Name

Reg. No Class

MAYFLOWER SECONDARY SCHOOL MAYFLOWER SECONDARY SCHOOL MAYFLOWER SECONDARY SCHOOL MAYFLOWER SECONDARY SCHOOL

4E/5N

ADDITIONAL MATHEMATICS

4049/01

Paper 1 [90 marks]

PRELIMINARY EXAMINATION

20 August 2024

2 hours 15 minutes

Candidates answer on the question paper.

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.
Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer ALL questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **90**.
Write the brand and model of your calculator in the space provided below.

Brand/Model of Calculator

For Examiner's Use	
Total	90

This question paper consists of **16** printed pages and **2** blank pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

[Turn over

1 Given that $4(5^{x+3}) = 20^{3-x}$, evaluate 10^x **without using a calculator**. [4]

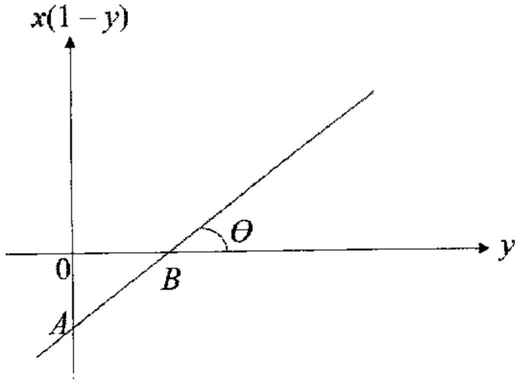
2 Solve the equations.

(a) $\log_2(x+4) = 2\log_2 x - 1$ [4]

(b) $10\log_y 5 + 3 = \log_5 y$ [4]

[Turn over

3 The variables x and y are related by $y = \dfrac{2x+8}{2x+1}$. When values of $x(1-y)$ are plotted against y, a straight line is obtained. The straight line intersects the vertical and horizontal axes at A and B respectively.



(i) Find the coordinates of A and of B. [4]

(ii) State the value of $\tan\theta$. [1]

[Turn over

4 (i) Factorise completely $2x^3 - 3x^2 - 5x + 6$. [4]

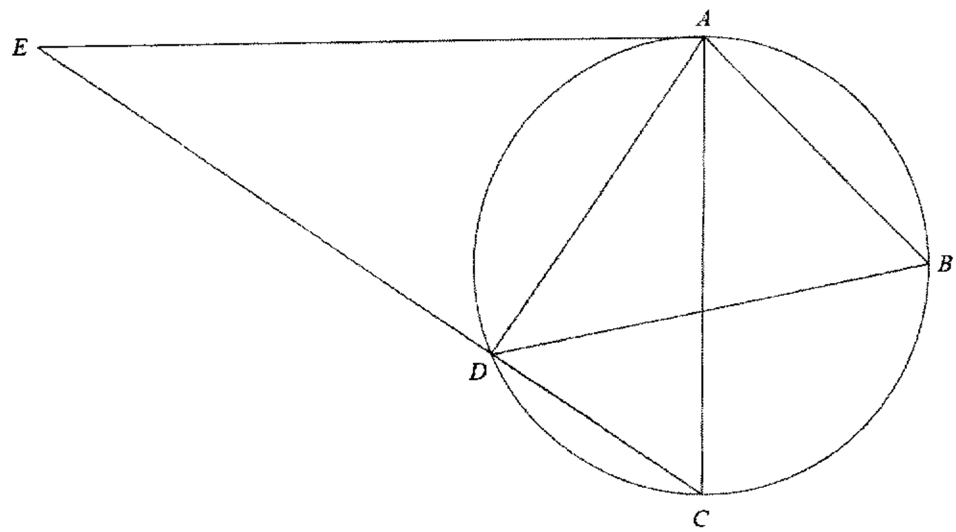
(ii) **Hence**, solve $2e^{3y} - 3e^{2y} - 5e^y + 6 = 0$. [3]

5 (i) Prove that $\frac{2-\sec^2\theta}{\sec\theta(\sin\theta+\cos\theta)} = 1-\tan\theta$. [4]

