SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

SECONDARY 4
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS
Paper 1

4049/01

29 August 2025 (Friday)

2 hours 15 minutes

CANDIDATE NAME

CLASS | 4 | - | | |

INDEX NUMBER

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number in the spaces above.
Write in dark blue or black pen in the space provided for each question.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown in the space below the question.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use		
Q1	3	
Q2	4	
Q3	5	
Q4	5	
Q5	6	
Q6	7	
Q7	8	
Q8	8	
Q9	8	
Q10	8	
Q11	8	
Q12	10	
Q13	10	
Total	**90**	

This document consists of **18** printed pages including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

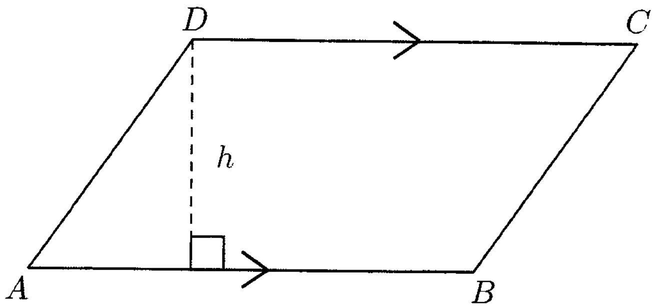
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer all the questions.

1 A parallelogram of length AB $\left(\sqrt{6}-1\right)$ cm has an area of $\left(2\sqrt{6}+3\right)$ cm^2.



Without using a calculator, find the perpendicular height, h, in the form $\left(a+b\sqrt{6}\right)$ cm, where a and b are integers. [3]

Perpendicular height h

$$=\frac{2\sqrt{6}+3}{\sqrt{6}-1}$$

$$=\frac{2\sqrt{6}+3}{\sqrt{6}-1}\times\frac{\sqrt{6}+1}{\sqrt{6}+1}$$

$$=\frac{12+3+5\sqrt{6}}{5}$$

$$=\left(3+\sqrt{6}\right)\text{ cm}$$

2 The function f is given by $f(x) = \sin^2 2x \tan 2x$.

(i) Find $f'(x)$. [2]

$$f'(x) = (2\sin 2x)(2\cos 2x)\tan 2x + \sin^2 2x\left(2\sec^2 2x\right)$$
$$= 4(\sin 2x\cos 2x)\tan 2x + 2\sin^2 2x\left(\sec^2 2x\right)$$
$$= 4\sin^2 2x + 2\tan^2 2x$$

(ii) Explain why f is an increasing function for $0 < x < \frac{\pi}{4}$. [2]

Given $0 < x < \frac{\pi}{4}$
$\sin^2 2x > 0$ and $\tan^2 2x > 0$,
hence $4\sin^2 2x + 2\tan^2 2x > 0 \Rightarrow f'(x) > 0$.

Since $f'(x) > 0$, f is an increasing function.

3 The equation of a curve is $y = (4 + p)x^2 - 2x + p$, where p is a constant.

The curve has a maximum point and intersects the line $y = 2x - 1$.

Find the range of values of p. [5]

$$(4 + p)x^2 - 2x + p = 2x - 1$$
$$(4 + p)x^2 - 4x + p + 1 = 0$$

$$a = 4 + p, b = -4, c = p + 1$$

Since the curve intersects the line, $b^2 - 4ac \geq 0$

$$16 - 4(4 + p)(p + 1) \geq 0$$
$$4 - \left(p^2 + 5p + 4\right) \geq 0$$
$$-p^2 - 5p \geq 0$$
$$p^2 + 5p \leq 0$$
$$p(p + 5) \leq 0$$
$$-5 \leq p \leq 0$$

Since curve has a maximum point, $p + 4 < 0$

$$p < -4$$

Hence

$$-5 \leq p < -4$$

4 Show that the curve $y = -x^2 + (m+4)x - (m^2+6)$ is always negative for all real values of m. [5]

$$\text{Discriminant} = (m+4)^2 - 4(-1)(-m^2-6)$$
$$= m^2 + 8m + 16 - 4m^2 - 24$$
$$= -3m^2 + 8m - 8$$
$$= -3\left(m^2 - \frac{8}{3}m\right) - 8$$
$$= -3\left[\left(m - \frac{4}{3}\right)^2 - \frac{16}{9}\right] - 8$$
$$= -3\left(m - \frac{4}{3}\right)^2 + \frac{16}{3} - \frac{24}{3}$$
$$= -3\left(m - \frac{4}{3}\right)^2 - \frac{8}{3}$$

Since $\left(m - \frac{4}{3}\right)^2 \geq 0$ for all real values of m,

$$-3\left(m - \frac{4}{3}\right)^2 \leq 0$$
$$-3\left(m - \frac{4}{3}\right)^2 - \frac{8}{3} < 0$$
$$\therefore b^2 - 4ac < 0$$

Since $b^2 - 4ac < 0$ and coefficient of x^2 is negative,
$y = -x^2 + (m+4)x - (m^2+6)$ is always negative for all real values of m. (shown)

ALTERNATIVE

$$\frac{dy}{dx} = -2x + m + 4$$
$$= 0$$
$$2x = m + 4$$
$$x = \frac{m}{2} + 2$$

At $x = \frac{m}{2} + 2$

$$y = -\left(\frac{m+4}{2}\right)^2 + (m+4)\left(\frac{m+4}{2}\right) - m^2 - 6$$
$$= \frac{1}{4}\left(m^2 + 8m + 16\right) - m^2 - 6$$
$$= -\frac{3}{4}m^2 + 2m - 2$$
$$= -\frac{3}{4}\left(m^2 - \frac{8}{3}m\right) - 2$$
$$= -\frac{3}{4}\left[\left(m - \frac{4}{3}\right)^2 + \frac{16}{9}\right] - 2$$
$$= -\frac{3}{4}\left(m - \frac{4}{3}\right)^2 - 3\frac{1}{3}$$

Since $\left(m - \frac{4}{3}\right)^2 \geq 0$ for all real values of m,

$$-\frac{3}{4}\left(m - \frac{4}{3}\right)^2 \leq 0$$
$$-\frac{3}{4}\left(m - \frac{4}{3}\right)^2 - 3\frac{1}{3} < 0$$
$$\therefore y - \text{coordinate} < 0$$

Since the y-coordinate of the turning point is negative and the graph has a maximum point, $y = -x^2 + (m+4)x - \left(m^2 + 6\right)$ is always negative for all real values of m. (shown)

