SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

SECONDARY 4
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS
Paper 2

4049/02

2 September 2025 (Tuesday) **2 hours 15 minutes**

CANDIDATE NAME	

CLASS | 4 | - | | |

INDEX NUMBER | | |

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number in the spaces above.
Write in dark blue or black pen in the space provided for each question.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown in the space below the question.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use		
Q1	8	
Q2	10	
Q3	9	
Q4	9	
Q5	5	
Q6	6	
Q7	11	
Q8	8	
Q9	8	
Q10	6	
Q11	10	
Total	**90**	

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

Answer all the questions.

1 **(i)** It is given that $f(x) = x^4 - px^3 + 7x^2 + x - q$ has a quadratic factor $x^2 - 2x - 3$.
Show that $p = 5$ and $q = 12$. [5]

(ii) Hence, solve the equation $x^4 - px^3 + 7x^2 + x - q = 0$. [3]

2 (a) **Without using a calculator,**

find the integer value of $\left(5^{\lg 2}\right)\left(2^{\lg 3}\right)\left(5^{\lg 9}\right)\left(2^{\lg 6}\right)$. [3]

(b) Given that $\log_2\left(y^2\right) = 4 - \log_{0.5} x$, express y in terms of x. [3]

(c) Solve the equation $2e^x = 3 - 5\sqrt{e^x}$. [4]

3 A metal cube is heated to a temperature of 205 °C before being dropped into a liquid. As the cube cools, its temperature, T °C, t minutes after it enters the liquid is given by $T = K + 175e^{-mt}$, where K and m are constants.

(i) Show that the value of K is 30. [1]

When $t = 3$, the temperature of the cube reaches 128 °C.

(ii) Find the value of m. [3]

(iii) Find the rate at which the temperature of the cube is decreasing at the instant when $t = 8$. [3]

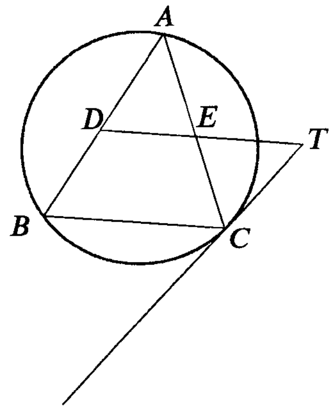
(iv) Explain why the temperature of the cube can never fall below 30 °C. [2]

4 (a) In the expansion of $\left(2 + x^2\right)\left(\frac{1}{2x} - ax\right)^5$, there is no term in x. Given that $a \neq 0$, find the value of the constant a. [5]

(b) Write down and simplify the first three terms in the expansion of $\left(2+\frac{x^2}{2}\right)^5$ in ascending powers of x. Hence find the estimated value of $(2.005)^5$, showing all your workings clearly. [4]

[Turn over

5



The diagram shows a circle passing through the vertices of a triangle ABC. Points D and E are the midpoints of AB and AC respectively. The tangent to the circle at C meets DE extended at the point T. Prove that points A, D, C and T lie on a circle. [5]

6 The expression $3\cos\theta + 5\sin\theta$ is defined for $0 \leq \theta \leq \frac{\pi}{2}$ radians.

(i) Express $3\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle in radian. [2]

Hence,

(ii) solve the equation $3\cos\theta + 5\sin\theta = 4$, [2]

(iii) find the minimum value of $\frac{1}{(3\cos\theta+5\sin\theta)^2+1}$. [2]

[Turn over

7 **(a)** The mass, y mg, of a radioactive substance decreases with time, x hours, after the start of the experiment. The research analyst claims that the data can be modelled by an equation of the form $x^2y = a + bx^2$, where a and b are constants. Values of y for different values of x have been collected. Explain how a straight line graph can be drawn to represent the formula, and state how the values of a and b could be obtained from the line. [4]

(b) The table shows experimental values of two variables R and t.

t	10	20	30	40	50
R	2.32	2.79	3.31	4.74//	4.90

It is known that R and t are related by the equation $R = R_0(3^{-kt})$, where R_0 and k are constants. An error was made in recording one of the values of R.

