BP~167

GAN ENG SENG SCHOOL
Preliminary Examination 2024

CANDIDATE NAME	

CLASS | | | **INDEX NUMBER** | | |

ADDITIONAL MATHEMATICS
Paper 1

4049/01

27 August 2024
2 hours 15 minutes

Sec 4 Express/ 5 Normal (Academic)

Candidates answer on the Question Paper

No Additional Materials are required

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

	For Examiner's Use
Total	90

This paper consists of 21 printed pages (including the cover page).

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The equation of a curve is $y=-2x^2+3x+5$.

(a) Write $y=-2x^2+3x+5$ in the form $y=a(x-h)^2+k$, where a, h and k are constants. [3]

(b) Using your results in part (a), explain clearly why the maximum value of y is k when $x = h$. [2]

2 (a) Factorise $64-(x+1)^3$. [2]

(b) Hence, solve $64-(x+1)^3=15(3-x)$, expressing non-integer roots in surd form. [2]

3 Express $\dfrac{x+7}{(x^2-9)(x-3)}$ as the sum of 3 partial fractions. [6]

4 A curve has the equation $y = x^3 + kx^2 + kx + 8$. Find the set of values of k such that

(a) the curve is an increasing function, [3]

(b) the curve has exactly 1 stationary point. [2]

5 (a) Find the term independent of x in the expansion of $\left(2x^2+\frac{3}{x}\right)^9$. [3]

(b) (i) Find the first three terms of the expansion of $(2-3x)^5$ in ascending powers of x. [2]

(ii) Hence, find the value of n such that the coefficient of x in the expansion of $(2-3x)^5(1-3x)^n$ is -720. [3]

6 The gradient function of a curve $y = f(x)$ is given by $\frac{36}{(2x+1)^2}$ for $x > -\frac{1}{2}$.

The curve passes through the point $\left(\frac{1}{2}, 2\right)$.

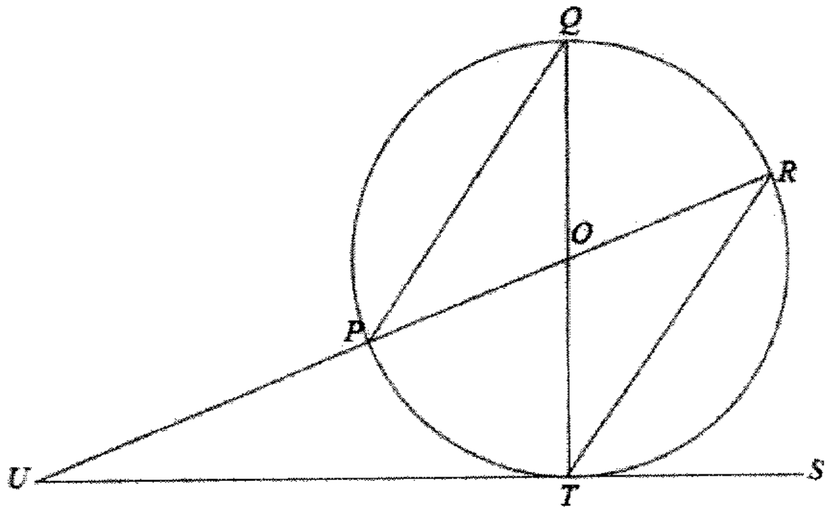
(a) Find the equation of the curve. [3]

(b) Find the angle, in degrees, that the tangent to the curve at $x = 1$ makes with the x-axis. [2]

7 The point P lies on the curve $y=\ln\left(\frac{x+1}{x-1}\right)$ for $x>1$. The normal to the curve at P is parallel to the line $2y=3x+2$. The equation of the tangent to the curve at P cuts the y axis at Q. Find the area of the triangle POQ, where O is the origin.