(ii) Hence solve the equation $\frac{4-2\sec^2\theta}{\sec\theta(\sin\theta+\cos\theta)} = \sec^2\theta - 2$ for $0 \le \theta \le 2\pi$. [5]

6 The first two non-zero terms in the expansion of $(1+bx)(1+ax)^6$ in ascending powers of x are 1 and $-\frac{21}{4}x^2$. Find the value of each of the constants a and b, where $a<b$. [7]

7

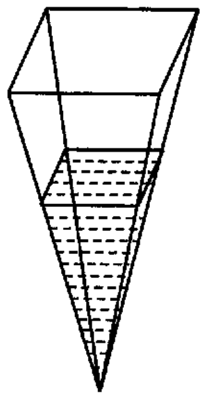


The diagram shows a circle passing through the points A, B, C and D. AC is a diameter of the circle. The line EA is a tangent to the circle and it intersects the straight line EDC at E.

(i) Show that angle AED = angle DAC. [2]

(ii) Show that $AD^2 = CD \times DE$. [4]

8 A vessel in the shape of an inverted right pyramid has a square base of side 12 cm and a height of 30 cm. Water is leaking from the vessel at a constant rate of 5 cm^3/s.



(i) Show that the volume of water in the vessel, V cm^3, is given by $V = \frac{4h^3}{75}$, where h is the depth of the water. [2]

(ii) Find the rate of change of the depth of water when the water is 6 cm deep. [3]

9 f(x) is such that $f'(x)=\sin\frac{1}{4}x-\cos 4x$. Given that $f(2\pi)=1$, show that

$16f''(x)+f(x)=a\sin 4x+b$, where a and b are constants. [6]

10 (a) (i) Find the range of values of the constant m for which the curve $y = x^2 - 5x + m$ meets the line $y = mx - 8$. [4]

(ii) Hence state the values of m for which the line is a tangent. [1]

(b) Given that $px^2 + 5x - q$ is always positive, what conditions must apply to the constants p and q? [3]

11 **(i)** Express $\frac{-x^2+2x+1}{(x-1)(x^2+1)}$ in the form $\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$ where A, B and C are constants.

[4]

(ii) Differentiate $\ln(x^2+1)$ with respect to x. [1]

(iii) Using your results from parts **(i)** and **(ii)**, find $\int \frac{-2x^2+4x+2}{(x-1)(x^2+1)}\,\mathrm{d}x$. [2]

[Turn over

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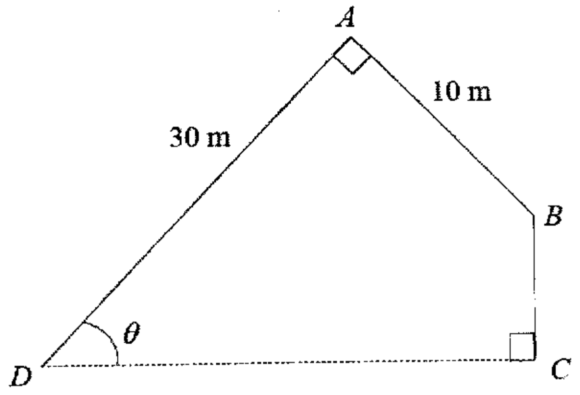
12



The diagram shows an area that is enclosed by glass panels at *AB*, *BC* and *AD*. $AB = 10$ m, $AD = 30$ m, angle DAB = angle $BCD = 90°$. The glass panel *AD* makes an acute angle θ with *CD*.

