BP~441

MARK SCHEME Register no: .... Class: ....

NGEE ANN SECONDARY SCHOOL

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PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS

4049/01

Paper 1

1 September 2025
2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

Total	**/90**

Checked by student: .... Date: ....

This document consists of **24** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n\,!}{r\,!\,(n-r)\,!} = \frac{n\,(n-1)\ldots(n-r+1)}{r\,!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Determine with working, whether the function $f(x)=\frac{7}{(3x+1)}$, $x>0$, is an increasing or decreasing function. [3]

$$f'(x)=\frac{d}{dx}[f(x)]$$

$$=7(-1)(3x+1)^{-2}(3) \text{------M1}$$

$$=-\frac{21}{(3x+1)^2}$$

$$\left.\begin{array}{l}\text{For } x>0,\\ (3x+1)^2>0\\ -\frac{21}{(3x+1)^2}<0\end{array}\right] \text{------M1}$$

Since $f'(x)<0$, for $x>0$, $f(x)$ is a decreasing function. ------A1

2 The equation of a quadratic curve is given by $y=-x^2+px-17$ and it has a maximum point $(4,q)$ where p and q are constants.

(a) By expressing y in the form of $-(x-a)^2+b$, find the value of p and of q. [4]

$$y=-x^2+px-17$$
$$=-(x^2-px)-17$$
$$=-\left[\left(x-\frac{p}{2}\right)^2-\left(-\frac{p}{2}\right)^2\right]-17 \text{ -----M1 for completing the square}$$
$$=-\left(x-\frac{p}{2}\right)^2+\left(-\frac{p}{2}\right)^2-17$$
$$=-\left(x-\frac{p}{2}\right)^2+\frac{p^2}{4}-17$$

Since maximum point is (4, q),

$$\frac{p}{2}=4 \qquad \& \qquad q=\frac{p^2}{4}-17 \text{ -----M1}$$
$$p=8\text{------A1} \qquad =\frac{8^2}{4}-17$$
$$=-1\text{------A1}$$

Alternatively,

Since maximum point is (4, q),

$$y=-(x-4)^2+q\text{------M1}$$
$$= -x^2+8x-16+q\text{------M1}$$
$$p=8\text{------A1} \qquad \& \qquad -17=-16+q$$
$$q=-1\text{------A1}$$

(b) **Hence,** write down the nature of the turning point of $y=\frac{5}{-x^2+px-17}$ and state the coordinates of the turning point. [2]

Minimum point B1 at $(4,-5)$------B1

3 A line that is not parallel to the $x-$axis has the equation $a^2x+y=3a-b-3$, where a and b are constants. A curve has the equation $2x^2+y^2=b+3$. Given that the line and the curve intersect at the point $(0,a)$, find the value of a and of b. [5]

$a^2x+y=3a-b-3$
$y=-a^2x+3a-b-3------(1)$
$2x^2+y^2=b+3------(2)$

When $x=0, y=a$,
From (1), $a=3a-b-3------$M1 for substituting point into line
$b=2a-3------(3)$
From (2), $a^2=b+3-------$M1 for substituting point into curve
$a^2=b+3------(4)$

Substitute (3) into (4),
$a^2=2a-3+3------$M1 for solving simultaneously
$a^2-2a=0$
$a(a-2)=0$
$a=0$ (rej, since not parallel) or $a=2------$A1 with rejection
$b=1------$A1

4 The first three terms in the expansion of $(1+ax)^n$ are $1-3x+\frac{33}{8}x^2$, where a and n are constants. Find the value of a and of n. [5]

$$(1+ax)^n = 1+\binom{n}{1}(ax)+\binom{n}{2}(ax)^2+\cdots = 1+nax+\binom{n}{2}a^2x^2+\cdots$$

By comparing the coefficients of:

$x:\binom{n}{1}a=-3$ ------ M1

$na=-3$

$a=-\frac{3}{n}$ ------ (1)

$x^2:\binom{n}{2}a^2=\frac{33}{8}$ ------ M1

$\frac{n(n-1)}{2}a^2=\frac{33}{8}$ ------ (2)

Sub. (1) into (2):

$\frac{n(n-1)}{2}\left(-\frac{3}{n}\right)^2=\frac{33}{8}$ ------ M1

$\frac{9n^2-9n}{2n^2}=\frac{33}{8}$

$72n^2-72n=66n^2$

$6n^2-72n=0$

$n^2-12n=0$

$n(n-12)=0$

$n=0$ (rej, since $n\neq 0$) or $n-12=0$

$n=12$ ------ A1

Sub. $n=12$ into (1):

$a=-\frac{3}{12}$

$a=-\frac{1}{4}$

$\therefore$ $a=-\frac{1}{4}$ and $n=12$ ------ A1 with rejection

Note: if solve for a first, no need for rejection of value as there is only one value of n.

5 **(a)** Prove that $\dfrac{1-\cos x+\sin x}{1+\cos x+\sin x}=\tan\dfrac{x}{2}$. [4]

$$\frac{1-\cos x+\sin x}{1+\cos x+\sin x}$$

$$=\frac{2\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}\text{------B1 for } 1-\cos x=2\sin^2\frac{x}{2}\text{ and B1 for }\sin x=2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$=\frac{\sin\frac{x}{2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)}{\cos\frac{x}{2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)}\text{------M1 for factorising}$$

$$=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\text{------A1}$$

$$=\tan\frac{x}{2}\text{------[AG]}$$

(b) It is given that $f(x) = 1 - 2\sin 3x$.