[9]

8

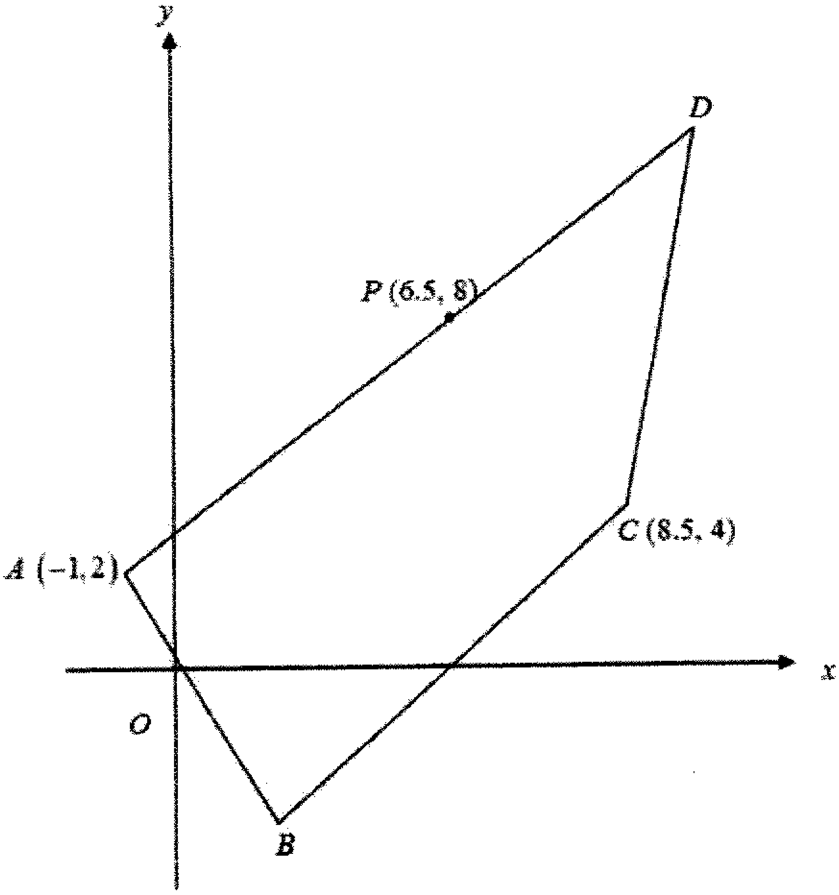


In the diagram, TQ and PR are the diameters of the circle with centre O.
The tangent at T meets RP produced at U. Prove that

(a) $TR = PQ$, [3]

(b) $QP \times UT = TP \times UR$. [3]

9 The diagram shows the quadrilateral $ABCD$ in which point A is $(-1, 2)$ and point C is $(8.5, 4)$. The point $P(6.5, 8)$ lies on AD such that $AP : PD = 3 : 2$. The midpoint of BC, point M lies on the x-axis and directly below point P.



(a) Find the coordinates of D, M and B. [5]

(b) Calculate the area of the quadrilateral $ABCD$. [2]

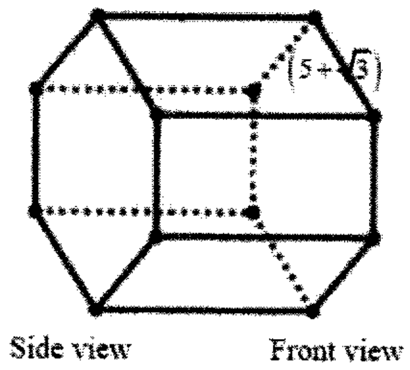
10 (a) Sketch, on the same diagram, the curves $y = 3\cos x$ and $y = 1 - 2\sin\left(\frac{2x}{3}\right)$ for $0 \le x \le 3\pi$. [3]

(b) Hence, find the number of distinct values of x for $0 \le x \le 3\pi$, for which

(i) $3\cos x + 2\sin\left(\frac{2x}{3}\right) = 1$. [1]

(ii) $3\cos 3x + 2\sin 2x = 1$. [1]

11 The diagram shows a prism in which its cross-section is a regular hexagon of side $(5+\sqrt{3})$ cm.



(a) Find an expression for the cross-sectional area of the prism in the form $q\sqrt{3}+p$ cm^2, where p and q are integers. [3]

(b) Given that the volume of the prism is $3\left(32\sqrt{3}+138\right)\text{cm}^3$, find the height of the prism in the form $\left(a\sqrt{3}+b\right)\text{cm}$, where a and b are integers. [3]

12 A particle passes a fixed point O and moves in a straight line such that, t s after leaving O, its velocity, v m/s, is given by $v=\dfrac{3}{t+2}-\dfrac{t+2}{3}$. The particle reaches its greatest distance from O at P.