(i) Plot $\ln R$ against t and draw a straight line graph to illustrate the information. [2]

7 **(b)** **(ii)** From your straight line graph, identify the incorrect reading and suggest a corrected value for *R*. [2]

(iii) Use your graph to estimate the values of R_0 and k. [3]

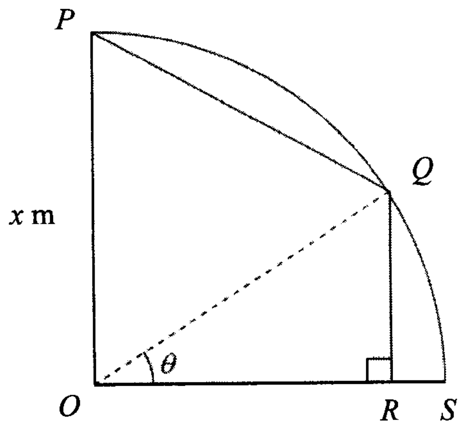
8 **(a)** Given that $\int_1^6 \mathrm{f}(x)\,\mathrm{d}x = 14$ and $\int_1^3 \mathrm{f}(x)\,\mathrm{d}x = 8$, find

(i) $\int_3^6 \mathrm{f}(x)\,\mathrm{d}x$, [1]

(ii) $\int_1^6 \left(4x - 3\mathrm{f}(x)\right)\,\mathrm{d}x$. [2]

(b) It is given that $\mathrm{f}(x) = \ln(\sin 2x)$. Show that $\mathrm{f}''(x) + \left[\mathrm{f}'(x)\right]^2 + 4 = 0$. [5]

9 The diagram shows a garden plot designed in the shape of a trapezium *OPQR* inscribed in a quadrant *OPQS* with a fixed radius x metres and angle $QOR = \theta$ radians.



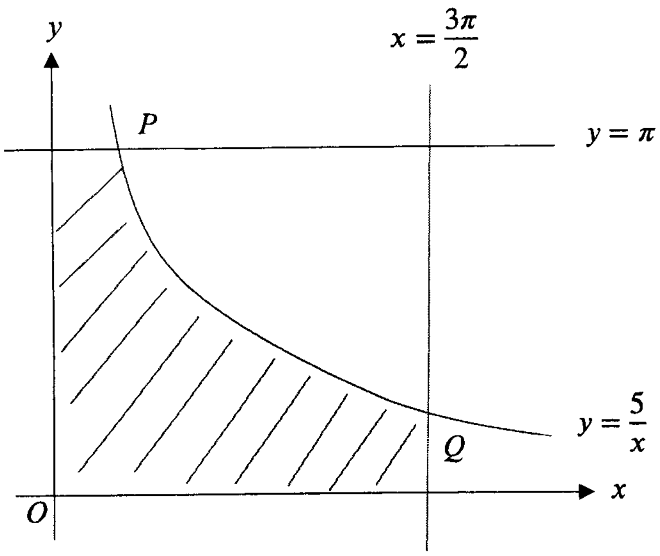
(i) Show that the area, A m^2, of trapezium *OPQR* is given by
$A = \frac{1}{4}x^2(\sin 2\theta + 2\cos\theta)$. [3]

(ii) Given that θ can vary, find

(a) the value of θ for which A has a stationary value. [3]

(b) the stationary value of A at $x = 10$ and determine whether it is a maximum or a minimum. [2]

10 The diagram shows part of the curve $y = \frac{5}{x}$, $y = \pi$ and $x = \frac{3\pi}{2}$. The curve $y = \frac{5}{x}$ cuts the line $y = \pi$ at P and cuts the line $x = \frac{3\pi}{2}$ at Q.



(i) Find the coordinates of P and of Q in terms of π. [2]

(ii) Find the area of the shaded region bounded by $y = \frac{5}{x}$, $y = \pi$, $x = \frac{3\pi}{2}$, the y-axis and the x-axis. [3]

(iii) Explain why the area of the shaded region is between 5 units^2 and $\frac{3\pi^2}{2}$ units^2. [1]

11 A particle, travelling in a straight line, has velocity, v cm/s, given by $v = 3t^2 - pt + 4$, where p is a constant and t seconds is the time after passing through a fixed point O. The particle comes to instantaneous rest when $t = 2$.

(i) Show that $p = 8$. [1]

(ii) Find the range of values of t during which the particle moves in the positive direction. [3]

(iii) Find the minimum velocity of the particle. [3]

(iv) Calculate the total distance travelled by the particle for the first 3 seconds. [3]

----- **End of Paper** -----

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