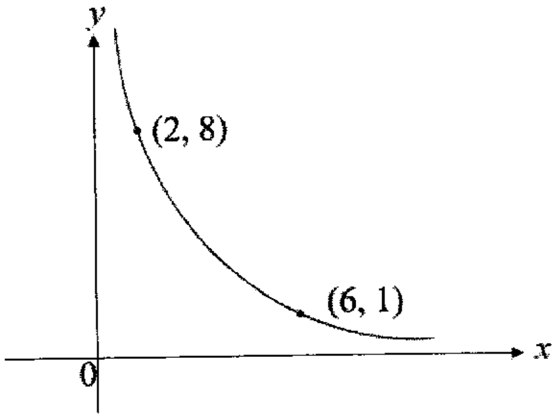
(i) Show that L m, the length of the glass panels, can be expressed as $40 + 30\sin\theta - 10\cos\theta$. [2]

(ii) Express L in the form $40 + R\sin(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) The total length of the glass panels is 65 m. Find the value of θ. [2]

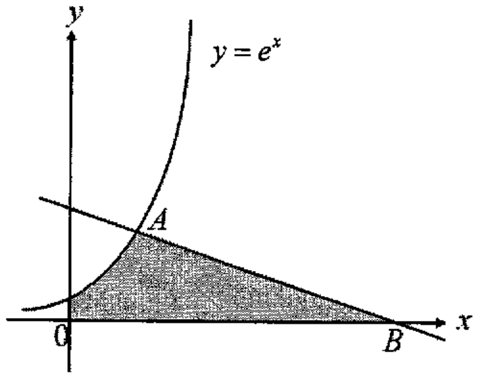
(iv) Explain whether it is possible to build an area where the length of the glass panels is 90 m. [1]

13 (a) The figure shows part of the curve $y = g(x)$. (2, 8) and (6, 1) are two points on the curve.



Given that $\int_2^6 y \, dx = 32$, find the value of $\int_1^8 x \, dy$. [2]

(b)



The diagram shows part of the curve $y = e^x$. The normal to the curve at point A where $x = 1$ intersects the x-axis at point B. Find the area of the shaded region.
Leave your answer in **exact form**. [7]

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Sec 4 Express A Math Prelim Paper 1 2024 Marking Scheme

1 Given that $4(5^{x+3}) = 20^{3-x}$, evaluate 10^x **without using a calculator**. [4]

	$4(5^{x+3}) = 20^{3-x}$ $4(5^x)(5^3) = \frac{20^3}{20^x}$ $5^x(20^x) = \frac{20^3}{4(5^3)}$ $100^x = 16$ $10^x = 4$

2 Solve the equations.

(a) $\log_2(x+4) = 2\log_2 x - 1$ [4]

(b) $10\log_y 5 + 3 = \log_5 y$ [4]

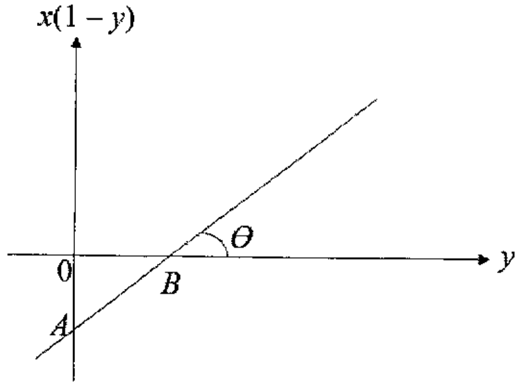
a	$\log_2(x+4) = 2\log_2 x - 1$ $2\log_2 x - \log_2(x+4) = 1$ $\log_2 x^2 - \log_2(x+4) = 1$ $\log_2 \frac{x^2}{x+4} = 1$ $\frac{x^2}{x+4} = 2^1$ $x^2 - 2x - 8 = 0$ $(x-4)(x+2) = 0$ $x = 4, \quad x = -2$ (rej)
b	$10\log_y 5 + 3 = \log_5 y$ $10\left(\frac{\log_5 5}{\log_5 y}\right) + 3 = \log_5 y$ $\frac{10}{\log_5 y} + 3 = \log_5 y$ Let $\log_5 y = u$ $\frac{10}{u} + 3 = u$ $10 + 3u = u^2$ $u^2 - 3u - 10 = 0$ $(u-5)(u+2) = 0$ $u = 5, \qquad u = -2$ $\log_5 y = 5 \qquad \log_5 y = -2$ $y = 5^5 \qquad y = 5^{-2}$ $y = 3125 \qquad y = \frac{1}{25}$

[Turn over

3 The variables x and y are related by $y=\frac{2x+8}{2x+1}$. When values of $x(1-y)$ are plotted against y, a straight line is obtained. The straight line intersects the vertical and horizontal axes at A and B respectively.