(a) Find the time when the particle reaches P. [2]

(b) Find the distance travelled by the particle when $t = 2$. [5]

(c) Show that the velocity of the particle is decreasing at point P. [2]

13 (a) Given that $\log_a 125 - 3\log_a \sqrt{b} + \log_a c = 3$, express a in terms of b and c. [3]

(b) Show that $5^{n+1} - 4(5^n) + 5^{n-1}$ is divisible by 2 for all positive integers of n. [3]

14 A metal container of hot liquid is left to cool in a laboratory. The temperature of the liquid, T °C, after n minutes is given by $T = ae^{-kn} + 15$, where a and k are constants. The table below shows the measured values of T and n.

n (minutes)	10	20	30	40	50
T °C	66.9	46.5	35.1	27.2	22.4

(a) Plot $\ln(T - 15)$ against n and draw a straight line graph on page 21 to illustrate the information. [3]

(b) Use your graph to estimate

(i) the value of k, [2]

(ii) the number of minutes it takes for the temperature of the liquid to drop to 40% of its initial value. [3]

End of paper

GAN ENG SENG SCHOOL
PRELIM 2024 S4EXP_5NA AM1_OLP

Marking Scheme
2024 Sec 4 Express PRELIM AM Paper 1

1(a)	$y=-2x^2+3x+5$ $=-2\left[x^2-\frac{3}{2}x\right]+5$---M1 $=-2\left[\left(x-\frac{3}{4}\right)^2-\frac{9}{16}\right]+5$---M1 $=-2\left(x-\frac{3}{4}\right)^2+\frac{18}{16}+5$ $=-2\left(x-\frac{3}{4}\right)^2+\frac{49}{8}$---A1		
(b)	For all real values of x, $\left(x-\frac{3}{4}\right)^2\geq 0$---M1 $-2\left(x-\frac{3}{4}\right)^2\leq 0$ $-2\left(x-\frac{3}{4}\right)^2+\frac{49}{8}\leq\frac{49}{8}$---A1 $\therefore x=\frac{3}{4}, y_{max}=\frac{49}{8}$		
	Alternative method $y=-2\left(x-\frac{3}{4}\right)^2+k$ When $x=\frac{3}{4}$, $y=-2(0)+k$ $=k$ $\frac{dy}{dx}=-4(x-\frac{3}{4})$ When $\frac{dy}{dx}=0$---M1 $x=\frac{3}{4}$ $\frac{d^2y}{dx^2}=-4(<0)$---A1 $\therefore y$ is a maximum value, k		