	ALTERNATIVE
	$$\begin{aligned} y &= -x^2 + (m+4)x - (m^2+6) \\ &= -[x^2 - (m+4)x] - (m^2+6) \\ &= -\left[\left(x - \frac{m+4}{2}\right)^2 - \left(\frac{m+4}{2}\right)^2\right] - (m^2+6) \\ &= -\left(x - \frac{m+4}{2}\right)^2 + \frac{m^2}{4} + 2m + 4 - m^2 - 6 \\ &= -\left(x - \frac{m+4}{2}\right)^2 - \frac{3m^2}{4} + 2m - 2 \end{aligned}$$ y-coordinate of maximum point is $-\frac{3m^2}{4} + 2m - 2$ Discriminant of this function $= 2^2 - 4\left(-\frac{3}{4}\right)(-2) = 4 - 6 = -2 < 0$ Since discriminant of function is less than zero, and coefficient of m^2 is negative, the y-coordinate of maximum point is always negative. Therefore, $= -x^2 + (m+4)x - (m^2+6)$ is always negative for all real values of m. (shown)

5 The line $4y = 5x - 6$ cuts the curve $x + 4xy - 3y^2 = 7$ at the points P and Q.

Find the length of the line PQ in the form $a\sqrt{b}$, where a and b are integers. [6]

$$5x = 4y + 6$$
$$x = \frac{4y}{5} + \frac{6}{5} \text{ --- (1)}$$
$$x + 4xy - 3y^2 = 7 \text{ --- (2)}$$

Substitute (1) into (2) :

$$\left(\frac{4y}{5} + \frac{6}{5}\right)(1 + 4y) - 3y^2 = 7$$
$$(4y + 6)(1 + 4y) - 15y^2 = 35$$
$$16y^2 + 28y + 6 - 15y^2 - 35 = 0$$
$$y^2 + 28y - 29 = 0$$
$$(y - 1)(y + 29) = 0$$
$$y = 1 \text{ or } -29$$
$$x = 2 \text{ or } -22$$

Point P is $(2,1)$ and Point Q is $(-22, -29)$.

Length of line PQ

$$= \sqrt{(2 - (-22))^2 + (1 + 29)^2}$$
$$= \sqrt{1476}$$
$$= 6\sqrt{41} \text{ units}$$

6 Express $\dfrac{x^3+x^2-10x-22}{(x-3)(x+1)^2}$ in partial fractions. [7]

$$(x-3)(x+1)^2 = (x-3)\left(x^2+2x+1\right)$$
$$= x^3 - x^2 - 5x - 3$$

$$\begin{array}{rrrrrrrrrrrr} & & & & & & & & & & & 1 \\ \cline{6-12} x^3 & - & x^2 & - & 5x - 3 & \big) & x^3 & + & x^2 & - & 10x & - \; 22 \\ & & & & (-) & & x^3 & - & x^2 & - & 5x & - \; 3 \\ \cline{7-12} & & & & & & & & 2x^2 & - & 5x & - \; 19 \end{array}$$

$$\frac{x^3+x^2-10x-22}{(x-3)(x+1)^2} = 1 + \frac{2x^2-5x-19}{(x-3)(x+1)^2}$$

Let $\dfrac{2x^2-5x-19}{(x-3)(x+1)^2} = \dfrac{A}{x-3} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$

$$= \frac{A(x+1)^2 + B(x-3)(x+1) + C(x-3)}{(x-3)(x+1)^2}$$

$$2x^2 - 5x - 19 = A(x+1)^2 + B(x-3)(x+1) + C(x-3)$$

Sub $x = -1$

$$2(-1)^2 - 5(-1) - 19 = C(-1-3)$$
$$-12 = -4C$$
$$C = 3$$

Sub $x = 3$

$$2(3)^2 - 5(3) - 19 = A(3+1)^2$$
$$-16 = 16A$$
$$A = -1$$

Sub $x = 0$

$$-19 = A + B(-3)(1) + C(-3)$$
$$-19 = -1 - 3B + 3(-3)$$
$$3B = 9$$
$$B = 3$$

Hence

$$\frac{x^3+x^2-10x-22}{(x-3)(x+1)^2} = 1 - \frac{1}{x-3} + \frac{3}{x+1} + \frac{3}{(x+1)^2}$$

7 **(i)** Prove that $\dfrac{1}{\operatorname{cosec} x - \cot x} - \dfrac{1}{\sin x} = \cot x$. [4]

$$\begin{aligned} LHS &= \frac{1}{\operatorname{cosec} x - \cot x} - \frac{1}{\sin x} \\ &= \frac{1}{\frac{1}{\sin x} - \frac{\cos x}{\sin x}} - \frac{1}{\sin x} \\ &= \frac{\sin x}{1 - \cos x} - \frac{1}{\sin x} \\ &= \frac{\sin x (1 + \cos x)}{1 - \cos^2 x} - \frac{1}{\sin x} \\ &= \frac{\sin x (1 + \cos x)}{\sin^2 x} - \frac{1}{\sin x} \\ &= \frac{1 + \cos x}{\sin x} - \frac{1}{\sin x} \\ &= \cot x \\ &= RHS \text{ (shown)} \end{aligned}$$

(ii) Hence solve the equation $\dfrac{1}{\operatorname{cosec} x - \cot x} - \dfrac{1}{\sin x} = \cot^2 x - 2$ for $0 < x < 2\pi$. [4]

$$\cot x = \cot^2 x - 2$$
$$\cot^2 x - \cot x - 2 = 0$$
$$(\cot x - 2)(\cot x + 1) = 0$$
$$\cot x = -1 \quad \text{or} \quad \cot x = 2$$
$$\tan x = -1 \quad \text{or} \quad \tan x = 0.5$$
$$x = \frac{3\pi}{4}, \frac{7\pi}{4} \quad \text{or} \quad \text{Basic angle} = 0.46364$$
$$x = 0.464, 3.61 \text{ radians (3 s.f.)}$$
$$x = 0.464, 2.36, 3.61, 5.50 \text{ radians (3 s.f.)}$$

8 **(i)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^7 \ln 7x\right)$. [3]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^7 \ln 7x\right) = 7x^6 \ln 7x + \frac{7x^7}{7x}$$
$$= 7x^6 \ln 7x + x^6$$

(ii) Hence find $\int x^6 \ln 7x \,\mathrm{d}x$. [4]

$$\int \frac{\mathrm{d}}{\mathrm{d}x}\left(x^7 \ln 7x\right) dx = \int 7x^6 \ln 7x + x^6\, dx$$
$$x^7 \ln 7x = \int 7x^6 \ln 7x\, dx + \int x^6\, dx$$
$$= \int 7x^6 \ln 7x\, dx + \frac{x^7}{7} + c_1$$

, where c_1 is an arbitrary constant

$$\int 7x^6 \ln 7x\, dx = x^7 \ln 7x - \frac{x^7}{7} + c_2$$

, where c_2 is an arbitrary constant

$$\int x^6 \ln 7x\, dx = \frac{x^7 \ln 7x}{7} - \frac{x^7}{49} + c_3$$

, where c_3 is an arbitrary constant

(Accept $\int x^6 \ln 7x\, dx = \frac{x^7 \ln 7x}{7} - \frac{x^7}{49} + c$ as students may only integrate at the final step.)