(i) Find the coordinates of A and of B. [4]

(ii) State the value of $\tan\theta$. [1]



i	$y=\frac{2x+8}{2x+1}$ $2xy+y=2x+8$ $2x-2xy=y-8$ $2x(1-y)=y-8$ $x(1-y)=\frac{1}{2}y-4$ $A=(0,-4)$ sub $x(1-y)=0$, $0=\frac{1}{2}y-4$ $y=8$ $B=(8,0)$
ii	$\tan\theta=\frac{1}{2}$

4 **(i)** Factorise completely $2x^3-3x^2-5x+6$. [4]

(ii) **Hence**, solve $2e^{3y}-3e^{2y}-5e^y+6=0$. [3]

i	$f(x)=2x^3-3x^2-5x+6$ $f(1)=2-3-5+6=0$ $(x-1)$ is a factor $x-1\overline{)2x^3-3x^2-5x+6}$ quotient $2x^2-x-6$ $-(2x^3-2x^2)$ $-x^2-5x+6$ $-(-x^2+x)$ $-6x+6$ $-(-6x+6)$ 0 $2x^3-3x^2-5x+6=(x-1)(2x^2-x-6)$ $=(x-1)(2x+3)(x-2)$
ii	$2e^{3y}-3e^{2y}-5e^y+6=0$ $e^y=1,\ e^y=-\frac{3}{2}(rej),\ e^y=2$ $y=0,\quad y=\ln 2$

5 (i) Prove that $\frac{2-\sec^2\theta}{\sec\theta(\sin\theta+\cos\theta)}=1-\tan\theta$. [4]

(ii) Hence solve the equation $\frac{4-2\sec^2\theta}{\sec\theta(\sin\theta+\cos\theta)}=\sec^2\theta-2$ for $0\le\theta\le 2\pi$. [5]

i	LHS $=\frac{2-\sec^2\theta}{\sec\theta(\sin\theta+\cos\theta)}$ $=\frac{2-(1+\tan^2\theta)}{\frac{1}{\cos\theta}(\sin\theta+\cos\theta)}$ $=\frac{1-\tan^2\theta}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\cos\theta}}$ $=\frac{1-\tan^2\theta}{\tan\theta+1}$ $=\frac{(1+\tan\theta)(1-\tan\theta)}{\tan\theta+1}$ $=1-\tan\theta$ $=$ RHS OR LHS $=\frac{2-\sec^2\theta}{\sec\theta(\sin\theta+\cos\theta)}$ $=\left(2-\frac{1}{\cos^2\theta}\right)\div[\frac{1}{\cos\theta}(\sin\theta+\cos\theta)]$ $=\frac{2\cos^2\theta-1}{\cos^2\theta}\times\frac{\cos\theta}{\sin\theta+\cos\theta}$ $=\frac{2\cos^2\theta-1}{\cos\theta(\sin\theta+\cos\theta)}$ $=\frac{2\cos^2\theta-(\sin^2\theta+\cos^2\theta)}{\cos\theta(\sin\theta+\cos\theta)}$ $=\frac{\cos^2\theta-\sin^2\theta}{\cos\theta(\sin\theta+\cos\theta)}$ $=\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\cos\theta(\sin\theta+\cos\theta)}$ $=\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\cos\theta(\sin\theta+\cos\theta)}$ $=\frac{\cos\theta-\sin\theta}{\cos\theta}$ $=1-\frac{\sin\theta}{\cos\theta}$ $=1-\tan\theta$ $=$ RHS