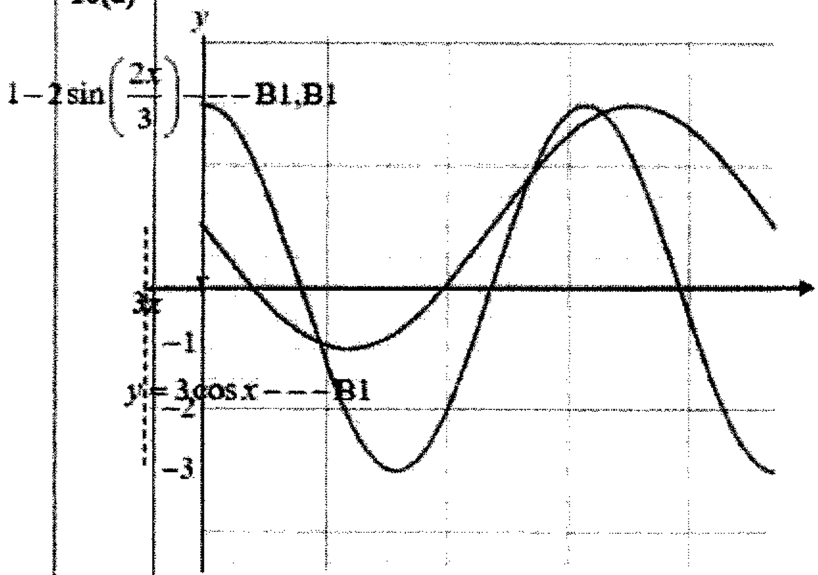
2(a)	$64-(x+1)^3$ $=4^3-(x+1)^3$ $=[4-(x+1)][4^2+4(x+1)+(x+1)^2]$ ---M1 $=(3-x)(16+4x+4+x^2+2x+1)$ $=(3-x)(x^2+6x+21)$ ---A1		
(b)	$(3-x)(x^2+6x+21)=15(3-x)$ $(3-x)(x^2+6x+21)-15(3-x)=0$ $(3-x)(x^2+6x+21-15)=0$ $(3-x)(x^2+6x+6)=0$ ---M1 $(3-x)=0$ or $(x^2+6x+6)=0$ $x=3$ or $x=\frac{-6\pm\sqrt{36-4(6)}}{2}$ $x=3$ or $x=\frac{-6\pm\sqrt{12}}{2}$ $x=3$ or $x=\frac{-6\pm2\sqrt{3}}{2}$ $x=3$ or $x=-3\pm\sqrt{3}$ ---A1,0		
3.	$\frac{x+7}{(x^2-9)(x-3)}$ $=\frac{x+7}{(x+3)(x-3)(x-3)}$ $\frac{x+7}{(x+3)(x-3)^2}=\frac{A}{(x+3)}+\frac{B}{(x-3)}+\frac{C}{(x-3)^2}$ ----M1 $A(x-3)^2+B(x+3)(x-3)+C(x+3)=x+7$ ------M1 Sub $x=3$ $6C=10$ $C=\frac{5}{3}$ ----A1 Sub $x=-3$ $36A=4$ $A=\frac{1}{9}$ ---A1 Sub $x=0$ $7=9A-9B+3C$ $9\left(\frac{1}{9}\right)-9B+3\left(\frac{5}{3}\right)=7$ $B=-\frac{1}{9}$ ---A1 $\frac{x+7}{(x-3)(x-3)(x+3)}=\frac{1}{9(x+3)}-\frac{1}{9(x-3)}+\frac{5}{3(x-3)^2}$ ----A1		

4(a)	$y = x^3 + kx^2 + kx + 8$ $\frac{dy}{dx} = 3x^2 + 2kx + k$ ---M1 Let $\frac{dy}{dx} > 0$ $3x^2 + 2kx + k > 0$ Let discriminant < 0 $(2k)^2 - 4(3)(k) < 0$ ---M1 $4k^2 - 12k < 0$ $k^2 - 3k < 0$ $k(k-3) < 0$ $0 < k < 3$ ---A1		
(b)	$\frac{dy}{dx} = 3x^2 + 2kx + k$ $\frac{dy}{dx} = 0$ $3x^2 + 2kx + k = 0$ Let discriminant = 0 $(2k)^2 - 4(3)(k) = 0$ $4k^2 - 12k = 0$ $k^2 - 3k = 0$ $k(k-3) = 0$ } M1 $k = 0$ or $k = 3$ ---A1		
5(a)	$\left(2x^2 + \frac{3}{x}\right)^9$ $T_{r+1} = \binom{9}{r}(2x^2)^{9-r}\left(\frac{3}{x}\right)^r$ $= \binom{9}{r}(2)^{9-r}(x^2)^{9-r}(3)^r x^{-r}$ $= \binom{9}{r}(2)^{9-r}(3)^r x^{18-3r}$ ---M1 Let $18 - 3r = 0$ $r = 6$ ---A1 $T_7 = \binom{9}{6}(2)^3(3)^6$ $= 489888$ ---A1		

5(b)	$(2-3x)^5$ $=2^5+\binom{5}{1}(2)^4(-3x)+\binom{5}{2}(2)^3(-3x)^2+....$ $=32-240x+720x^2+....$ -----B2,1,0		
(c)	$(2-3x)^5(1-3x)^n$ $=(32-240x+720x^2+...)(1-3x)^n$ $=(32-240x+720x^2+...)\left(1+\binom{n}{1}(-3x)+...\right)$---M1 Coefficient of $x=32\binom{n}{1}(-3x)-240x$---M1 $32\times n\times(-3x)-240x=-720x$ $-96nx-240x=-720x$ $-96n=-720+240$ $n=5$---A1		
6(a)	$\frac{dy}{dx}=36(2x+1)^{-2}$ $y=\int 36(2x+1)^{-2}\,dx$ $=36\times\frac{(2x+1)^{-1}}{-1(2)}+c$ (c : constant) $=\frac{-18}{(2x+1)}+c$---M1 Sub $\left(\frac{1}{2},2\right)$ $2=\frac{-18}{(1+1)}+c$---M1 $2=-9+c$ $c=11$ $y=\frac{-18}{(2x+1)}+11$---A1		
(b)	$\frac{dy}{dx}=36(2x+1)^{-2}$ At $x=1$ $\frac{dy}{dx}=\frac{36}{3^2}$ $=4$---M1 $\tan\theta=4$ $\theta=76.0^\circ$---A1		