(iii) Without additional working, write what the integral will be for $x^{k-1} \ln kx$, where k is an integer. [1]

$$\int x^{k-1} \ln kx\, dx = \frac{x^k \ln kx}{k} - \frac{x^k}{k^2} + c_4,$$

where c_4 is an arbitrary constant

Similar to part (ii), accept

$$\int x^{k-1} \ln kx\, dx = \frac{x^k \ln kx}{k} - \frac{x^k}{k^2} + c,$$

where c is an arbitrary constant.

9 A curve has equation $y = 0.2x^5 + 3x^3 - 10x$.

(i) Find the x-coordinates of the stationary points, A and B, on the curve. [4]

$$\frac{dy}{dx} = x^4 + 9x^2 - 10$$

For stationary points,

$$\frac{dy}{dx} = 0$$
$$x^4 + 9x^2 - 10 = 0$$

$$(x^2 + 10)(x^2 - 1) = 0$$
$$x = 1 \text{ or } x = -1 \text{ or } x^2 = -10$$
$$(\text{rej. as } x^2 \neq -10)$$

The x – coordinates are $x = 1$ and $x = -1$.

It is given that P is a point on the curve where the **gradient is a minimum**.

(ii) Show that P and the midpoint of AB are the same point. [4]

$$x = 1 \text{ or } x = -1$$
$$y = -6.8 \text{ or } 6.8$$
$$\text{Midpoint of } AB = \left(\frac{1-1}{2}, \frac{-6.8+6.8}{2}\right)$$
$$= (0,0)$$

$$\text{Let } m = x^4 + 9x^2 - 10$$
$$\frac{dm}{dx} = 4x^3 + 18x$$
$$\text{For max./min. gradient, } \frac{dm}{dx} = 0.$$
$$2x(2x^2 + 9) = 0$$
$$x = 0 \text{ or } 2x^2 + 9 = 0 \text{ (rej. as } 2x^2 \neq -9)$$
$$\Rightarrow y = 0$$

$$\frac{d^2m}{dx^2} = 12x^2 + 18$$
$$\text{Subt. } x = 0, \frac{d^2m}{dx^2} = 18 > 0$$

Therefore, $P = (0,0)$ is a point on the curve where the **gradient is a minimum**.

Since the midpoint of $AB = (0,0)$, P and the midpoint of AB are the same. (shown)

10 Points $P(-6,-12)$ and $Q(2,-20)$ lie on a circle C_1 with centre A.

The line $7y+4x+87=0$ passes through the centre of the circle.

(i) Find the coordinates of A and the radius of the circle, C_1. [5]

$$\text{Midpoint of } PQ = \left(\frac{-6+2}{2}, \frac{-12+(-20)}{2}\right)$$
$$= (-2, -16)$$
$$\text{Gradient } PQ = \frac{-12+20}{-6-2} = -1$$

Equation of perpendicular bisector of PQ is

$$y+16 = 1(x+2)$$
$$y = x-14 \;---\; (1)$$
$$7y+4x+87 = 0 \;---\; (2)$$

Substitute (1) into (2) :

$$7x-98+4x+87=0$$
$$x = 1 \Rightarrow y = -13$$
$$A = (1,-13)$$

$$\text{Radius} = \sqrt{(1-2)^2+\left(-13-(-20)\right)^2}$$
$$= \sqrt{1^2+7^2}$$
$$= \sqrt{50}$$
$$= 5\sqrt{2} \text{ units}$$

(ii) Hence find the equation of the circle C_1. [1]

$$(x-1)^2+(y+13)^2 = 50$$

(iii) Another circle C_2 is obtained by reflecting circle C_1 in the line $y = x$.

Find the equation of the circle, C_2. [2]

New centre = $(-13, 1)$

Equation of circle C_2 is
$$(x+13)^2+(y-1)^2 = 50$$

11 A spherical scoop of ice cream with radius x cm rests on top of an inverted cone with height 18 cm and radius 7.5 cm.

As the ice cream scoop melts, the radius of the ice cream scoop decreases at a constant rate of 0.00167 cm/s.

(i) Assuming that the scoop of ice cream retains its shape, find the rate at which the volume of the ice cream scoop is decreasing at the instant when $x = 8$. [3]

$$V = \frac{4}{3}\pi x^3$$

$$\frac{dV}{dx} = 4\pi x^2$$

$$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$$

$$\frac{dV}{dt} = 4\pi x^2(-0.00167)$$

$$At\ x = 8, \frac{dV}{dt} = -1.34309$$

$$= -1.34\text{cm}^3/\text{s (3 s.f.)}$$

Volume of the ice cream scoop is decreasing at a rate of $1.34\ \text{cm}^3$ per second.

(ii) Show that the volume of the melted ice cream in the cone, C, is $C = \frac{25\pi}{432}h^3$, where h cm is the depth of the melted ice cream in the cone. [1]

Let radius of cone be r,

Given radius is 7.5 cm and height is 18 cm,

$$\frac{h}{r} = \frac{18}{7.5}$$

$$r = \frac{5}{12}h$$

$$C = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{5}{12}h\right)^2 h$$

$$= \frac{25\pi}{432}h^3 \text{ (shown)}$$

(iii) The cone was observed to be 12% filled at some point in time. It is given that the rate of increase of the volume of melted ice cream in the cone is the same rate as the rate of decrease of the volume of the ice cream found in part **(i)**. Assuming no melted ice cream drips outside the cone, find the rate of change of the depth of the melted ice cream in the cone at that instant. [4]

Let h_1 be unknown height.

Using volume of similar figures,

$$\left(\frac{h_1}{18}\right)^3 = \left(\frac{V_1}{V_2}\right)$$

Since cone was filled to 12%, $\frac{V_1}{V_2} = 0.12$

$$h_1 = \sqrt[3]{0.12} \times 18$$
$$= 8.8783$$

$$\frac{dC}{dh} = \frac{25\pi}{144}h^2$$

$$\frac{dh}{dt} = \frac{1}{\frac{dC}{dh}} \times \frac{dC}{dt}$$

Since rate of increase of the volume of melted ice cream in the cone is the same rate as the rate of decrease of volume of the ice cream found in part **(i)**,

At $h = 8.8783$

$$\frac{dh}{dt} = \frac{144}{25\pi(8.8783)^2} \times 1.34309$$
$$= 0.031240$$
$$= 0.0312 \text{ cm/s (3 s.f.)}$$

12 An arrow is shot towards a target in an archery competition.

Its height, h m, above the ground at time t seconds after being shot is given by the formula $h = 1.69 + 0.9t - 0.25t^2$.