(i) State the least and greatest values of $f(x)$. [2]

Least value of $f(x) = 1 - 2(1) = -1$ – – – – – B1

Greatest value of $f(x) = 1 - 2(-1) = 3$ – – – – – B1

(ii) State the period of $f(x)$. [1]

Period of $f(x) = \frac{360°}{3} = 120° \left(\text{or } \frac{2\pi}{3} \right)$ – – – – – B1

(iii) Solve $1 - 2\sin 3x = 0$ for $0 \le x \le \pi$. [3]

$1 - 2\sin 3x = 0,$ $\quad 0 \le x \le \pi$

$\sin 3x = \frac{1}{2}$ $\quad 0 \le 3x \le 3\pi$

Basic angle, $\alpha = \sin^{-1}\frac{1}{2}$ – – – – – M1

$\alpha = \frac{\pi}{6}$

$3x = \alpha,\ \pi - \alpha,\ 2\pi + \alpha,\ 3\pi - \alpha$

$3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

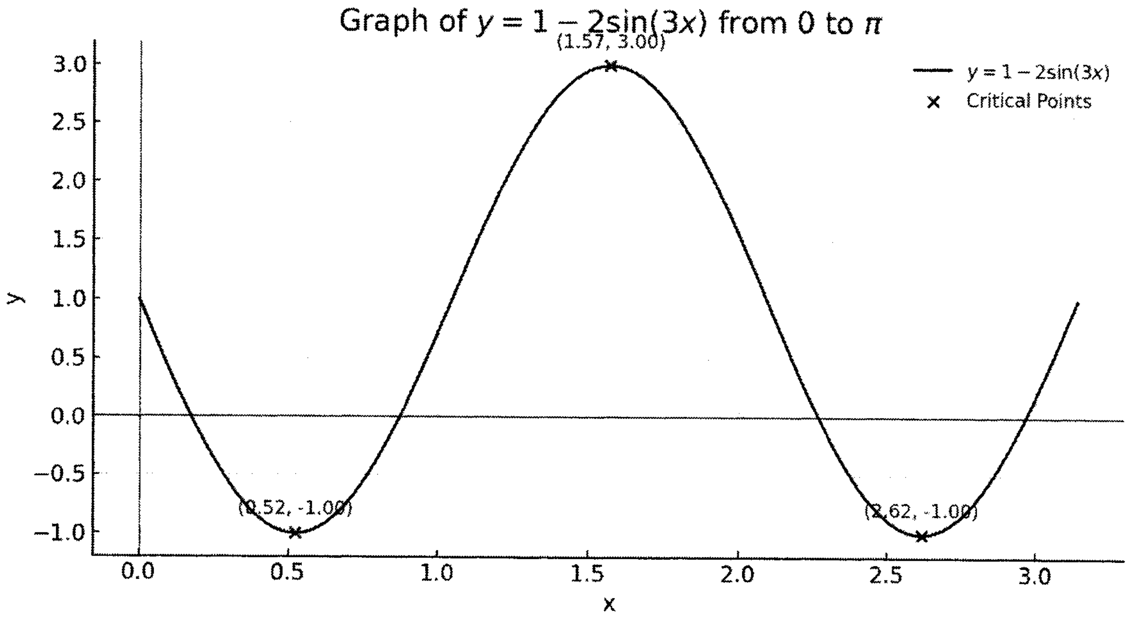
$x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$ – – – – – A2 $\left(\begin{array}{c} \text{4 correct answers: 2 marks} \\ \text{2 or 3 correct answers: 1 mark} \\ \text{otherwise: 0 mark} \end{array} \right)$

(iv) Sketch the graph of $y = \mathrm{f}(x)$ for $0 \le x \le \pi$. [3]

Correct Shape with 1.5 cycle for $y = \mathrm{f}(x)$ – B1

y-axis readings $(y = -1,\ 1 \text{ and } 3)$ – B1

Critical points readings ($x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi$) – B1



6 (a) It is given that $\sin A = -\frac{1}{\sqrt{5}}$ and $\tan B = -\frac{3}{4}$, where A and B are in the same quadrant. **Without using a calculator,** find the exact value of

(i) $\tan(-A)$, [2]

$\tan(-A)$

$= -\tan A$ ------M1

$= \frac{1}{2}$ ------A1

(ii) $\sin(A+B)$, [2]

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \left(-\frac{1}{\sqrt{5}}\right)\left(\frac{4}{5}\right) + \left(\frac{2}{\sqrt{5}}\right)\left(-\frac{3}{5}\right)$ ------M1

$= -\frac{10}{5\sqrt{5}}$

$= -\frac{2}{\sqrt{5}}$

$= -\frac{2\sqrt{5}}{5}$ ------A1

(iii) $\cos\frac{B}{2}$. [2]

$$\cos B = 2\cos^2\left(\frac{B}{2}\right) - 1 = \frac{4}{5} \text{------M1}$$

$$\cos^2\left(\frac{B}{2}\right) = 0.9$$

$$\cos\left(\frac{B}{2}\right) = \pm\frac{3}{\sqrt{10}}$$

Since $135° \leq \left(\frac{B}{2}\right) \leq 180°$,

$$\cos\left(\frac{B}{2}\right) = -\frac{3\sqrt{10}}{10} \text{------A1}$$

(b) Given that $\sin A = q$, where A is an obtuse angle, express $\tan^2 A$ in terms of q. [2]

$$\tan A = -\frac{q}{\sqrt{1-q^2}} \text{------M1}$$

$$\tan^2 A = \left(-\frac{q}{\sqrt{1-q^2}}\right)^2$$

$$= \frac{q^2}{(1+q)(1-q)} \text{------A1} \quad \text{or } \frac{q^2}{1-q^2}$$

If seen $\tan A = \frac{q}{\sqrt{1-q^2}}$, award zero marks.