7	$y=\ln\left(\frac{x+1}{x-1}\right)$ $y=\ln(x+1)-\ln(x-1)$ $\frac{dy}{dx}=\frac{1}{x+1}-\frac{1}{x-1}$ ---M1 $2y=3x+2$ $y=\frac{3}{2}x+1$ Gradient of normal at $A=\frac{3}{2}$ $\therefore$ Gradient of tangent = $-\frac{2}{3}$ ---M1 $\frac{1}{x+1}-\frac{1}{x-1}=-\frac{2}{3}$ $\frac{x-1-(x+1)}{(x+1)(x-1)}=\frac{-2}{3}$ $\frac{-2}{(x+1)(x-1)}=\frac{-2}{3}$ ---M1 $x^2-1=3$ $x^2=4$ $x=2$ or $x=-2$ $\therefore x=2$ ---A1 At $x=2$, $y=\ln\left(\frac{2+1}{2-1}\right)$ $y=\ln 3$ ---A1 $P(2,\ln 3)$ Equation of tangent $y=-\frac{2}{3}x+c$ Sub $(2,\ln 3)$ $\ln 3=-\frac{2}{3}(2)+c$ ---M1 $c=\ln 3+\frac{4}{3}$ $y=-\frac{2}{3}x+\ln 3+\frac{4}{3}$ ---A1		

	At $x=0$ $y=\ln 3+\frac{4}{3}$ $Q(0, \ln 3+\frac{4}{3})$ Area of triangle POQ $=\frac{1}{2}\times 2\times\left(\ln 3+\frac{4}{3}\right)$---M1 $=\ln 3+\frac{4}{3}$ $=2.43$---A1		
8(a)	$OQ=OP=OR=OT$ (radii of a circle)---M1 $\angle QOP=\angle ROT$ (vertically opp angles)---M1 ΔQOP is congruent to ΔROT (SAS congruency)---A1 Hence, $TR=PQ$ (Shown)		
(b)	$\angle TUR=\angle PUT$ (Common angle) $\angle PTU=\angle TRU$ (Tangent-chord theorem)---M1 ΔUTR is similar to ΔUPT (AA similarity)---M1 $\frac{TP}{RT}=\frac{UT}{UR}=\frac{PU}{TU}$ $RT=QP$ (From (i)) $\frac{TP}{QP}=\frac{UT}{UR}$---M1 $QP\times UT=TP\times UR$ (Shown)		
9(a)	$D\left(6.5+\frac{7.5}{3}\times 2, 8+\frac{6}{3}\times 2\right)$---M1 $=D(11.5,12)$---A1 $M(6.5,0)$---B1 $\left(\frac{x+8.5}{2},\frac{y+4}{2}\right)=(6.5,0)$---M1 $x=4.5, y=-4$ $B(4.5,-4)$---A1		

(b)	$A(-1,2), B(4.5,-4), C(8.5,4), D(11.5,12)$ Area $= \frac{1}{2}\begin{bmatrix} -1 & 4.5 & 8.5 & 11.5 & -1 \\ 2 & -4 & 4 & 12 & 2 \end{bmatrix}$ $= \frac{1}{2}\left[4+4.5(4)+8.5(12)+2(11.5)-(2(4.5)-4(8.5)+4(11.5)+12(-1))\right]$----M1 $= \frac{1}{2}(147-9)$ $= \frac{138}{2}$ $= 69$ units^2 -----A1		
10(a)	 B1 - $y = 3\cos x$, shape, 1.5 oscillations over 3π 3 and -3 to be marked on y axis π, 2π and 3π to be marked on x axis	**B1 Shape** **B1 Shifting curve vertically up by 1 unit**	
(b)(i)	3		
(b)(ii)	9		

