BP~395

Name: .... Register no: .... Class: ....

NGEE ANN SECONDARY SCHOOL

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PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS

4049/01

Paper 1

1 September 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

Total	**/90**

Checked by student: .... Date: ....

This document consists of **24** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n\,!}{r\,!\,(n-r)\,!} = \dfrac{n\,(n-1)\ldots(n-r+1)}{r\,!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Determine with working, whether the function $f(x)=\frac{7}{(3x+1)}$, $x>0$, is an increasing or decreasing function. [3]

2 The equation of a quadratic curve is given by $y = -x^2 + px - 17$ and it has a maximum point $(4, q)$ where p and q are constants.

(a) By expressing y in the form of $-(x-a)^2 + b$, find the value of p and of q. [4]

(b) **Hence,** write down the nature of the turning point of $y = \dfrac{5}{-x^2 + px - 17}$ and state the coordinates of the turning point. [2]

3 A line that is not parallel to the $x-$axis has the equation $a^2x + y = 3a - b - 3$, where a and b are constants. A curve has the equation $2x^2 + y^2 = b + 3$. Given that the line and the curve intersect at the point $(0, a)$, find the value of a and of b. [5]

4 The first three terms in the expansion of $(1+ax)^n$ are $1-3x+\frac{33}{8}x^2$, where a and n are constants. Find the value of a and of n. [5]

5 **(a)** Prove that $\dfrac{1-\cos x+\sin x}{1+\cos x+\sin x}=\tan\dfrac{x}{2}$. [4]

(b) It is given that $f(x)=1-2\sin 3x$.

(i) State the least and greatest values of $f(x)$. [2]

(ii) State the period of $f(x)$. [1]

(iii) Solve $1-2\sin 3x=0$ for $0\le x\le \pi$. [3]

(iv) Sketch the graph of $y = f(x)$ for $0 \le x \le \pi$. [3]

6 (a) It is given that $\sin A = -\frac{1}{\sqrt{5}}$ and $\tan B = -\frac{3}{4}$, where A and B are in the same quadrant. **Without using a calculator,** find the exact value of

(i) $\tan(-A)$, [2]

(ii) $\sin(A+B)$, [2]

(iii) $\cos\frac{B}{2}$. [2]

(b) Given that $\sin A = q$, where A is an obtuse angle, express $\tan^2 A$ in terms of q. [2]

7 When the function $f(x) = 2x^3 + ax^2 + bx + 6$ is divided by $x^2 + x - 2$, the remainder is $4 - 8x$.

(a) Show that $a = 1$ and $b = -13$. [3]

(b) Show that $(x - 2)$ is a factor of $f(x)$ and **hence**, solve $f(x) = 0$. [4]

(c) By using a suitable substitution, solve the equation $6y^3 - 13y^2 + y = -2$. [2]

8 $f(x)$ is such that $f''(x) = -2\cos x + 12\sin 2x$. The graph of $y = f(x)$ passes through the origin and $\left(\frac{\pi}{2}, -2\right)$. Show that $f(\pi) = -4$. [5]

Continuation of working space for question 8.

9 The number of cells in an experiment can be modelled by a function $N(t) = 50000\left(1.6^{\frac{1}{2}t}\right)$, where t is the number of hours that have passed from the start of the experiment.

(a) Find the number of cells present at the start of the experiment. [1]

(b) Find the time taken for the number of cells to be ten times the initial number of cells. Give your answer to the nearest hour. [3]

(c) It is given that the number of cells at time $t = t_2$ is double the number of cells at the time $t = t_1$. Show that the difference between the two timings $(t_2 - t_1)$ is approximately 2.95 hours, corrected to 3 significant figures. [3]

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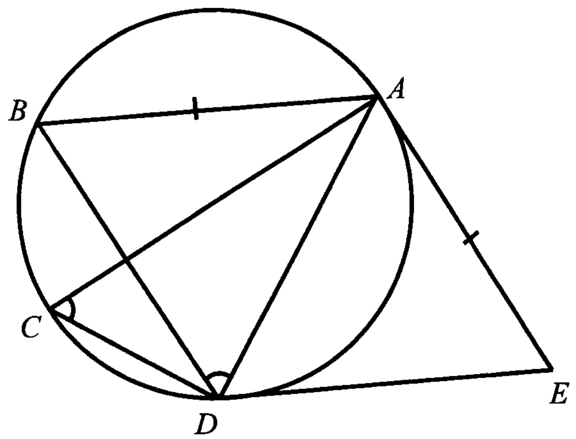

In the diagram, A, B, C and D lie on a circle. The tangents to the circle at A and D meet at E. It is given that angle ACD = angle BDA and $BA = AE$.