(i) State the height above the ground from which the arrow is shot. [1]

$$h(0) = 1.69 \text{ m}$$

(ii) Express h in the form $a + b(t + c)^2$, where a, b and c are constants. [3]

$$\begin{aligned} h &= -0.25t^2 + 0.9t + 1.69 \\ &= -0.25\left(t^2 - 3.6t\right) + 1.69 \\ &= -0.25\left[(t - 1.8)^2 - 3.24\right] + 1.69 \\ &= -0.25(t - 1.8)^2 + 0.81 + 1.69 \\ &= 2.5 - 0.25(t - 1.8)^2 \end{aligned}$$

(iii) Hence state the maximum height attained by the arrow and the time at which this occurs. [2]

greatest height = 2.5 m at 1.8 seconds

(iv) The arrow missed the target and hit the ground. Explain why the time taken for the arrow to hit the ground is **not** twice the time found in part **(iii)**. [1]

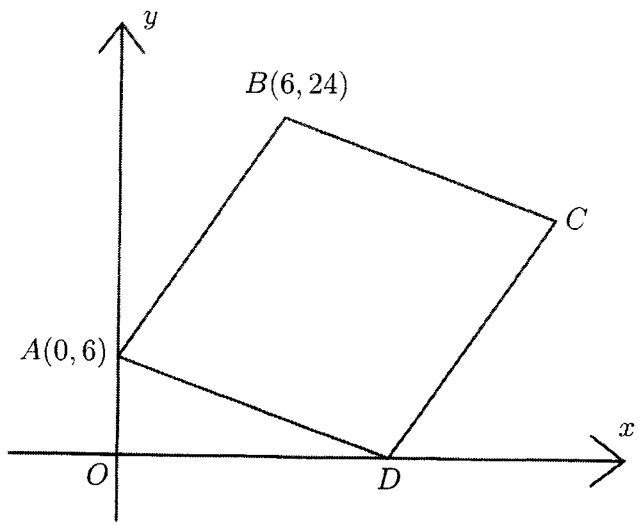
The arrow was shot at 1.69 m above the ground instead of from the ground.

(v) Find the length of time for which the arrow is at least 2.19 m above the ground. [3]

$$h = 2.5 - 0.25(t - 1.8)^2 \geq 2.19$$
$$0.25(t - 1.8)^2 \leq 0.31$$
$$(t - 1.8)^2 \leq 1.24$$
$$-\sqrt{1.24} \leq t - 1.8 \leq \sqrt{1.24}$$
$$0.68644 \leq t \leq 2.9135$$

Length of time $= 2.9135 - 0.68644$
$= 2.22$ s (3 s.f.)

13 The diagram, which is not drawn to scale, shows a rhombus *ABCD* in which the point *A* is (0, 6) and point *B* is (6, 24). The point *D* lies on the *x*-axis.



(i) Find the coordinates of *D*. [3]

Let D be $(d, 0)$

$$\text{length } AB = \sqrt{(6-0)^2 + (24-6)^2}$$
$$= \sqrt{360} \text{ units}$$
$$\sqrt{(6-0)^2 + (0-d)^2} = \sqrt{360}$$
$$d^2 = 324$$
$$d = 18 \text{ or } -18$$
$$\text{(rejected because } d > 0\text{)}$$
$$D = (18, 0)$$

Point D is $(18, 0)$.

(ii) Find the coordinates of C. [4]

$$\text{midpoint of } BD = \left(\frac{6+18}{2}, \frac{24+0}{2}\right) = (12, 12)$$

Let C *be* (a, b).

$$\text{midpoint of } AC = \left(\frac{0+a}{2}, \frac{6+b}{2}\right)$$

Since it is a rhombus, midpoint BD is the same as midpoint AC.

$$\left(\frac{0+a}{2}, \frac{6+b}{2}\right) = (12, 12)$$
$$(a, b) = (24, 18)$$

Point C is $(24, 18)$.

[NOTE: if students use coordinate adding, they must state the reason why they are able to use it, i.e. parallel sides / equal lengths due to rhombus]

(iii) Find the ratio of the area of triangle AOD to the area of rhombus $ABCD$. [3]

$$\text{Area of triangle } AOD = \frac{1}{2}(6)(18) = 54 \text{ units}^2$$

Area of rhombus $ABCD$

$$= \frac{1}{2}\sqrt{(-12)^2 + 24^2}\sqrt{24^2 + (12)^2}$$
$$= 360 \text{ units}^2$$

$$Ratio = 54 : 360 = 3 : 20$$

ALTERNATIVE METHOD

$$\text{Area of triangle } AOD = \frac{1}{2}(6)(18) = 54 \text{ units}^2$$

Area of rhombus $ABCD$

$$= \frac{1}{2}\begin{vmatrix} 18 & 24 & 6 & 0 & 18 \\ 0 & 18 & 24 & 6 & 0 \end{vmatrix}$$
$$= \frac{1}{2}[(324 + 576 + 36) - (108 + 108)]$$
$$= 360 \text{ units}^2$$

$$Ratio = 54 : 360 = 3 : 20$$

---- **End of Paper** ----

BP~521

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

SECONDARY 4
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS
Paper 2

4049/02

2 September 2025 (Tuesday)

2 hours 15 minutes

CANDIDATE NAME	**Solutions**
CLASS	4 -
INDEX NUMBER	

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number in the spaces above.
Write in dark blue or black pen in the space provided for each question.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown in the space below the question.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use		
Q1	8	
Q2	10	
Q3	9	
Q4	9	
Q5	5	
Q6	6	
Q7	11	
Q8	8	
Q9	8	
Q10	6	
Q11	10	
Total	**90**	

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n *is a positive integer and* $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\text{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for Δ *ABC*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

Answer all the questions.

A4.2 Use of factor theorem, including factorising polynomials and solving cubic equations

1 **(i)** It is given that $f(x) = x^4 - px^3 + 7x^2 + x - q$ has a quadratic factor $x^2 - 2x - 3$.

Show that $p = 5$ and $q = 12$. [5]

$x^2 - 2x - 3 = (x-3)(x+1)$

Given $f(-1) = 0$ and $f(3) = 0$

$$(-1)^4 - p(-1)^3 + 7(-1)^2 + (-1) - q = 0$$
$$(3)^4 - p \cdot (3)^3 + 7 \cdot (3)^2 + 3 - q = 0$$

$q - p = 7$ ---- (1)

$27p + q = 147$ ---- (2)

(2) − (1): $28p = 140$

$p = 5$ and $q = 12$ (shown)

(ii) Hence, solve the equation $x^4 - px^3 + 7x^2 + x - q = 0$. [3]

$f(x) = x^4 - 5x^3 + 7x^2 + x - 12$
$= (x^2 - 2x - 3)(x^2 + bx + 4)$ [by inspection]

By comparing coefficients of x^2: $7 = 4 - 2b - 3$
$b = -3$

$$(x^2 - 2x - 3)(x^2 - 3x + 4) = 0$$
$$(x-3)(x+1)(x^2 - 3x + 4) = 0$$
$$x - 3 = 0 \text{ or } x + 1 = 0 \text{ or } x^2 - 3x + 4 = 0$$

Since discriminant of $x^2 - 3x + 4 = 0$ is $(-3)^2 - 4(1)(4) = -7 < 0$, there are no real solutions for $x^2 - 3x + 4 = 0$.