7 When the function $f(x)=2x^3+ax^2+bx+6$ is divided by x^2+x-2, the remainder is $4-8x$.

(a) Show that $a=1$ and $b=-13$. [3]

$x^2+x-2=(x+2)(x-1)$

Let $f(x)=2x^3+ax^2+bx+6=(x+2)(x-1)Q(x)+4-8x$

when $x=-2$,

$f(-2)=20$

$2(-8)+4a-2b+6=20$

$b=2a-15---(1)------$M1 for either (1) or (2)

when $x=1$,

$f(1)=-4$

$2(1)+a+b+6=-4$

$b=-a-12---(2)$

$(1)=(2):------$M1

$2a-15=-a-12$

$a=1, b=-13$ (shown)$------$A1 with working

(b) Show that $(x-2)$ is a factor of $f(x)$ and **hence**, solve $f(x)=0$. [4]

$f(x)=2x^3+x^2-13x+6$

When $x=2$,

$f(2)=2(8)+4-13(2)+6=0$

Hence, by Factor Theorem, $(x-2)$ is a factor of $f(x)$. (Shown)$------$B1

By long division or comparing coefficients,

$f(x)=(x-2)(2x^2+5x-3)------$M1

$=(x-2)(2x-1)(x+3)$

When $f(x)=0$.

$x=2------$A1

$x=\frac{1}{2},-3------$A1

(c) By using a suitable substitution, solve the equation $6y^3 - 13y^2 + y = -2$. [2]

$6y^3 - 13y^2 + y + 2 = 0$

$2 + y - 13y^2 + 6y^3 = 0$

$\frac{2}{y^3} + \frac{1}{y^2} - \frac{13}{y} + 6 = 0$

$\frac{1}{y} = x$---M1

$y = \frac{1}{2}, -\frac{1}{3}, 2$---A1

8 $\mathrm{f}(x)$ is such that $\mathrm{f}''(x)=-2\cos x+12\sin 2x$. The graph of $y=\mathrm{f}(x)$ passes through the origin and $\left(\frac{\pi}{2},-2\right)$. Show that $\mathrm{f}(\pi)=-4$. [5]

$\mathrm{f}''(x)=-2\cos x+12\sin 2x$

$\mathrm{f}'(x)=\int\left(-2\cos x+12\sin 2x\right)\,\mathrm{d}x$

$=-2\sin x+\dfrac{12\left(-\cos 2x\right)}{2}+c$

$=-2\sin x-6\cos 2x+c$------B1

$\mathrm{f}(x)=\int\left(-2\sin x-6\cos 2x+c\right)\,\mathrm{d}x$

$=-2\left(-\cos x\right)-\dfrac{6\sin 2x}{2}+cx+d$

$=2\cos x-3\sin 2x+cx+d$------B1

At $(0,\ 0),0=2\cos 0-3\sin 2(0)+c(0)+d$------M1

$2+d=0$

$d=-2$

At $\left(\frac{\pi}{2},-2\right),-2=2\cos\frac{\pi}{2}-3\sin 2\left(\frac{\pi}{2}\right)+\frac{\pi}{2}c-2$------M1

$-3\sin\pi+\frac{\pi}{2}c=0$

$\frac{\pi}{2}c=0$

$c=0$

$\mathrm{f}(x)=2\cos x-3\sin 2x-2$

$\mathrm{f}(\pi)=2\cos\pi-3\sin 2\pi-2$------A1

$=2(-1)-3(0)-2$

$=-4$ (shown)------[AG]

Continuation of working space for question 8.

9 The number of cells in an experiment can be modelled by a function $\mathrm{N}(t)=50000\left(1.6^{\frac{1}{2}t}\right)$, where t is the number of hours that have passed from the start of the experiment.

(a) Find the number of cells present at the start of the experiment. [1]

When $t=0$,
number of cells $=\mathrm{N}(0)$
$=50000\left(1.6^{0}\right)$
$=50000$------B1

(b) Find the time taken for the number of cells to be ten times the initial number of cells. Give your answer to the nearest hour. [3]

$500000=50000\left(1.6^{0.5t}\right)$------B1
$10=1.6^{0.5t}$
$\lg 10=0.5t\lg 1.6$------M1
$0.5t=\dfrac{\lg 10}{\lg 1.6}$
$t=\dfrac{1}{\lg 1.6}\div\dfrac{1}{2}$
$=9.798$ h
$=10$ h (correct to the nearest h)------A1

(c) It is given that the number of cells at time $t = t_2$ is double the number of cells at the time $t = t_1$. Show that the difference between the two timings $(t_2 - t_1)$ is approximately 2.95 hours, corrected to 3 significant figures. [3]

Note that the prove needs to be in general and not for specific values of t

$N(t_2) = 2N(t_1)$

$50000(1.6^{0.5t_2}) = 2(50000)(1.6^{0.5t_1})$ ------B1

$(1.6^{0.5t_2}) = 2(1.6^{0.5t_1})$

$\frac{(1.6^{0.5t_2})}{(1.6^{0.5t_1})} = 2$

$\lg 1.6^{0.5(t_2-t_1)} = \lg 2$ ------M1 for manipulation of exp to log equation

$0.5(t_2 - t_1) = \frac{\lg 2}{\lg 1.6}$

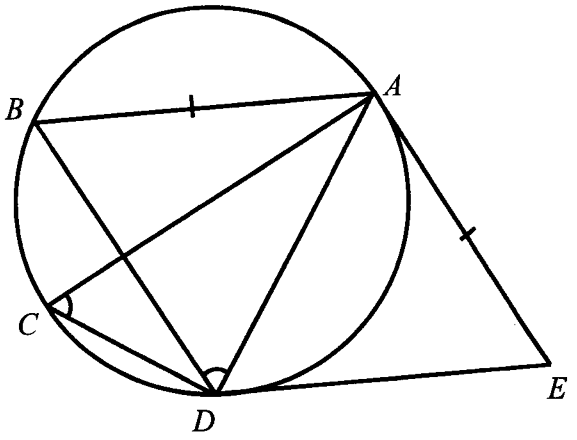
$(t_2 - t_1) = \frac{1}{0.5}\left(\frac{\lg 2}{\lg 1.6}\right)$

$= \frac{2\lg 2}{\lg 1.6}$

$= 2.9495$ (5 s.f.) ------A1

$= 2.95$ ------[AG]

10



In the diagram, A, B, C and D lie on a circle. The tangents to the circle at A and D meet at E. It is given that angle ACD = angle BDA and $BA = AE$.

