**(d)** **(i)** On your graph in **part (b)**, draw the graph of $h = t + 15$. [1]

**(ii)** State for the interval $0 \le t \le 30$, the number of solutions of

$$t = 40\cos\left(\frac{\pi}{10}t\right) + 35.$$ [1]

**11** An architect is designing a building, *ABCD,* using the model shown below.
To allow more natural light into the building, a 12 m glass roof, *BC*, is installed at an angle of $\theta$ to the vertical. Another glass roof, *AB*, of 7 m, is installed such that it is perpendicular to *BC*. The vertical height of the brick wall, *AD*, is given as $h$ m.



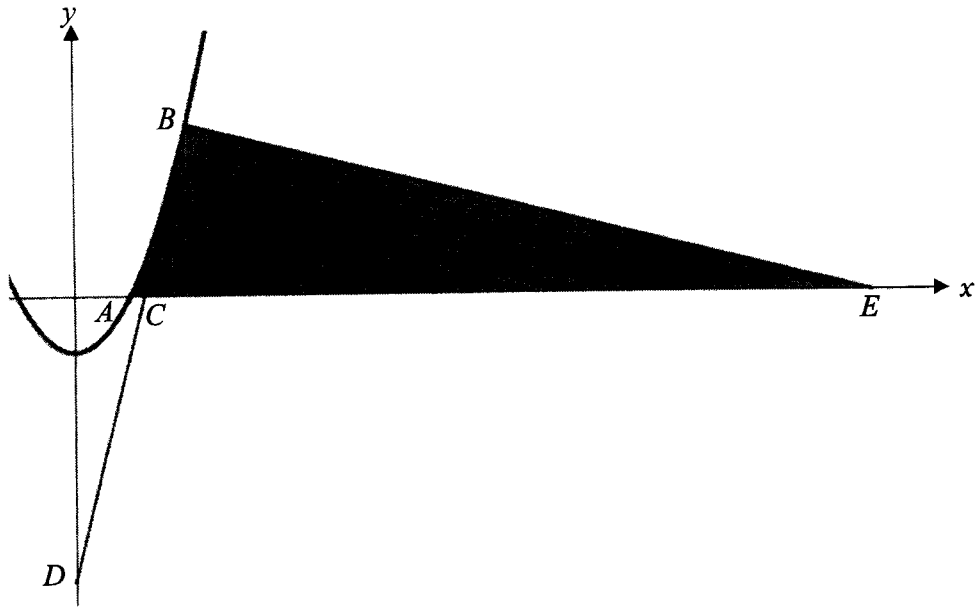

**(a)** Show that $h = 12\cos\theta - 7\sin\theta$. [2]

**(b)** Express $h$ in the form $R\cos(\theta+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

**(c)** Find the value of $\theta$ for which $h = 8\,\text{m}$. [2]

**(d)** The building owner makes a request to have the brick wall to have a height of at least 14 m. Is the architect able to meet the building owner's request? Explain your answer. [2]

**12** The diagram shows part of the curve $y = x^2 - 1$, cutting the $x$-axis at $A(1,0)$. The tangent at point $B$ on the curve cuts the $x$-axis at $C\left(\frac{5}{4},0\right)$ and $y$-axis at $D(0,-5)$ respectively. The normal at $B$ meets the $x$-axis at $E$.




**(a)** Find the coordinates of $B$. [3]

**(b)** Find the coordinates of $E$. [2]

**(c)** Find the area of the shaded region. [3]

**END OF PAPER**

Name : _____

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# METHODIST GIRLS' SCHOOL

Founded in 1887

## PRELIMINARY EXAMINATION 2025
## Secondary 4

**Wednesday** 27 August 2025

## ADDITIONAL MATHEMATICS
## PAPER 2

**4049/02**

2 hours 15 mins

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The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

| |
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| /90 |

**This question paper consists of 22 printed pages and 2 blank pages.**

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where $n$ is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$.

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**1** A piece of raw meat is accidentally left out at room temperature, and bacteria begin to grow on it. Initially, there are 500 bacteria presents, and the number of bacteria doubles every 3 hours. The number of bacteria, $N$, after $t$ hours can be modelled by the formula $N = 500(2)^{kt}$.

**(i)** Show that $k = \frac{1}{3}$. [2]

**(ii)** It is considered unsafe for consumption when the number of bacteria on the meat exceeds 3 000. Determine the maximum number of hours that it will still be safe to consume the meat.