11(a)	Area of cross-sectional area of prism $=6\times\frac{1}{2}\times\left(5+\sqrt{3}\right)^2\sin 60^\circ$ ---M1 $=3\times\left(25+10\sqrt{3}+3\right)\times\frac{\sqrt{3}}{2}$ $=3\left(28+10\sqrt{3}\right)\times\frac{\sqrt{3}}{2}$---M1 $=\frac{3}{2}\left(28\sqrt{3}+30\right)$ $=42\sqrt{3}+45$---A1		
(b)	Volume of the prism $=3\left(32\sqrt{3}+138\right)\text{ cm}^3$ $3\left(14\sqrt{3}+15\right)\times h=3\left(32\sqrt{3}+138\right)$ $\left(14\sqrt{3}+15\right)\times h=\left(32\sqrt{3}+138\right)$ $h=\frac{32\sqrt{3}+138}{14\sqrt{3}+15}\times\frac{14\sqrt{3}-15}{14\sqrt{3}-15}$---M1 $=\frac{32\sqrt{3}\left(14\sqrt{3}-15\right)+138\left(14\sqrt{3}-15\right)}{\left(14\sqrt{3}\right)^2-15^2}$ $=\frac{448(3)-480\sqrt{3}+1932\sqrt{3}-2070}{\left(14\sqrt{3}\right)^2-15^2}$---M1 $=\frac{1452\sqrt{3}-726}{363}$ $=4\sqrt{3}-2$---A1		
12(a)	$v=\frac{3}{t+2}-\frac{t+2}{3}$ Let $v=0$ $\frac{3}{t+2}-\frac{t+2}{3}=0$---M1 $(t+2)^2=9$ $t+2=3$ or $t+2=-3$ $t=1$ or $t=-5$(rej)---A1		

(b)	$s=\int \frac{3}{t+2}-\frac{t+2}{3}\,dt$ $=3\ln(t+2)-\frac{t^2}{6}-\frac{2}{3}t+c$ ---M1 At $t=0, s=0$ $3\ln(0+2)+c=0$ $c=-3\ln 2$ $s=3\ln(t+2)-\frac{t^2}{6}-\frac{2}{3}t-3\ln 2$ --A1		
(b)	At $t=1$ $s=3\ln 3-\frac{1}{6}-\frac{2}{3}-3\ln 2$ $=0.383$ m ----A1 At $t=2$ $s=3\ln 4-\frac{4}{6}-\frac{4}{3}-3\ln 2$ $=0.0794$ ---A1 Distance travelled $=0.383+(0.383-0.0794)$ $=0.687$ m ---A1		
(c)	$a=\frac{dv}{dt}$ $=\frac{-3}{(t+2)^2}-\frac{1}{3}$ ---M1 At $t=1$ $a=-\frac{3}{9}-\frac{1}{3}$ $=-\frac{2}{3}<0$ ---A1 Since $a<0$, the velocity of the particle is decreasing.		
13(a)	$\log_a 125-3\log_a\sqrt{b}+\log_a c=3$ $\log_a 125-\log_a b^{\frac{3}{2}}+\log_a c=3$ $\log_a \frac{125c}{b^{\frac{3}{2}}}=3$ ---M1 $a^3=\frac{125c}{b^{\frac{3}{2}}}$ ---M1 $a=\frac{5\sqrt[3]{c}}{\sqrt{b}}$ ---A1		

(b)	$5^{n+1}-4\left(5^{n}\right)+5^{n-1}$ $=5^{n-1}\left(5^{2}\right)-4\left(5^{n-1}\right)(5)+5^{n-1}$ ---M1 $=5^{n-1}\left(5^{2}-20+1\right)$ ---M1 $=6\left(5^{n-1}\right)$ ---A1 **Alternative Method** $5\times5^{n}-4\left(5^{n}\right)+\frac{5^{n}}{5}$ ---M1 $=\frac{1}{5}\left(25\left(5^{n}\right)-20\left(5^{n}\right)+5^{n}\right)$ $=\frac{6}{5}\left(5^{n}\right)$ ---M1 $=6\left(5^{n-1}\right)$ ---A1 Since the expression has a factor of 6 which is divisible by 2, hence $5^{n+1}-4\left(5^{n}\right)+5^{n-1}$ is divisible by 2 for all positive integers of n		
14	Refer to solution on the graph paper.		