(a) Show, giving all reasons, that

(i) triangle ABD is isosceles, [2]

$\angle ABD = \angle ACD$ ($\angle$s in the same segment) – – – – – – B1
$\angle ACD = \angle BDA$ (given)
$\therefore \angle ABD = \angle BDA$

Since $\angle ABD = \angle BDA$, using base angles of isosceles triangle, triangle ABD is isosceles. B1

(ii) triangle BDA and triangle ADE are congruent. [3]

$\angle EAD = \angle ABD$ (alt. segment theorem) – – – – – – B1

$\angle ADE = \angle ABD$ (alt. segment theorem)
$= \angle BDA$ (base $\angle$s of isos. Δ) – – – – – – B1 for both
$BA = AE$ (given)
$\therefore \Delta BDA$ is congruent to ΔADE (AAS). – – – – – – B1

(b) Given that BD is parallel to AE, prove that $ABDE$ is a rhombus. [3]

$$\left.\begin{array}{l} \Delta BDA \text{ is congruent to } \Delta ADE \\ \quad BA = AD\ (\Delta ABD \text{ is isos. } \Delta) \\ \quad BD = AD\ (\Delta BDA \equiv \Delta ADE) \\ \quad AB = AE\ \text{(given)} \\ \therefore BD = AE \end{array}\right] \text{B1}$$

Since $BD//AE$ and $BD = AE, ABDE$ is a parallelogram.------B1

$BA = AE,\ \therefore\ ABDE$ is a rhombus.------B1

OR

ΔBDA is congruent to ΔADE

$BA = AD$ (ΔABD is isos. Δ)

$BD = AD$ ($\Delta BDA \equiv \Delta ADE$) $\quad \therefore AB = BD = DE = AE$

$AD = DE$ ($\Delta BDA \equiv \Delta ADE$)

$AB = AE$ (given)

Since $ABDE$ is a quadrilateral with 4 equal sides, $ABDE$ is a rhombus.

11 A particle starts from rest and travels in a straight line such that t seconds after leaving a fixed point O, the acceleration a m/s^2, is given by $a = 4t - 3$. Find

(a) the velocity of the particle when $t = 1$, [2]

$a = 4t - 3$

$v = \int 4t - 3 \, dt$

$= 2t^2 - 3t + c$ ------M1

At $t = 0, v = 0, c = 0$

$v = 2t^2 - 3t$

At $t = 1$,

$v = 2(1)^2 - 3(1)$

$= -1$ m/s ------A1

(b) the distance of the particle from O when it comes to an instantaneous rest, [4]

$v = 2t^2 - 3t$

At $v = 0$,

$2t^2 - 3t = 0$

$t(2t - 3) = 0$

$t = 0$ or $t = \frac{3}{2}$

$s = \int 2t^2 - 3t \, dt$ ------M1

$= \frac{2}{3}t^3 - \frac{3}{2}t^2 + c$

At $t = 0, s = 0, c = 0$

$s = \frac{2}{3}t^3 - \frac{3}{2}t^2$ ------B1

At $t = \frac{3}{2}$,

$s = \frac{2}{3}\left(\frac{3}{2}\right)^3 - \frac{3}{2}\left(\frac{3}{2}\right)^2$ -----M1

$= -1\frac{1}{8}$ m

The distance is $1\frac{1}{8}$ m. ------A1

(c) the total distance travelled by the particle in the first 5 seconds. Leave your answer in exact form. [3]

$$\text{Total distance} = \left|\int_0^{1.5} 2t^2 - 3t\ \mathrm{d}t\right| + \int_{1.5}^{5} 2t^2 - 3t\ \mathrm{d}t \text{------M1}$$

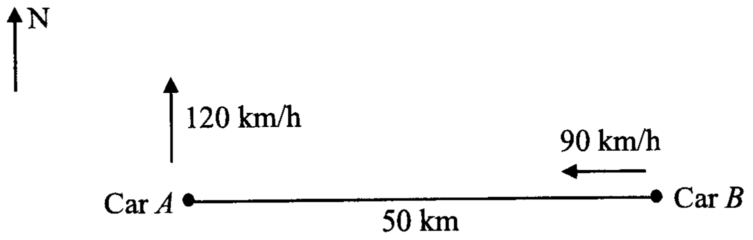
$$= \left|\left[\frac{2}{3}t^3 - \frac{3}{2}t^2\right]_0^{1.5}\right| + \left[\frac{2}{3}t^3 - \frac{3}{2}t^2\right]_{1.5}^{5}$$

$$= \left|\left[\frac{2}{3}(1.5)^3 - \frac{3}{2}(1.5)^2 - 0\right]\right| + \left[\frac{2}{3}(5)^3 - \frac{3}{2}(5)^2\right] - \left[\frac{2}{3}(1.5)^3 - \frac{3}{2}(1.5)^2\right] \text{------M1}$$