Solution: $x = 3$ and $x = -1$

A6.2 Simplifying expressions involving exponential and logarithmic functions

2 **(a)** **Without using a calculator,**

find the integer value of $(5^{\lg 2})\ (2^{\lg 3})\ (5^{\lg 9})\ (2^{\lg 6})$. [3]

$$\begin{aligned}(5^{\lg 2})(2^{\lg 3})(5^{\lg 9})(2^{\lg 6}) &= (5^{\lg 2+\lg 9})(2^{\lg 3+\lg 6})\\ &= (5^{\lg(2\times 9)})(2^{\lg(3\times 6)})\\ &= (5^{\lg 18})(2^{\lg 18})\\ &= (5\times 2)^{\lg 18}\\ &= 10^{\lg 18}\\ &= 18\end{aligned}$$

A6.1 Logarithmic functions including laws of logarithms and change of base

(b) Given that $\log_2(y^2) = 4 - \log_{0.5} x$, express y in terms of x. [3]

$\log_2(y^2) = 4\log_2 2 - \frac{\log_2 x}{\log_2 2^{-1}}$

$\log_2(y^2) = \log_2 2^4 + \frac{\log_2 x}{1}$

$\log_2(y^2) = \log_2 2^4 + \log_2 x$

$\log_2(y^2) = \log_2(16x)$

$y^2 = 16x$

$y = \pm 4\sqrt{x}$

A6.3 Solving simple equations involving exponential and logarithmic functions

(c) Solve the equation $2e^x = 3 - 5\sqrt{e^x}$. [4]

$2e^x = 3 - 5e^{\frac{x}{2}}$

Let $y = e^{\frac{x}{2}}$.

$2y^2 = 3 - 5y$

$2y^2 + 5y - 3 = 0$

$(2y - 1)(y + 3) = 0$

Since $e^{\frac{x}{2}} > 0$ for all x, $y = 0.5$

$e^{\frac{x}{2}} = 0.5$

$\frac{x}{2} = \ln 0.5$

$x = 2\ln 0.5 = -1.39$ (3 s.f.)

A6.3 Using exponential and logarithmic functions as models

3 A metal cube is heated to a temperature of 205°C before being dropped into a liquid. As the cube cools, its temperature, T°C, t minutes after it enters the liquid is given by $T = K + 175e^{-mt}$, where K and m are constants.

(i) Show that the value of K is 30. [1]

At $t = 0$, $T = 205^oC$.
$K + 175e^{-m(0)} = 205$
$K = 205 - 175$
$K = 30$ (shown)

When $t = 3$, the temperature of the cube reaches 128°C.

(ii) Find the value of m. [3]

When $t = 3$, $T = 128$°C.
$30 + 175e^{-m(3)} = 128$
$175e^{-3m} = 98$
$e^{-3m} = \frac{98}{175}$
$-3m = \ln\left(\frac{98}{175}\right)$
$m = 0.19327$
$m = 0.193$ (3 s.f.)

(iii) Find the rate at which the temperature of the cube is decreasing at the instant when $t = 8$. [3]

$T = 30 + 175e^{-0.19327t}$
$\frac{dT}{dt} = -33.82225e^{-0.19327t}$

When $t = 8$,
$\frac{dT}{dt} = -33.82225e^{-0.19327(8)}$
$= -7.2063$
$= -7.21$ (3 s.f.)

Temperature of the cube is decreasing at a rate of 7.21°C per minute.

(iv) Explain why the temperature of the cube can never fall below 30°C. [2]

Since $e^{-0.19327t} > 0$ for $t \geq 0$,
$175e^{-0.19327t} > 0$,
$30 + 175e^{-0.19327t} > 30$
Therefore, $T > 30$

Hence the temperature of the cube can never fall below 30°C.

A5.3 Use of the general term]

4 **(a)** In the expansion of $(2+x^2)\left(\frac{1}{2x}-ax\right)^5$, there is no term in x. Given that $a \neq 0$, find the value of the constant a. [5]

General term for $\left(\frac{1}{2x}-ax\right)^5 = \binom{5}{r}\left(\frac{1}{2x}\right)^{5-r}(-ax)^r$

$= \binom{5}{r}\left(\frac{1}{2}\right)^{5-r}(-a)^r x^{2r-5}$

$2r-5=1$ and $2r-5=-1$

$r=3$ and $r=2$

Term with $x^{-1} = \binom{5}{2}\left(\frac{1}{2}\right)^{5-2}(-a)^2x^{4-5}$

$= \frac{5}{4}a^2\left(\frac{1}{x}\right)$

Term with $x = \binom{5}{3}\left(\frac{1}{2}\right)^{5-3}(-a)^3x^{6-5}$

$= -\frac{5}{2}a^3x$

$(2+x^2)\left(\frac{1}{2x}-ax\right)^5$

$= (2+x^2)\left(-\frac{5}{2}a^3x+\frac{5}{4}a^2\left(\frac{1}{x}\right)+\cdots\right)$

$= \left(-5a^3+\frac{5}{4}a^2\right)x+\cdots$

$-5a^3+\frac{5}{4}a^2=0$

$\frac{5}{4}a^2(-4a+1)=0$

$a=0$ (rejected) or $a=\frac{1}{4}$

[redacted]

(b) Write down and simplify the first three terms in the expansion of $\left(2+\frac{x^2}{2}\right)^5$ in ascending powers of x. Hence find the estimated value of $(2.005)^5$, showing all your workings clearly. [4]

$\left(2+\frac{x^2}{2}\right)^5 = 2^5+\binom{5}{1}2^4\left(\frac{x^2}{2}\right)+\binom{5}{2}2^3\left(\frac{x^2}{2}\right)^2+\cdots$

$= 32+40x^2+20x^4+\cdots$ ---- (1)

Solving $\left(2+\frac{x^2}{2}\right)^5 = 2.005^5$ gives $x=0.1$

Substitute $x=0.1$ into (1):

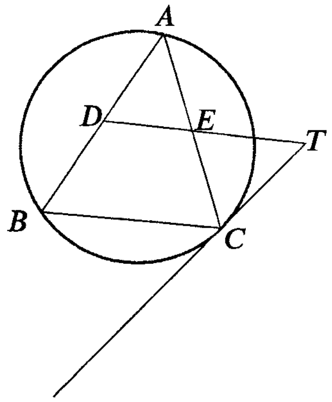
$2.005^5 = 32+40(0.1)^2+20(0.1)^4$

$= 32+40(0.01)+20(0.0001)$

$= 32.402$

G3.1 Use of tangent-chord theorem

5



The diagram shows a circle passing through the vertices of a triangle ABC. Points D and E are the midpoints of AB and AC respectively. The tangent to the circle at C meets DE extended at the point T. Prove that points A, D, C and T lie on a circle. [5]

By midpoint theorem, DE is parallel to BC.