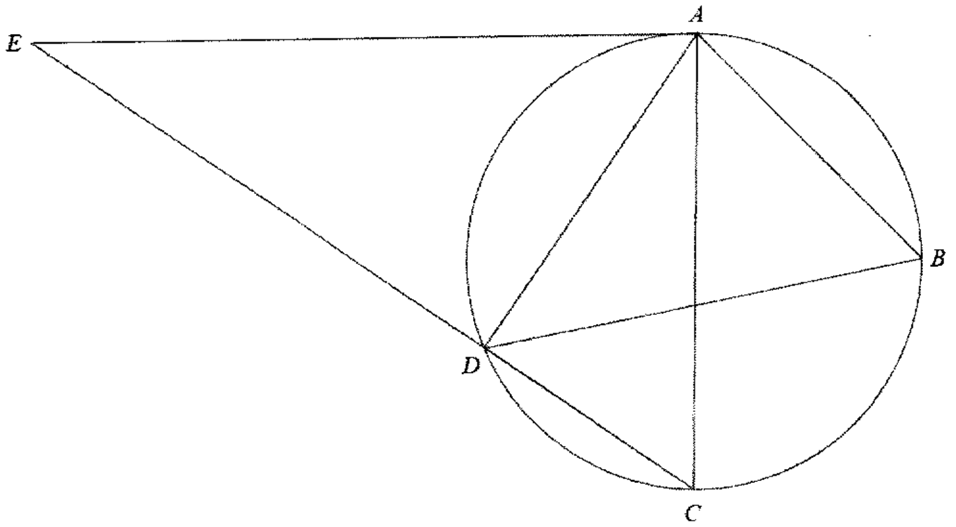
[Turn over

ii	$\dfrac{4-2\sec^2\theta}{\sec\theta(\sin\theta+\cos\theta)}=\sec^2\theta-2$ $2(1-\tan\theta)=\sec^2\theta-2$ $2-2\tan\theta=1+\tan^2\theta-2$ $\tan^2\theta+2\tan\theta-3=0$ $(\tan\theta+3)(\tan\theta-1)=0$ $\tan\theta=-3,$ $\tan\theta=1$ $\alpha=1.2490$ $\alpha=\dfrac{\pi}{4}$ $\theta=\pi-1.2490, 2\pi-1.2490$ $\theta=\dfrac{\pi}{4},\pi+\dfrac{\pi}{4}$ $\theta=1.8925, 5.0341$ $\theta=\dfrac{\pi}{4},\dfrac{5\pi}{4}$ $\theta=1.89, 5.03$ (3sf)

6 The first two non-zero terms in the expansion of $(1+bx)(1+ax)^6$ in ascending powers of x are 1 and $-\frac{21}{4}x^2$. Find the value of each of the constants a and b, where $a<b$. [7]

	$(1+ax)^6 = 1^6 + \binom{6}{1}(1^5)(ax) + \binom{6}{2}(1^4)(ax)^2 + \ldots$ $= 1 + 6ax + 15a^2x^2 + \ldots$ $(1+bx)(1+ax)^6 = (1+bx)(1+6ax+15a^2x^2+\ldots)$ $= 1+6ax+bx+15a^2x^2+6abx^2+\ldots$ $6a+b=0$ $b=-6a$ ---(1) $15a^2+6ab=-\frac{21}{4}$ ---(2) Sub (1) into (2): $15a^2+6a(-6a)=-\frac{21}{4}$ $a^2=\frac{1}{4}$ $a=-\frac{1}{2}$, $a=\frac{1}{2}$ (rej) $b=3$, $b=-3$ (rej)

7



The diagram shows a circle passing through the points A, B, C and D. AC is a diameter of the circle. The line EA is a tangent to the circle and it intersects the straight line EDC at E.