$$= \left|-\frac{9}{8}\right| + \frac{275}{6} - \left(-\frac{9}{8}\right)$$

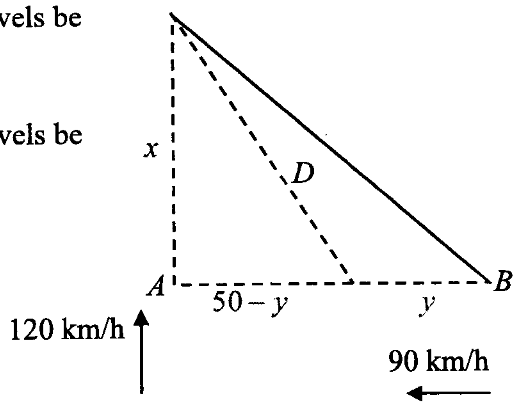
$$= 48\frac{1}{12} \text{------A1}$$ (either mixed fraction or 3 s.f. / no A1 for improper fraction)

12



The diagram shows Car B, 50 km due east of Car A. Both cars start moving at the same time. Car A travels due north at a constant speed of 120 km/h while Car B travels due west at a constant speed of 90 km/h.

(a) The distance between Car A and Car B at time t hours after the cars started moving is noted by D km. Show that $D=\sqrt{14400t^2+(50-90t)^2}$. [2]



Let the distance that Car A travels be x km.

Let the distance that Car B travels be y km.

For Car A,

$$120=\frac{x}{t}$$
$$x=120t \text{ km}$$

For Car B,

$$90=\frac{y}{t}$$
$$y=90t \text{ km}$$

By Pythagoras' Theorem,

$$D=\sqrt{x^2+(50-y)^2} \text{ (since D > 0)------M1}$$
$$=\sqrt{(120t)^2+(50-90t)^2}$$
$$=\sqrt{14400t^2+(50-90t)^2} \text{ ------A1}$$

(b) Given that t can vary, find the stationary value of D. [4]

$$\frac{\mathrm{d}D}{\mathrm{d}t}=\frac{1}{2}\left[14400t^2+(50-90t)^2\right]^{-\frac{1}{2}}\ \left[28800t+2(50-90t)(-90)\right]\text{------B1}$$

$$=\frac{45000t-9000}{2\sqrt{14400t^2+(50-90t)^2}}$$

$$\frac{\mathrm{d}D}{\mathrm{d}t}=0\text{------M1}$$

$$45000t-9000=0$$

$$t=0.2\text{ h}$$

$$D=\sqrt{14400(0.2)^2+\left[50-90(0.2)\right]^2}\text{------M1}$$

$$=40\text{------A1}$$

(c) Show that this stationary value of D gives the minimum distance between the two cars. [1]

Using the First Derivative Test,

t	0.19	0.2	0.21
Sign of $\frac{\mathrm{d}D}{\mathrm{d}t}$	$\frac{45000(0.19)-9000}{2\sqrt{14400(0.19)^2+(50-90(0.19))^2}}$ $=-5.62<0$	0	$\frac{45000(0.21)-9000}{2\sqrt{14400(0.21)^2+(50-90(0.21))^2}}$ $=5.62>0$
Sketch of tangent	$\backslash$	—	/

Hence, D is minimum. (shown) ------B1 for above table

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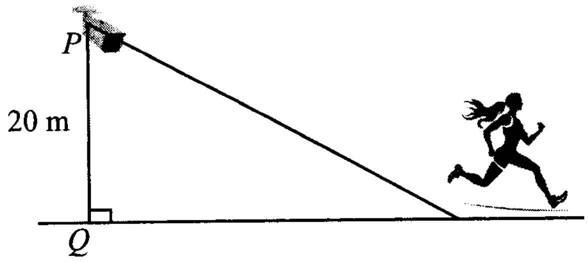
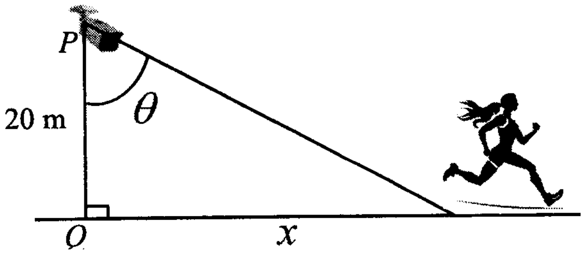


In the diagram, a surveillance camera is mounted at point P, 20 m vertically above point Q. A runner runs from point Q along a straight road at a speed of 4 m/s. The surveillance camera tracks the motion of the runner by panning upwards and downwards at point P. Find the rate of change of the angle that the surveillance camera makes with PQ when the runner is 15 m from Q. Give your answer in radians per second. [5]

Let the distance of the runner from Q be x m.

Let the angle of rotation of the surveillance camera from PQ when the runner is x m from Q be θ.



$$\tan\theta = \frac{x}{20}$$

$$x = 20\tan\theta$$

$$\frac{dx}{d\theta} = 20\sec^2\theta \text{------M1}$$

Using $\sec^2\theta = 1 + \tan^2\theta$ ------M1

$$\sec^2\theta = 1 + \left(\frac{x}{20}\right)^2$$

Given $\frac{dx}{dt} = 4\,\text{m/s}$

$$\frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$$

$$4 = (20\sec^2\theta) \times \frac{d\theta}{dt} \text{------M1}$$

$$\frac{d\theta}{dt} = \frac{4}{20\sec^2\theta}$$

$$= \frac{4}{20\left(1 + \left(\frac{x}{20}\right)^2\right)}$$

$$= \frac{80}{400 + x^2}$$

When $x = 15$ m,

$$\frac{d\theta}{dt} = \frac{80}{400+15^2} ------\text{M1}$$

$$= \frac{16}{125}$$

$$= 0.128 \ \ (3 \text{ s.f}) ------\text{A1}$$

$\therefore$ the rate of change of when the runner is 15m from Q is 0.128 rad / s (3 s.f.).