Thus, $\measuredangle ADE = \measuredangle ABC$. (corresponding angles)

By tangent-chord theorem,
$\measuredangle ABC = \measuredangle ECT$

Since $\measuredangle ADE = \measuredangle ECT$,
they are angles in same segment,
hence, points A, D, C and T lie on a circle. (shown)

G1.6 Use of the expressions of $a\cos\theta + b\sin\theta$ in the form $R\cos(\theta \pm \alpha)$ or $R\sin(\theta \pm \alpha)$

6 The expression $3\cos\theta + 5\sin\theta$ is defined for $0 \leq \theta \leq \frac{\pi}{2}$ radians.

(i) Express $3\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle in radian. [2]

$3\cos\theta + 5\sin\theta = R\cos(\theta - \alpha)$

$3\cos\theta + 5\sin\theta = \sqrt{3^2 + 5^2}\cos(\theta - \alpha)$, where $\alpha = \tan^{-1}\left(\frac{5}{3}\right)$

$= \sqrt{34}\cos(\theta - 1.0303)$ (working value)

$= 5.83\cos(\theta - 1.03)$ (3 s.f.)

Hence,

(ii) solve the equation $3\cos\theta + 5\sin\theta = 4$, [2]

$3\cos\theta + 5\sin\theta = 4$

$\sqrt{34}\cos(\theta - 1.0303) = 4$

$\cos(\theta - 1.0303) = \frac{4}{\sqrt{34}}$

Basic angle $= \cos^{-1}\left(\frac{4}{\sqrt{34}}\right) = 0.81482$

$\theta - 1.0303 = 0.81482, -0.81482$

$\theta = 1.84512, 0.21548$

As θ is acute, $\theta = 0.215$ radians (3 s.f.)

(accept 0.216 rad if exact form is used)

(iii) find the minimum value of $\frac{1}{(3\cos\theta + 5\sin\theta)^2 + 1}$. [2]

$-\sqrt{34} \leq 3\cos\theta + 5\sin\theta \leq \sqrt{34}$

$0 \leq (3\cos\theta + 5\sin\theta)^2 \leq 34$

$1 \leq (3\cos\theta + 5\sin\theta)^2 + 1 \leq 35$

$$\frac{1}{35} \leq \frac{1}{(3\cos\theta + 5\sin\theta)^2 + 1} \leq 1$$

Minimum value $= \frac{1}{35}$

G2.5 Transformation of given relationships, including $y = ax^n$ and $y = kb^x$, to linear form to determine the unknown constants from a straight line graph

7 **(a)** The mass, y mg, of a radioactive substance decreases with time, x hours, after the start of the experiment. The research analyst claims that the data can be modelled by an equation of the form $x^2y = a + bx^2$, where a and b are constants. Values of y for different values of x have been collected. Explain how a straight line graph can be drawn to represent the formula, and state how the values of a and b could be obtained from the line. [4]

Rearrange the given formula into a straight-line graph equation as follows, $x^2y = bx^2 + a$ where x^2y is plotted against x^2. When line is drawn, the value of b is the gradient and the value of a is the **vertical intercept** of the graph.

Alternatively,
$y = a\left(\frac{1}{x^2}\right) + b$ where y is plotted against $\frac{1}{x^2}$.

(b) The table shows experimental values of two variables R and t.

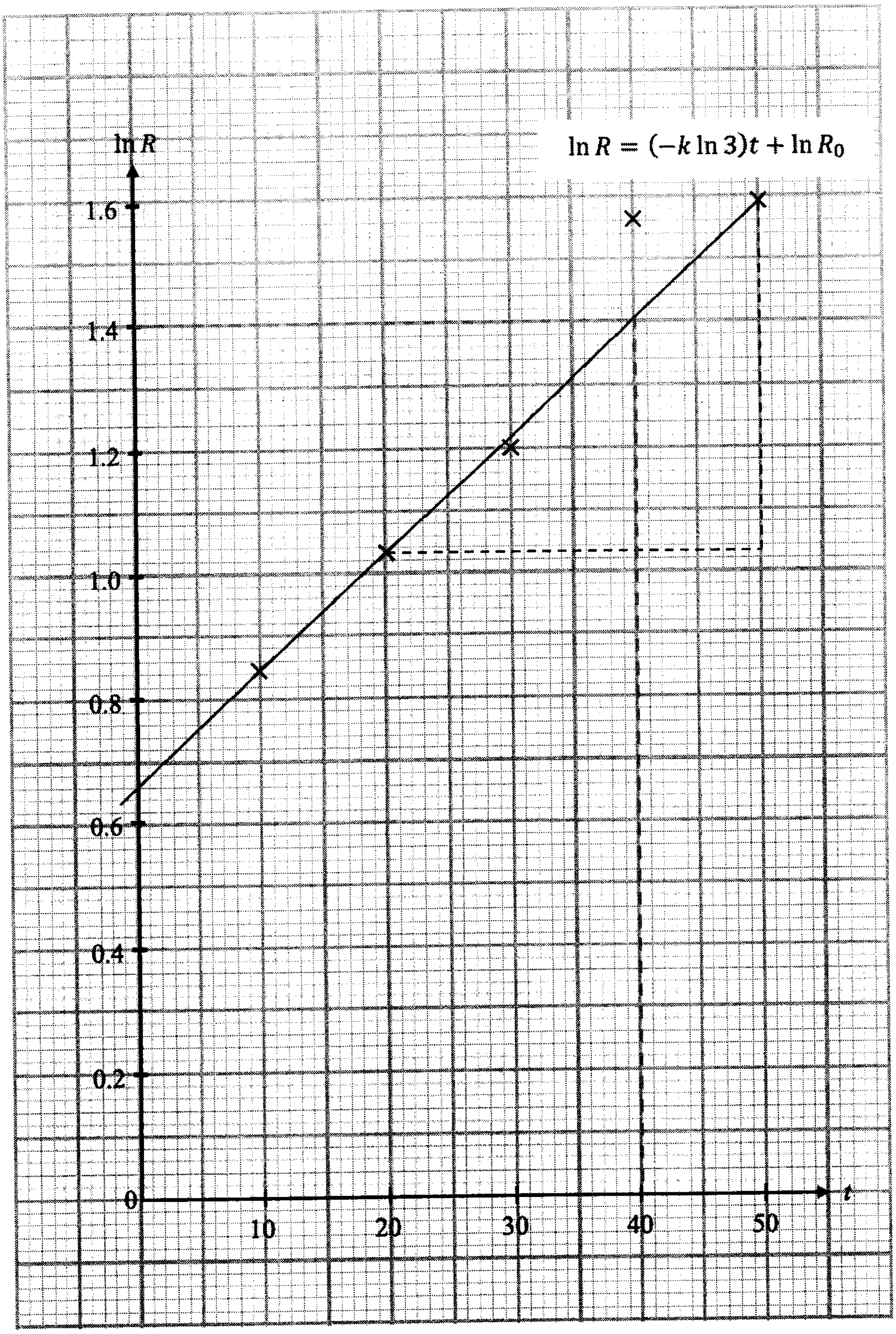
t	10	20	30	40	50
R	2.32	2.79	3.31	4.74	4.90

It is known that R and t are related by the equation $R = R_0(3^{-kt})$, where R_0 and k are constants. An error was made in recording one of the values of R.