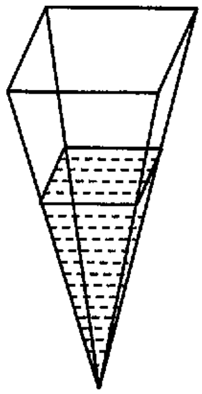
(i) Show that angle AED = angle DAC. [2]

(ii) Show that $AD^2 = CD \times DE$. [4]

i	Let $\angle AED = \theta$ $\angle EAD = 180 - 90 - \theta$ $= 90 - \theta$ ($\angle$ in semicircle or sum of $\angle$s in a Δ) $\angle DAC = 90 - (90 - \theta)$ $= \theta$ (tangent $\perp$ radius) $= \angle AED$
ii	$\angle EAD = \angle ACD$ ($\angle$s in alternate segment) $\angle ADE = \angle CDA = 90°$ ($\angle$ in semicircle) $\angle DEA = \angle DAC$ (sum of $\angle$s in a Δ) ΔDEA similar to ΔDAC (AAA) $\frac{DE}{DA} = \frac{EA}{AC} = \frac{DA}{DC}$ $\frac{DE}{DA} = \frac{DA}{DC}$ $AD^2 = CD \times DE$ OR $\angle DEA = \angle DAC$ (from i) $\angle ADE = \angle CDA = 90°$ ($\angle$ in semicircle) $\angle EAD = \angle ACD$ (sum of $\angle$s in a Δ) ΔDEA similar to ΔDAC (AAA)

[Turn over

8 A vessel in the shape of an inverted right pyramid has a square base of side 12 cm and a height of 30 cm. Water is leaking from the vessel at a constant rate of 5 cm^3/s.



(i) Show that the volume of water in the vessel, V cm^3, is given by $V = \frac{4h^3}{75}$, where h is the depth of the water. [2]

(ii) Find the rate of change of the depth of water when the water is 6 cm deep. [3]

i	Let x be the length of the side of the water surface. $\frac{x}{12} = \frac{h}{30}$ $x = \frac{2h}{5}$ $V = \frac{1}{3}(\frac{2h}{5})^2(h)$ $V = \frac{4h^3}{75}$
ii	$V = \frac{4h^3}{75}$ $\frac{dV}{dh} = \frac{4}{25}h^2$ $\frac{dV}{dh} = \frac{dV}{dt} \div \frac{dh}{dt}$ $\frac{4}{25}(6)^2 = -5 \div \frac{dh}{dt}$ $\frac{dh}{dt} = -\frac{125}{144}$ cm/s

9 f(x) is such that $f'(x)=\sin\frac{1}{4}x-\cos 4x$. Given that $f(2\pi)=1$, show that

$16f''(x)+f(x)=a\sin 4x+b$, where a and b are constants. [6]

	$f(x)=-4\cos\frac{1}{4}x-\frac{1}{4}\sin 4x+c$ Sub $f(2\pi)=1$, $-4\cos\frac{2\pi}{4}-\frac{1}{4}\sin 8\pi+c=1$ $c=1$ $f(x)=-4\cos\frac{1}{4}x-\frac{1}{4}\sin 4x+1$ $f''(x)=\frac{1}{4}\cos\frac{1}{4}x+4\sin 4x$ $16f''(x)+f(x)$ $=16(\frac{1}{4}\cos\frac{1}{4}x+4\sin 4x)-4\cos\frac{1}{4}x-\frac{1}{4}\sin 4x+1$ $=4\cos\frac{1}{4}x+64\sin 4x-4\cos\frac{1}{4}x-\frac{1}{4}\sin 4x+\frac{1}{4}$ $=63\frac{3}{4}\sin 4x+1$

10 **(a)** **(i)** Find the range of values of the constant m for which the curve $y = x^2 - 5x + m$ meets the line $y = mx - 8$. [4]

(ii) Hence state the values of m for which the line is a tangent. [1]

(b) Given that $px^2 + 5x - q$ is always positive, what conditions must apply to the constants p and q? [3]

ai	$y = x^2 - 5x + m$ --- (1) $y = mx - 8$ --- (2) $(1) = (2)$: $x^2 - 5x + m = mx - 8$ $x^2 - 5x - mx + m + 8 = 0$ $x^2 - (5+m)x + m + 8 = 0$ $b^2 - 4ac \geq 0$ $[-(5+m)]^2 - 4(1)(m+8) \geq 0$ $25 + 10m + m^2 - 4m - 32 \geq 0$ $m^2 + 6m - 7 \geq 0$ $(m-1)(m+7) \geq 0$ $m \leq -7$, $m \geq 1$
aii	$m = -7$, $m = 1$
b	$px^2 + 5x - q$ always positive: $p > 0$ $5^2 - 4p(-q) < 0$ $25 + 4pq < 0$ $pq < -\frac{25}{4}$