End of Paper

Ngee Ann Secondary School
Secondary 4 Additional Mathematics-O
2025 Preliminary Examination Paper 2
Marking Scheme

Qn No.	**Qn Part**	**Solutions**	**Marks (Remarks)**	**Total**
1	(a)	$5\sqrt{108}-\sqrt{243}+\frac{\sqrt{192}}{2}$ $=5\sqrt{36\times3}-\sqrt{81\times3}+\frac{\sqrt{64\times3}}{2}$ $=30\sqrt{3}-9\sqrt{3}+4\sqrt{3}$ $=25\sqrt{3}$	M1: At least 2 correct A1	2
	(b)	$\frac{p+2}{p-1}=\frac{\sqrt{5}+2+2}{\sqrt{5}+2-1}$ $=\frac{\sqrt{5}+4}{\sqrt{5}+1}$ $=\frac{\sqrt{5}+4}{\sqrt{5}+1}\times\frac{\sqrt{5}-1}{\sqrt{5}-1}$ $=\frac{5+3\sqrt{5}-4}{5-1}$ $=\frac{3}{4}\sqrt{5}+\frac{1}{4}$	M1: Attempt to rationalise M1: Expansion of surds A1	3

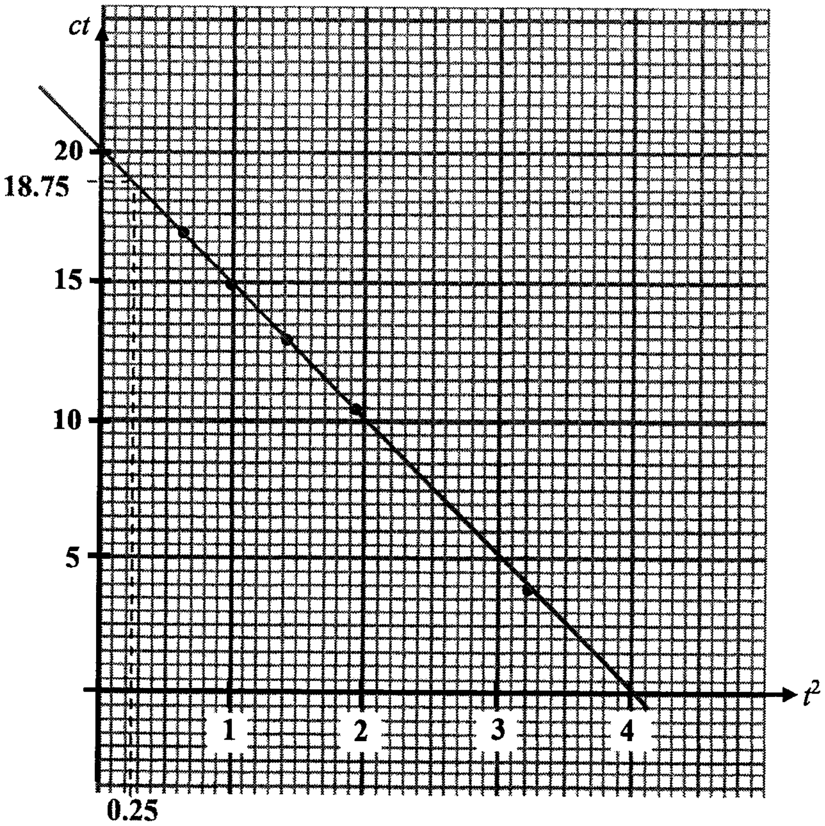
Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
2	(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\cos 2x}{1+\sin 2x}\right)$		
		$=\frac{-2\sin 2x(1+\sin 2x)-(\cos 2x)(2\cos 2x)}{(1+\sin 2x)^2}$	B1: numerator B1 for denominator	
		$=\frac{-2\sin 2x-2\sin^2 2x-2\cos^2 2x}{(1+\sin 2x)^2}$		
		$=\frac{-2\sin 2x-2(\sin^2 2x+\cos^2 2x)}{(1+\sin 2x)^2}$		
		$=-\frac{-2\sin 2x-2}{(1+\sin 2x)^2}$	M1: Realising $\sin^2 2x+\cos^2 2x=1$	
		$=\frac{-2(\sin 2x+1)}{(1+\sin 2x)^2}$	A1	
		$=\frac{-2}{1+\sin 2x}$	AG	4
	(b)	$\int_0^{\frac{\pi}{4}}\frac{-2}{1+\sin 2x}\,\mathrm{d}x=\left[\frac{\cos 2x}{1+\sin 2x}\right]_0^{\frac{\pi}{4}}$	M1: Reverse statement	
		$\frac{1}{-2\times 8}\int_0^{\frac{\pi}{4}}\frac{-2}{1+\sin 2x}\,\mathrm{d}x=-\frac{1}{16}\left[\frac{\cos 2x}{1+\sin 2x}\right]_0^{\frac{\pi}{4}}$	M1: Multiply by $-\frac{1}{16}$	
		$\int_0^{\frac{\pi}{4}}\frac{1}{8(1+\sin 2x)}\,\mathrm{d}x=-\frac{1}{16}\left(\frac{0}{2}-\frac{1}{1}\right)$	M1: Correct use of limits	
		$=\frac{1}{16}$	A1	4

Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
3	(a)	$x(x+1)^2 = x^3+2x^2+x$ $\begin{array}{rr} & 2 \\ x^3+2x^2+x \big) & \overline{2x^3+12x^2+14x+5} \\ & -(2x^3+4x^2+2x) \\ \hline & 8x^2+12x+5 \\ \hline \end{array}$ $\dfrac{2x^3+12x^2+14x+5}{x(x+1)^2} = 2+\dfrac{8x^2+12x+5}{x(x+1)^2}$	B1	1
	(b)	$\dfrac{8x^2+12x+5}{x(x+1)^2} = \dfrac{A}{x}+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}$ $8x^2+12x+5 = A(x+1)^2+Bx(x+1)+Cx$ When $x=0$, $A=5$. When $x=-1$, $C=-1$. When $x=1$, $2B=6$ $B=3$ $\dfrac{2x^3+12x^2+14x+5}{x(x+1)^2} = 2+\dfrac{5}{x}+\dfrac{3}{x+1}-\dfrac{1}{(x+1)^2}$	M1: Correct form of partial fractions M1: Remove denominator A1: Value of A A1: Value of C A1: Value of B	5

<table>
<tr><th>Qn No.</th><th>Qn Part</th><th>Solutions</th><th>Marks (Remarks)</th><th>Total</th></tr>
<tr><td>4</td><td>(a)</td><td>$10x^2-nx+2m=m-6x$
$10x^2+(6-n)x+m=0$
Since line is tangent to curve,
$(6-n)^2-4(10)m=0$
$(6-n)^2=40m$
Since $(6-n)^2\ge 0, 40m\ge 0, m\ge 0$
Hence *m* cannot be negative.</td><td>M1: Eliminate *x* or *y*
M1: Use of discriminant
M1
A1</td><td>4</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>(bi)</td><td>$2x^2-6x+1+k>0$
$(-6)^2-4(2)(1+k)<0$
$28-8k<0$
$k>3.5$</td><td>M1: Use of discriminant
A1</td><td>2</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>(bii)</td><td>$kx^2+(k+1)x+k<0$
Discriminant <0
$(k+1)^2-4k^2<0$
$-3k^2+2k+1<0$
$3k^2-2k-1>0$
$(3k+1)(k-1)>0$
$k<-\frac{1}{3}$ or $k>1$
Since $k<0$, $k<-\frac{1}{3}$.</td><td>M1: Use of discriminant
M1: Factorise quadratic
A1</td><td>3</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>5</td><td>(a)</td><td>$c=at+\frac{b}{t}$
$ct=at^2+b$
Draw *ct* against t^2.
<table><tr><td>t^2</td><td>0.64</td><td>1</td><td>1.44</td><td>1.96</td><td>3.24</td></tr><tr><td>ct</td><td>16.8</td><td>15.0</td><td>13.0</td><td>10.5</td><td>3.8</td></tr></table>See graph next page</td><td>B1: Correct statement
M1: Table of values
B1: Plot points
B1: Draw best fit line</td><td>4</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
</table>

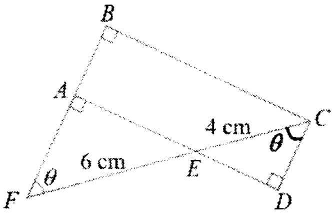
	(b)	$ct = at^2 + b$ $a =$ gradient $= -\frac{20}{4}$ $= -5$ $b =$ vertical intercept $= 20$	M1 A1 (Accept $-5.13 < a < -4.88$) B1 (Accept $19.5 < b < 20.5$)	3
	(c)	When $t = 0.5$, $t^2 = 0.25$. When $t^2 = 0.25$, $ct = 18.75$ $0.5c = 18.75$ $c = 37.5 > 30$ Since the concentration is greater than 30 mg/L, the chemical is not safe for plants when $t = 0.5$ h.	M1 (Accept $18.25 < ct < 19.25$) ($36.5 < c < 38.5$) A1	2

Graph for 5a



Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
6	(a)	$y=2\log_w wx+\log_w(3x-2)-2$ For y to be defined, $wx>0$ and $3x-2>0$ $x>0$ $x>\frac{2}{3}$ Therefore, $x>\frac{2}{3}$.	B1: Final answer only	1
	(b)	$y=2\log_w wx+\log_w(3x-2)-2$ $=\log_w(wx)^2+\log_w(3x-2)-\log_w w^2$ $=\log_w\left[\frac{w^2x^2(3x-2)}{w^2}\right]$ $=\log_w\left[3x^3-2x^2\right]$	M1: Use of power law M1: Realise $2=\log_w w^2$ M1: Use of product and quotient laws A1: Manipulation to correct format	4
	(c)	$\log_w(3x^3-2x^2)=0$ $3x^3-2x^2=w^0$ $3x^3-2x^2=1$ $3x^3-2x^2-1=0$ $(x-1)(3x^2+x+1)=0$ $3x^2+x+1=0$ $x=\frac{-1\pm\sqrt{1^2-4(3)(1)}}{2(3)}$ $=\frac{-1\pm\sqrt{-11}}{6}$ (no real solutions) or $x=1$ (only one real root)	M1 M1: Factorisation M1: Using formula or discriminant A1 A1: For value of x	5

Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
7	(a)	$x^3+3x^2-13x-15=9x+9$	M1: Eliminate x or y	
		$x^3+3x^2-22x-24=0$		
		$(x+6)(x-4)(x+1)=0$	M1: Factorise	
		$x=-6,\ x=4$ or $x=-1$		
		$A(-6,-45)$ and $B(-1,0)$	A1, A1	4
	(b)	Shaded region		
		$=\int_{-6}^{-1}\left[\left(x^3+3x^2-13x-15\right)-\left(9x+9\right)\right]\,dx$	M1	
		$-\int_0^3\left(x^3+3x^2-13x-15\right)\,dx$	M1	
		$=\int_{-6}^{-1}\left(x^3+3x^2-22x-24\right)\,dx$		
		$-\int_0^3\left(x^3+3x^2-13x-15\right)\,dx$		
		$=\left[\frac{x^4}{4}+x^3-11x^2-24x\right]_{-6}^{-1}$	M1: Correct integral	
		$-\left[\frac{x^4}{4}+x^3-\frac{13x^2}{2}-15x\right]_0^3$	M1: Correct integral	
		$=\left(\frac{1}{4}-1-11+24\right)-\left(324-216-396+144\right)$	M1: Substituting limits	
		$-\left[\left(\frac{81}{4}+27-\frac{117}{2}-45\right)-0\right]$		
		$=212.5\ \text{units}^2$	A1	6

Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
8				
	(a)	$\angle ECD = \angle AFE = \theta$ (alt. $\angle$s)		
		$\sin\theta = \frac{AE}{6} \Rightarrow AE = 6\sin\theta$		
		$\cos\theta = \frac{CD}{4} \Rightarrow CD = 4\cos\theta$		
		$\sin\theta = \frac{ED}{4} \Rightarrow ED = 4\sin\theta$		
		$P = 4 + 6\sin\theta + 4\cos\theta + 4\sin\theta + 6\sin\theta$	M1	
		$P = 4 + 16\sin\theta + 4\cos\theta$	A1	2
	(b)	$R = \sqrt{16^2 + 4^2} = 4\sqrt{17}$	B1	
		$\tan\alpha = \frac{4}{16}$	M1	
		$\alpha = 14.036° \approx 14.0°$		
		$P = 4 + 4\sqrt{17}\sin(\theta + 14.036°)$	A1	3
	(c)	Max $P = 4 + 4\sqrt{17}$	B1	
		$\sin(\theta + 14.036°) = 1$		
		$\theta + 14.036° = 90°$	M1	
		$\theta = 75.964° \approx 76.0°$	A1	3
	(d)	$4 + 4\sqrt{17}\sin(\theta + 14.036°) = 15$		
		$\sin(\theta + 14.036°) = 0.66697$	M1	
		$\theta + 14.036° = 41.834°$		2
		$\theta = 27.8°$	A1	
	(e)	Since the maximum $P = 4 + 4\sqrt{17} \approx 20.492$, the perimeter of $ABCE$ could be 20 cm.	B1	1

Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
9	(a)	$y=\frac{e^{2x}(3x+2)}{5}$		
		$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{5}e^{2x}+\frac{2}{5}e^{2x}(3x+2)$	M1, M1	
		$=\frac{7}{5}e^{2x}+\frac{6}{5}xe^{2x}$ At stationary points,		
		$\frac{7}{5}e^{2x}+\frac{6}{5}xe^{2x}=0$	M1	
		$e^{2x}(7+6x)=0$		
		$e^{2x}=0$ (N.A) or $x=-\frac{7}{6}$	A1, A1	5
	(b)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{14}{5}e^{2x}+\frac{6}{5}e^{2x}+\frac{12}{5}xe^{2x}$	M2	
		$=\frac{12xe^{2x}+20e^{2x}}{5}$ When $x=-\frac{7}{6}$,		
		$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{12\left(-\frac{7}{6}\right)e^{-\frac{7}{3}}+20e^{-\frac{7}{3}}}{5}$	M1	
		$=0.116>0$ The stationary point is a minimum point.	A1	4
	(c)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{5}e^{2x}(7+6x)$ When $x>-1$,		
		$\frac{1}{5}e^{2x}>0$, $7+6x>0$ and $\frac{1}{5}e^{2x}(7+6x)>0$.	B1	
		Since $\frac{\mathrm{d}y}{\mathrm{d}x}>0$, the gradient of the curve is positive when $x>-1$.	B1	2

Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
10	(a)	Gradient of $DE = \frac{-3+7}{-4-4} = -\frac{1}{2}$	M1	
		Gradient of normal to $DE = 2$		
		Midpoint of $DE = \left(\frac{-4+4}{2}, \frac{-3-7}{2}\right)$	M1	
		$= (0,-5)$		
		Equation of normal: $y = 2x-5$	M1	
		$4(2x-5) = 3x-15$	M1: Eliminate x or y	
		$5x = 5$		
		$x = 1$		
		$y = 2-5 = -3$		
		Centre = $(1,-3)$	B1	
		Radius = $\sqrt{(1+4)^2+(-3+3)^2} = 5$ units	B1	
		Equation of circle:		
		$(x-1)^2+(y+3)^2 = 5^2$		
		$(x-1)^2+(y+3)^2 = 25$	A1	7
	(b)	Distance between $(-1,1)$ and $(1,-3)$		
		$= \sqrt{(1+1)^2+(-3-1)^2}$		
		$= 4.47 < 5$	M1	
		Since the distance between $(-1,1)$ and $(1,-3)$ is lesser than the radius of the circle, the point $(-1,1)$ lies inside the circle.	A1	2
	(c)	Solving simultaneously $4y = 3x-15$ and $y = x-3$:		
		$4(x-3) = 3x-15$	M1	
		$x = -3$		
		$y = -6$		
		Since the point of intersection of $y = x-3$ and the normal to the circle is not $(1,-3)$ but $(-3,-6)$, AB is not a possible diameter of the circle.	A1	2

Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
10	**(c)**	**Alternatively,** As $(1,-3)$ is the centre of the circle, sub. $x=1$ into the line $y=x-3$. $y=1-3=-2$ Since $y\neq -3$, the line $y=x-3$ does not pass through the centre $(1,-3)$. Therefore, the line segment PQ is not a possible diameter of the circle.	M1: Sub. centre into the line A1: Conclusion	2