[3]

**2** **(a)** Given that $y = x^3 - x$, find the range of values of $x$ such that $x\frac{d^2y}{dx^2} + \frac{dy}{dx} > 0$. [4]

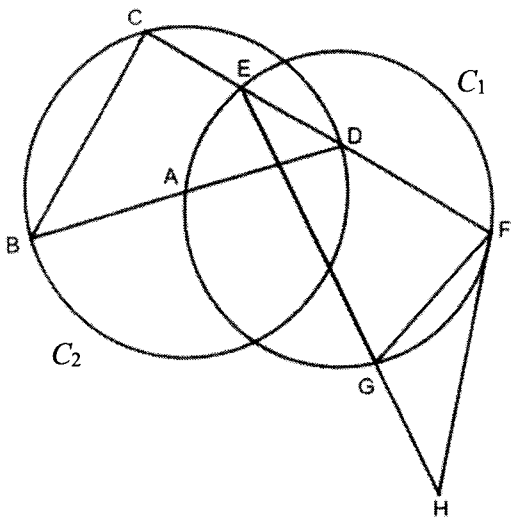
**2** **(b)** A curve is such that $\frac{d^2y}{dx^2} = \frac{1}{3\sqrt{2x-1}}$. Given that the curve has a gradient of 3 at the point $(1, 6)$, find the equation of the curve. [4]

3 A circle $C_1$ passes through $A(-1,-3)$. Its centre, $B$, lies on the line $4y+3x=10$. $AB$ is perpendicular to $4y+3x=10$.

**(a)** Find the equation of the circle. [5]

**3** **(b)** A second circle $C_2$ has centre at $A(-1,-3)$. The line $y=0$ is a tangent to circle $C_2$. Find the equation of the circle in the form of $x^2+y^2+px+qy+r=0$, where $p$, $q$ and $r$ are integers. [2]

4




The diagram shows two intersecting circles $C_1$ and $C_2$. The line $CF$ intersects $C_1$ at $E$, and $C_2$ at $D$, such that $CE = ED$. $BAD$ is a straight line, where $A$ is the centre of $C_2$. The tangent to $C_1$ at $F$ meets $EG$ produced at $H$.

Stating your reasons clearly, prove that

**(i)** $\Delta GFH$ and $\Delta FEH$ are similar, [2]

**4** **(ii)** $HF^2 = HE \times GH$, [2]

**(iii)** $AF^2 = AE^2 + EF^2$ [3]

5 **(a)** Show that $6\log_4 x - \log_2 y = 3$ can be expressed as $y = ax^n$, where $a$ and $n$ are constants to be determined. [4]

**(b)** Solve the equation $3e^y - 5 = 2e^{-y}$, giving values of $y$ in logarithmic form. [4]

**6** **(a)** Find the range of values of $m$ such that $(m-6)x^2-8x+m>0$ for all values of $x$. [4]

**(b)** Given that the quadratic equation $px^2-2(p-q)x-(r-p)=0$ has equal roots, prove that $p=\dfrac{q^2}{2q-r}$. [4]

7 A particle moves in a straight line so that, $t$ seconds after passing through a fixed point $O$, its velocity, $v$ m/s, is given by $v = k\cos 2t$, where $k$ is a constant. When $t = \frac{\pi}{12}$, the acceleration is - 6 m/s.

**(a)** Find the value of $k$. [2]

**(b)** The particle is first at rest at point $P$. Find the value of $t$ when the particle is at point $P$. [2]

7 **(c)** Find the distance travelled by the particle from $O$ to $P$. [3]

**8** The equation of the curve is $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (x+4)\sqrt{2x-3}$, $x > \frac{3}{2}$.

**(a)** Show that $\mathrm{f}'(x) = \frac{px+q}{\sqrt{2x-3}}$, where $p$ and $q$ are integers. [3]

**8** **(b)** Find the $x$-coordinates of the points where the gradient of the normal is $-\frac{1}{7}$. [5]

**(c)** Determine whether f is an increasing or a decreasing function. Explain your answer. [2]

**9** **(a)** Prove that $\cot x + \dfrac{\sin x}{1+\cos x} = \operatorname{cosec} x$. [3]

**9** **(b)** Hence solve $\cot x + \dfrac{\sin x}{1+\cos x} = 4\cos x$, for $0 \le x \le 2\pi$. [4]

**(c)** Using the result in part **(a)**, find the set of values of the $k$, such that the equation $\cot x + \dfrac{\sin x}{1+\cos x} = \dfrac{1}{k}$, $k \neq 0$has solutions. [2]

**10** **(i)** The function $f(x) = ax^3 + 4x^2 + bx - 2$, where $a$ and $b$ are constants, is exactly divisible by $x+2$ and leaves a remainder of 4 when divided by $x+1$.