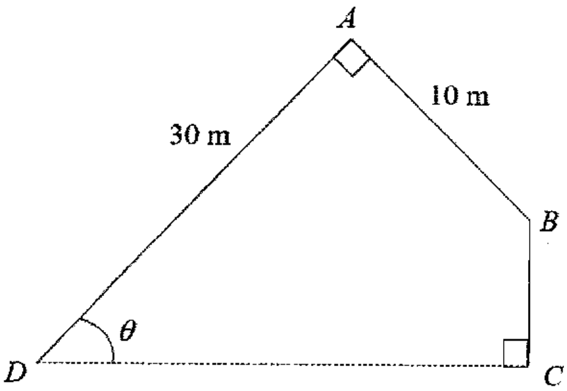
11 **(i)** Express $\frac{-x^2+2x+1}{(x-1)(x^2+1)}$ in the form $\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$ where A, B and C are constants. [4]

(ii) Differentiate $\ln(x^2+1)$ with respect to x. [1]

(iii) Using your results from parts **(i)** and **(ii)**, find $\int \frac{-2x^2+4x+2}{(x-1)(x^2+1)}\,dx$. [2]

i	$\frac{-x^2+2x+1}{(x-1)(x^2+1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$ $-x^2+2x+1=A(x^2+1)+(Bx+C)(x-1)$ Sub $x=1$, $2=A(2)+0$ $A=1$ Sub $x=0$, $1=1+C(-1)$ $C=0$ Sub $x=2$, $1=5+(2B)(1)$ $B=-2$ $\frac{-x^2+2x+1}{(x-1)(x^2+1)}=\frac{1}{x-1}-\frac{2x}{x^2+1}$
ii	$\frac{d}{dx}\ln(x^2+1)=\frac{2x}{x^2+1}$
iii	$\int\frac{-2x^2+4x+2}{(x-1)(x^2+1)}\,dx=2\int\frac{1}{x-1}-\frac{2x}{x^2+1}\,dx$ $=2[\ln(x-1)-\ln(x^2+1)]+c$

12



The diagram shows an area that is enclosed by glass panels at *AB*, *BC* and *AD*. $AB = 10$ m, $AD = 30$ m, angle DAB = angle $BCD = 90°$. The glass panel *AD* makes an acute angle θ with *CD*.

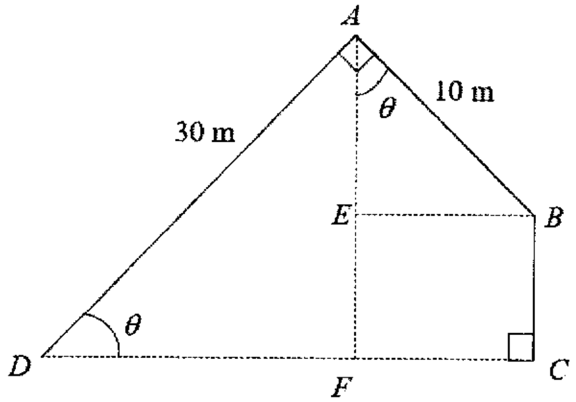
(i) Show that *L* m, the length of the glass panels, can be expressed as $40 + 30\sin\theta - 10\cos\theta$. [2]

(ii) Express *L* in the form $40 + R\sin(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) The total length of the glass panels is 65 m. Find the value of θ. [2]