(a) Show, giving all reasons, that

(i) triangle ABD is isosceles, [2]

(ii) triangle BDA and triangle ADE are congruent. [3]

(b) Given that *BD* is parallel to *AE*, prove that *ABDE* is a rhombus. [3]

11 A particle starts from rest and travels in a straight line such that t seconds after leaving a fixed point O, the acceleration a m/s^2, is given by $a = 4t - 3$. Find

(a) the velocity of the particle when $t = 1$, [2]

(b) the distance of the particle from O when it comes to an instantaneous rest, [4]

(c) the total distance travelled by the particle in the first 5 seconds.
Leave your answer in exact form. [3]

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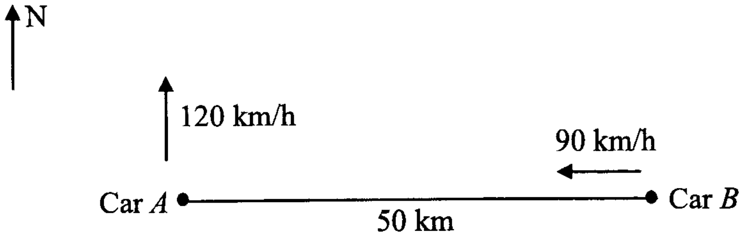

The diagram shows Car B, 50 km due east of Car A. Both cars start moving at the same time. Car A travels due north at a constant speed of 120 km/h while Car B travels due west at a constant speed of 90 km/h.

(a) The distance between Car A and Car B at time t hours after the cars started moving is noted by D km. Show that $D = \sqrt{14400t^2 + (50 - 90t)^2}$. [2]

(b) Given that t can vary, find the stationary value of D. [4]

(c) Show that this stationary value of D gives the minimum distance between the two cars. [1]

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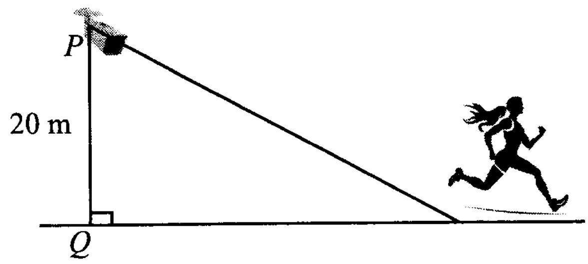

In the diagram, a surveillance camera is mounted at point P, 20 m vertically above point Q. A runner runs from point Q along a straight road at a speed of 4 m/s. The surveillance camera tracks the motion of the runner by panning upwards and downwards at point P. Find the rate of change of the angle that the surveillance camera makes with PQ when the runner is 15 m from Q. Give your answer in radians per second. [5]

End of Paper

Name: .... Register no: .... Class: ....

NGEE ANN SECONDARY SCHOOL

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PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4049/02**

Paper 2

2 September 2025

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1 **(a)** **Without using a calculator**, express $5\sqrt{108}-\sqrt{243}+\dfrac{\sqrt{192}}{2}$ in the form $a\sqrt{3}$, where a is an integer. [2]

(b) Given that $p=\sqrt{5}+2$, express $\dfrac{p+2}{p-1}$ in the form of $a\sqrt{5}+b$, where a and b are rational numbers. [3]

2 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\cos 2x}{1+\sin 2x}\right) = -\frac{2}{1+\sin 2x}$. [4]

(b) Hence find $\int_0^{\frac{\pi}{4}} \frac{1}{8(1+\sin 2x)} \mathrm{d}x$. [4]

3 **(a)** Express $\dfrac{2x^3+12x^2+14x+5}{x(x+1)^2}$ in the form $p+\dfrac{qx^2+rx+s}{x(x+1)^2}$. [1]

(b) Hence express $\dfrac{2x^3+12x^2+14x+5}{x(x+1)^2}$ in partial fractions. [5]

4 (a) Given that the line $y = m - 6x$ is a tangent to the curve $y = 10x^2 - nx + 2m$, show that m cannot be negative. [4]

(b) Find the range of values of k for which

(i) the graph of $y = 2x^2 - 6x + 1 + k$ lies completely above the x-axis, [2]

(ii) $kx^2 + (k+1)x + k$ is always negative. [3]

5 A scientist is studying how the concentration of a chemical solution (c, in mg/L) decreases over time (t, in hours) during a natural breakdown process.

The table below shows some recorded values of t and c.

t	0.8	1.0	1.2	1.4	1.8
c	21.0	15.0	10.8	7.5	2.1

It is believed that c and t are related by the equation $c = at + \frac{b}{t}$, where a and b are constants.