(i) Plot $\ln R$ against t and draw a straight line graph to illustrate the information. [2]

t	10	20	30	40	50
$\ln R$	0.842	1.03	1.20	1.56	1.59

$R = R_0(3^{-kt})$
$\ln R = \ln R_0(3^{-kt})$
$\ln R = \ln R_0 + \ln 3^{-kt}$
$\ln R = \ln R_0 - kt\ln 3$
$\ln R = (-k\ln 3)t + \ln R_0$
$\therefore$ Plot $\ln R$ against t


$\ln R$
$\ln R = (-k \ln 3)t + \ln R_0$
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
10
20
30
40
50
t

7 **(b)** **(ii)** From your straight line graph, identify the incorrect reading and suggest a corrected value for R. [2]

$R = 4.74$ is the incorrect reading

From the graph, $\ln R = 1.4$
$R = e^{1.4} = 4.06$ (3 s.f.)

(iii) Use your graph to estimate the values of R_0 and k. [3]

$R = R_0(3^{-kt})$
$\ln R = (-k \ln 3)t + \ln R_0$

Vertical intercept $= \ln R_0$
$= 0.66$ (from the graph)
Thus, $R_0 = e^{0.66} = 1.93$ (3 s.f.)

Gradient $= -k \ln 3$
$= \frac{1.59-1.03}{50-20}$
$= 0.018666$
Thus, $k = \frac{0.018666}{-\ln 3}$
$= -0.0170$ (3 s.f.)

C1.15 Evaluation of definite integrals

8 **(a)** Given that $\int_1^6 \mathrm{f}(x)\,\mathrm{d}x = 14$ and $\int_1^3 \mathrm{f}(x)\,\mathrm{d}x = 8$, find

(i) $\int_3^6 \mathrm{f}(x)\,\mathrm{d}x$, [1]

$$\int_3^6 \mathrm{f}(x)\,\mathrm{d}x = 14 - 8$$
$$= 6$$

(ii) $\int_1^6 (4x - 3\mathrm{f}(x))\,\mathrm{d}x$. [2]

$$\begin{aligned}\int_1^6 (4x - 3\mathrm{f}(x))\,\mathrm{d}x &= \int_1^6 4x\,\mathrm{d}x - 3\int_1^6 \mathrm{f}(x)\,\mathrm{d}x \\ &= [2x^2]_1^6 - 3(14) \\ &= 72 - 2 - 42 \\ &= 28\end{aligned}$$

C1.4 Derivatives of x^n, for any rational n, $\sin x$, $\cos x$, $\tan x$, e^x, and $\ln x$ together with constant multiples, sums and differences

8 **(b)** It is given that $f(x) = \ln(\sin 2x)$. Show that $f''(x) + [f'(x)]^2 + 4 = 0$. [5]

$$f(x) = \ln(\sin 2x)$$

$$f'(x) = \frac{2\cos 2x}{\sin 2x}$$

$$f''(x) = \frac{\sin 2x(-4\sin 2x) - 2\cos 2x\,(2\cos 2x)}{(\sin 2x)^2}$$

$$= \frac{-4\sin^2 2x - 4\cos^2 2x}{(\sin 2x)^2}$$

$$= \frac{-4(\sin^2 2x + \cos^2 2x)}{(\sin 2x)^2}$$

$$= -\frac{4}{(\sin 2x)^2}$$

$$f''(x) + [f'(x)]^2 + 4 = -\frac{4}{(\sin 2x)^2} + \left[\frac{2\cos 2x}{\sin 2x}\right]^2 + 4$$

$$= -\frac{4}{(\sin 2x)^2} + \frac{4\cos^2 2x}{\sin^2 2x} + 4$$

$$= \frac{-4 + 4\cos^2 2x}{\sin^2 2x} + 4$$

$$= \frac{-4(1 - \cos^2 2x)}{\sin^2 2x} + 4$$

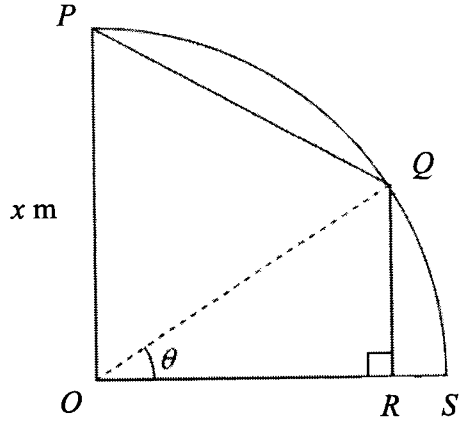
$$= \frac{-4\sin^2 2x}{\sin^2 2x} + 4$$

$$= -4 + 4$$

$$= 0 \text{ (shown)}$$

C1.10 Apply differentiation to maxima and minima problems

9 The diagram shows a garden plot designed in the shape of a trapezium $OPQR$ inscribed in a quadrant $OPQS$ with a fixed radius x metres and angle $QOR = \theta$ radians.



(i) Show that the area, A m^2, of trapezium $OPQR$ is given by $A = \frac{1}{4}x^2(\sin 2\theta + 2cos\theta)$. [3]

$A = \left(\frac{1}{2}\right)(x\cos\theta)(x\sin\theta + x)$
$= \frac{1}{2}x^2 \sin\theta\cos\theta + \frac{1}{2}x^2\cos\theta$
$= \frac{1}{2}x^2\left(\frac{\sin 2\theta}{2}\right) + \frac{1}{2}x^2\cos\theta$
$= \frac{1}{4}x^2\sin 2\theta + \frac{1}{2}x^2\cos\theta$
$= \frac{1}{4}x^2(\sin 2\theta + 2\cos\theta)$ (shown)

(ii) Given that θ can vary, find

(a) the value of θ for which A has a stationary value. [3]

$\frac{dA}{d\theta} = \frac{1}{4}x^2(2\cos 2\theta - 2\sin\theta)$
For stationary value of A, $\frac{dA}{d\theta} = 0$
$\frac{1}{4}x^2(2\cos 2\theta - 2\sin\theta) = 0$
$\frac{1}{2}x^2(\cos 2\theta - \sin\theta) = 0$
$\frac{1}{2}x^2(1 - 2\sin^2\theta - \sin\theta) -= 0$
$1 - 2\sin^2\theta - \sin\theta = 0$
$2\sin^2\theta + \sin\theta - 1 = 0$
$(2\sin\theta - 1)(\sin\theta + 1) = 0$
$\sin\theta = \frac{1}{2}$ or $\sin\theta = -1$
$\theta = \frac{\pi}{6}$ rad, or $\theta = \frac{3\pi}{2}$ rad. (rej, because $\theta < \frac{\pi}{2}$)

9 **(ii)** **(b)** the stationary value of A at $x = 10$ and determine whether it is a maximum or a minimum. [2]

$\frac{d^2A}{d\theta^2} = \frac{1}{4}x^2(-4\sin 2\theta - 2\cos\theta)$

Subt. $\theta = \frac{\pi}{6}$ and $x = 10$ into $\frac{d^2A}{d\theta^2}$,

$\frac{d^2A}{d\theta^2} = \frac{1}{4}(10)^2\left(-4\sin 2\left(\frac{\pi}{6}\right) - 2\cos\frac{\pi}{6}\right)$

$= \frac{1}{4}(100)(-5.1961)$

$= -129.90 \quad < 0$

Since $\frac{d^2A}{d\theta^2} < 0$, A is a maximum when $\theta = \frac{\pi}{6}$.