Find the value of $a$ and of $b$. [4]

**10** **(ii)** Factorise $f(x)$ completely. [3]

**(iii)** By substituting a suitable value of $x$, find the remainder when 3 003 994 998 is divided by 1002. [2]

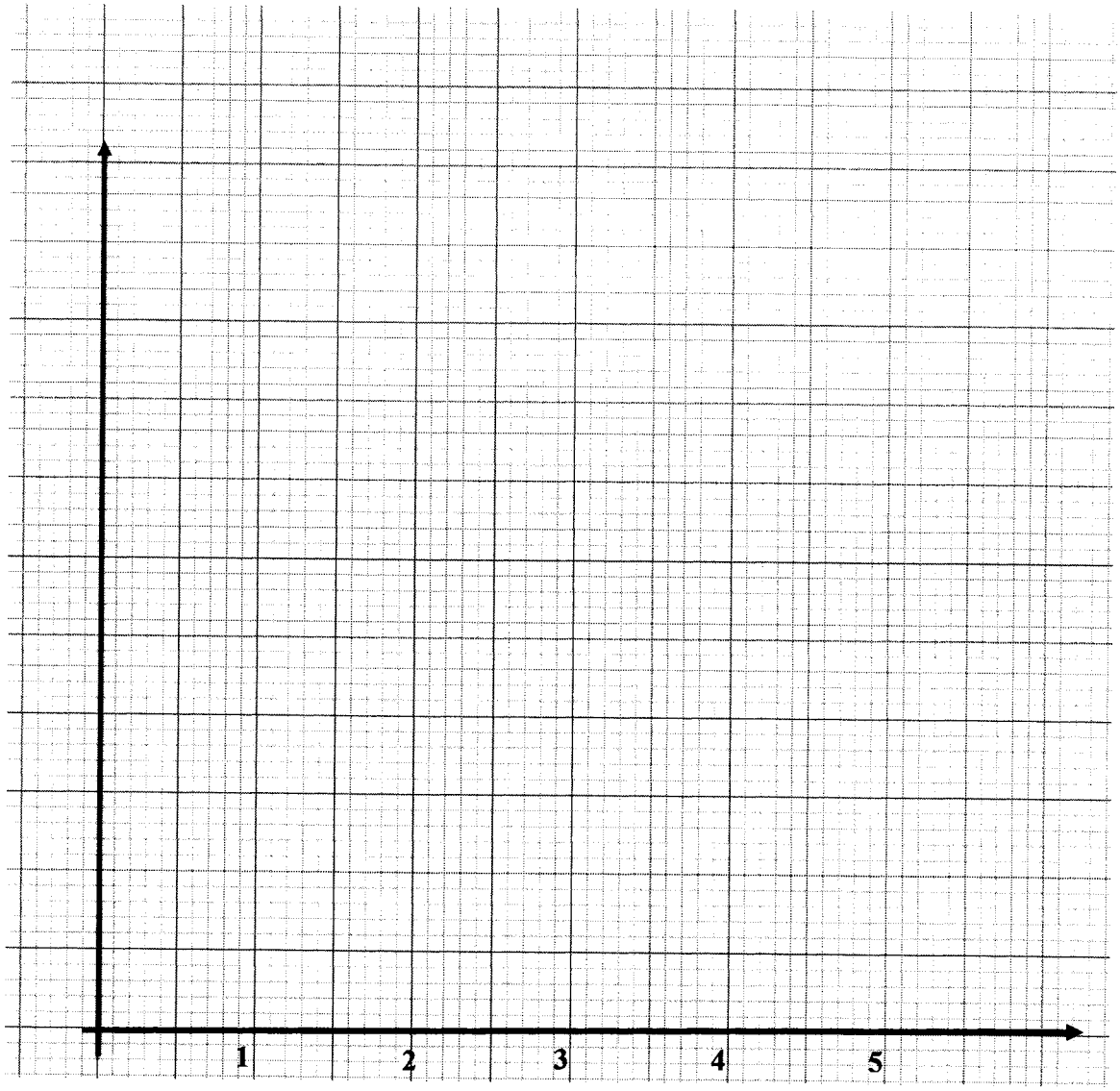
**11** It is believed that the number of views, $V$, of a viral video online after $t$ hours are related by the equation $V = ab^{2t}$, where $a$ and $b$ are constants.

The table below shows a set of survey results for the variables $V$ and $t$.

| Time **(hours)** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Views **(in thousands)** | 2.16 | 3.11 | 4.48 | 6.45 | 9.29 |

**(a)(i)** Explain how a straight line graph can be drawn to represent the formula and draw it for the given data. [4]

**(a)(ii)** On the grid below, draw the graph and estimate the value of $a$ and of $b$. [3]




**(iii)** Using your graph to estimate the number of views when time is 6 hours. [2]

**11** **(b)** The variables $x$ and $y$ are related in such a way that when a graph of $xy$ is plotted against $x^2$, a straight line graph which passes through the points (1, 9) and (5, 1) is obtained. Express $y$ in terms of $x$. [3]

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