(iv) Explain whether it is possible to build an area where the length of the glass panels is 90 m. [1]

i



$$\sin\theta = \frac{AF}{30}$$
$$AF = 30\sin\theta$$

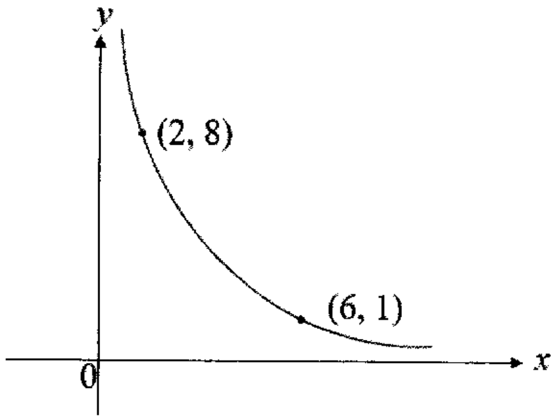
$$\cos\theta = \frac{AE}{10}$$
$$AE = 10\cos\theta$$

$$L = 30 + 10 + 30\sin\theta - 10\cos\theta$$
$$L = 40 + 30\sin\theta - 10\cos\theta$$

[Turn over

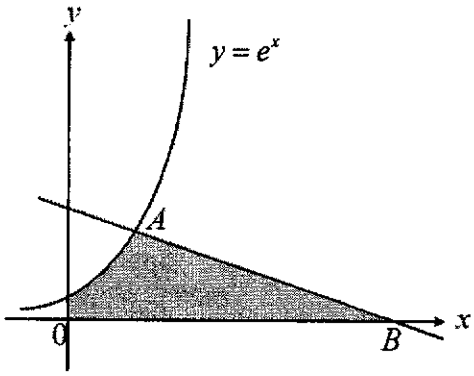
ii	$R=\sqrt{30^2+10^2}$ $R=\sqrt{1000}$ $R=10\sqrt{10}$ $\alpha=\tan^{-1}\dfrac{10}{30}$ $\alpha=18.434$ $L=40+10\sqrt{10}\sin(\theta-18.434)$ $L=40+10\sqrt{10}\sin(\theta-18.4)$ (1dp)
iii	$40+10\sqrt{10}\sin(\theta-18.434)=65$ $\sin(\theta-18.434)=\dfrac{25}{10\sqrt{10}}$ $\alpha=52.238$ $\theta-18.434=52.238$ $\theta=70.672$ $\theta=70.7°$ (1dp)
iv	Since the maximum value of $\underline{40+10\sqrt{10}\sin(\theta-18.434)=40+10\sqrt{10}<90}$, it is <u>not possible</u> to build the area. OR $40+10\sqrt{10}\sin(\theta-18.434)=90$ $\sin(\theta-18.434)=1.5811$ Since $\underline{\sin(\theta-18.434)\le 1}$, it is <u>not possible</u> to build the area.

13 **(a)** The figure shows part of the curve $y = g(x)$. (2, 8) and (6, 1) are two points on the curve.



Given that $\int_2^6 y \, dx = 32$, find the value of $\int_1^8 x \, dy$. [2]

(b)



The diagram shows part of the curve $y = e^x$. The normal to the curve at point A where $x = 1$ intersects the x-axis at point B. Find the area of the shaded region.
Leave your answer in **exact form.** [7]

i	$\int_2^8 x \, dy = 32 - (4\times1) + (7\times2)$ $= 42$
ii	$y = e^x$ $\frac{dy}{dx} = e^x$ $m_{normal} = -\frac{1}{e}$ Subs $(1,e), m_{normal} = -\frac{1}{e}$, $e = -\frac{1}{e}(1) + c$ $c = e + \frac{1}{e}$ Normal: $y = -\frac{1}{e}x + e + \frac{1}{e}$ Sub $y = 0$, $-\frac{1}{e}x + e + \frac{1}{e} = 0$ $\frac{1}{e}x = e + \frac{1}{e}$ $x = e^2 + 1$ $\int_0^1 e^x \, dx = [e^x]_0^1$ $= e - 1$ Area of triangle $= \frac{1}{2}(e^2 + 1 - 1)(e)$ $= \frac{1}{2}e^3$ Area of region $= \frac{1}{2}e^3 + e - 1$

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