Subt. $\theta = \frac{\pi}{6}$ into A,

Maximum area of A

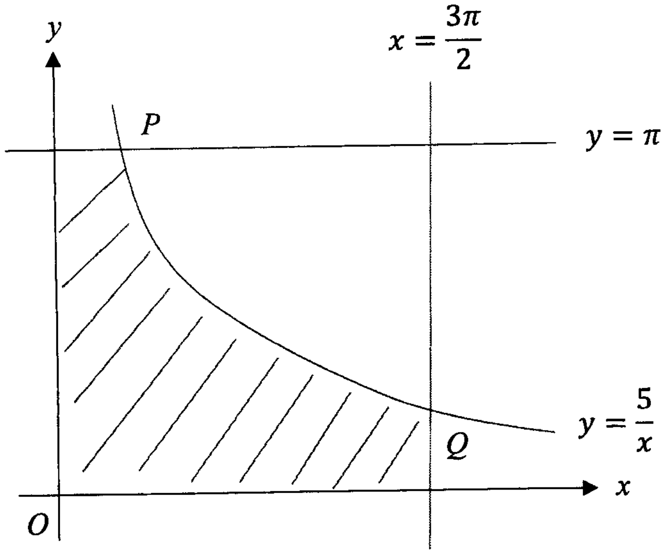
$= \frac{1}{4}(10)^2\left(\sin 2\left(\frac{\pi}{6}\right) + 2\cos\frac{\pi}{6}\right)$

$= 64.951$

$= 65.0\ \text{cm}^2$ (3 s.f.)

C1.16 Finding the area of a region bounded by a curve and line(s).

10 The diagram shows part of the curve $y = \frac{5}{x}$, $y = \pi$ and $x = \frac{3\pi}{2}$. The curve $y = \frac{5}{x}$ cuts the line $y = \pi$ at P and cuts the line $x = \frac{3\pi}{2}$ at Q.



(i) Find the coordinates of P and of Q in terms of π. [2]

When $y = \pi$, $\frac{5}{x} = \pi$,
Therefore, $x = \frac{5}{\pi}$
Point P is $\left(\frac{5}{\pi}, \pi\right)$.

When $x = \frac{3\pi}{2}$, $y = 5 \div \frac{3\pi}{2} = \frac{10}{3\pi}$
Point Q is $\left(\frac{3\pi}{2}, \frac{10}{3\pi}\right)$.

(ii) Find the area of the shaded region bounded by $y = \frac{5}{x}$, $y = \pi$, $x = \frac{3\pi}{2}$, the y-axis and the x-axis. [3]

$$\begin{aligned}\text{Shaded area} &= \pi\left(\frac{5}{\pi}\right) + \int_{\frac{5}{\pi}}^{\frac{3\pi}{2}} \frac{5}{x}\,dx \\ &= 5 + 5[\ln x]_{\frac{5}{\pi}}^{\frac{3\pi}{2}} \\ &= 5 + 5\left[\ln\left(\frac{3\pi}{2}\right) - \ln\left(\frac{5}{\pi}\right)\right] \\ &= 10.4 \text{ units}^2 \text{ (3 s.f.)}\end{aligned}$$

(iii) Explain why the area of the shaded region is between 5 units^2 and $\frac{3\pi^2}{2}$ units^2. [1]

Area of big rectangle $= \pi\left(\frac{3\pi}{2}\right)$

$= \frac{3\pi^2}{2}$ units^2

Area of small rectangle $= \left(\frac{3\pi}{2}\right)\left(\frac{10}{3\pi}\right)$

$= 5$ units^2

Therefore, area of shaded region is between 5 units^2 and $\frac{3\pi^2}{2}$ units^2.

C1.18 Application of differentiation and integration to problems involving displacement, velocity and acceleration of a particle moving in a straight line

11 A particle, travelling in a straight line, has velocity, v cm/s, given by $v = 3t^2 - pt + 4$, where p is a constant and t seconds is the time after passing through a fixed point O. The particle comes to instantaneous rest when $t = 2$.

(i) Show that $p = 8$. [1]

$v = 3t^2 - pt + 4$
When $v = 0$ and $t = 2$,
$3(2)^2 - p(2) + 4 = 0$
$2p = 16$
$p = 8$ (shown)

(ii) Find the range of values of t during which the particle moves in the positive direction. [3]

$3t^2 - 8t + 4 > 0$
$(3t - 2)(t - 2) > 0$
The range of values of t is $t < \frac{2}{3}$ or $t > 2$.
Since $t \geq 0$, the range of values of t is $0 \leq t < \frac{2}{3}$ or $t > 2$.

(iii) Find the minimum velocity of the particle. [3]

$\frac{dv}{dt} = 6t - 8$
For minimum/maximum velocity, $\frac{dv}{dt} = 0$

$$6t - 8 = 0$$
$$t = \frac{4}{3}$$

$\frac{d^2v}{dt^2} = 6 > 0$
At $t = \frac{4}{3}$, velocity is a minimum value.

At $t = \frac{4}{3}$, minimum velocity $= 3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) + 4$

$$= -\frac{4}{3} \text{ cm/s}$$

Accepts -1.33 cm/s (3 s.f.)

(iv) Calculate the total distance travelled by the particle for the first 3 seconds. [3]

$s = \int v\, dt$
$= t^3 - 4t^2 + 4t + c$, where c is an arbitrary constant

When $s = 0$ and $t = 0$, $c = 0$
Therefore, $s = t^3 - 4t^2 + 4t$

Particle is at rest when $v = 0$.
$3t^2 - 8t + 4 = 0$
$(3t - 2)(t - 2) = 0$
$t = \frac{2}{3}$ or $t = 2$

When $t = \frac{2}{3}$, $s = \frac{32}{27}$ cm
When $t = 2$, $s = 0$ cm
When $t = 3$, $s = 3$ cm
Total distance travelled for the first 3 seconds
$= \frac{32}{27} + \frac{32}{27} + 3$
$= 5\frac{10}{27}$ cm
Accepts 5.37 cm (3 s.f.)

----- **End of Paper** -----