(a) Express the given equation in a form suitable for drawing a straight line graph, and using suitable scales, draw the graph for the values given on a graph paper. [4]

(b) Use your graph to estimate the value of each of the constants a and b. [3]

(c) The chemical is considered harmful to plants when its concentration exceeds 30 mg/L. Use your graph to determine whether the chemical is safe for plants at time $t = 0.5$. Justify your answer. [2]

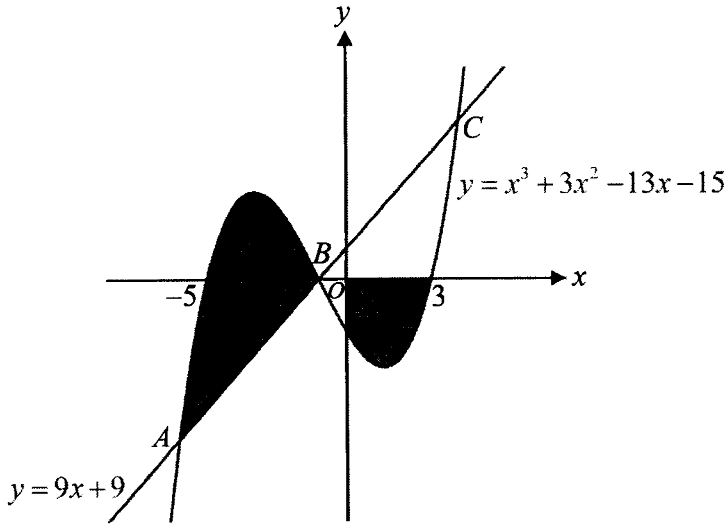
6 It is given that $y = 2\log_w wx + \log_w(3x-2) - 2$, where w is a positive integer and $w \neq 1$.

(a) Write down the range of values of x for which y is defined. [1]

(b) Show that y can be written as $\log_w(3x^3 - 2x^2)$. [4]

(c) Hence show that $y = 0$ has only one real root. [5]

7

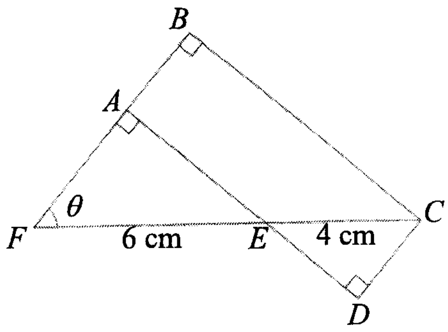


The diagram shows part of the curve $y = x^3 + 3x^2 - 13x - 15$ and the line $y = 9x + 9$. The curve intersects the line at the points A, B and C. The curve intersects the x-axis at points $(-5, 0)$, $(3, 0)$ and point B.

(a) Find the coordinates of points A and B. [4]

(b) Hence find the area of the shaded region. [6]

8



The diagram shows the top view of a garden layout, where FC is a pathway that cuts across the garden. It is given that angle $AFE = \theta$ and angle ABC = angle FAD = angle $EDC = 90°$. CF intersects AD at E, with $CE = 4$ cm and $FE = 6$ cm. $ABCE$ is a raised garden bed in the shape of a trapezium and $0° < \theta < 90°$.

(a) Show that the perimeter of $ABCE$, P cm, is given by $P = 4 + 16\sin\theta + 4\cos\theta$. [2]

(b) Express $P = 4 + 16\sin\theta + 4\cos\theta$ in the form $c + R\sin(\theta + \alpha)$, where c is a constant, R is a positive constant and α is acute. [3]

(c) Find the maximum exact value of P and the corresponding value of θ. [3]

(d) Find the value of θ when the perimeter of $ABCE$ is 15 cm. [2]

(e) Without calculating the exact value of θ, decide whether the perimeter of $ABCE$ could be 20 cm. Explain your reasoning. [1]

9 A curve has the equation $y = \dfrac{e^{2x}(3x+2)}{5}$.

(a) Find the x-coordinate(s) of the stationary point of the curve. [5]

(b) Find $\frac{d^2y}{dx^2}$ and hence determine the nature of the stationary point. [4]

(c) Explain why given that when $x > -1$, the gradient of the curve is positive. [2]

10 A circle passes through the points $D(-4,-3)$ and $E(4,-7)$.
The line with equation $4y = 3x - 15$ is a normal to the circle at a point F.

(a) Showing all your working, find the equation of the circle. [7]

(b) Explain why the point $(-1,1)$ lies inside the circle. [2]

(c) The line $y = x - 3$ intersects the circle at points P and Q. Determine, with working, whether the line segment PQ is a possible diameter of the circle.

[2]

End of Paper

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