BP~469

NAME		INDEX NO.		CLASS	

NORTHLAND SECONDARY SCHOOL
PRELIMINARY EXAMINATION
Secondary 4 Express / 5 Normal Academic

ADDITIONAL MATHEMATICS

4049/02

Paper 2

28 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

For Examiner's Use
90

This document consists of **19** printed pages and **1** blank page.

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1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 Express $\dfrac{1-8x-3x^2}{(x-1)(2x^2+3)}$ in partial fractions. [5]

2 (a) Find the smallest integer c for which the line $y = 7x + c$ intersects the curve $y = 2x^2 - 4$ at two distinct points. [4]

(b) Find the range of values of k for which the curve $y = kx^2 + 2(2k-5)x + 9k$ lies below the x-axis. [4]

3 (a) Prove the identity $\dfrac{1-\cos x}{\sin x - \operatorname{cosec} x + \cot x} = \tan x$. [4]

(b) Hence solve the equation $\dfrac{1-\cos 2x}{\sin 2x-\operatorname{cosec} 2x+\cot 2x}=-3$ for $0°<x<180°$. [3]

(c) Show that there are no solutions to the equation $\dfrac{1-\cos x}{\sin x-\operatorname{cosec} x+\cot x}=\tan 2x$ for $0°<x<180°$. [2]

4 **(a)** The equation $\log_2 x + \log_8 x = \log_4 2$ has the solution $x = 2^m$. Find the value of m. [4]

(b) Sketch the graph of $y = \log_3 x$. [2]

(c) Explain why the equation $\log_5(2x-11)-\log_5(x-4)=1$ has no real solutions. [4]

5 **(a)** The function f is defined by $f(x)=e^{x^2+x}$ where $x>0$. Explain, with working, whether f is an increasing or a decreasing function. [3]

(b) The equation of a curve is $y = \frac{x^2}{x+3}$.

(i) Find an expression for $\frac{dy}{dx}$ and obtain the coordinates of the stationary points of the curve. [5]

(ii) Find an expression for $\frac{d^2y}{dx^2}$ and hence determine the nature of these stationary points. [3]

6 The points P and Q both lie on a circle and have coordinates $(2, 7)$ and $(-6, 1)$ respectively. The line with equation $y+2x+4=0$ is a normal to the circle.

(a) Find the equation of the perpendicular bisector of PQ. [4]

(b) Find the equation of the circle. [5]

(c) Find the exact value of the coordinates of the point on the circle which is furthest from the y-axis. [2]

7 A particle moves in a straight line, such that its velocity, v m/s, t seconds after passing a fixed point O, is given by $v = -0.3(4-t)^2 + 1.2$.

(a) Find the acceleration of the particle when it first comes to instantaneous rest. [4]

After 1 second, its displacement, s m, is 2.5 m from O.

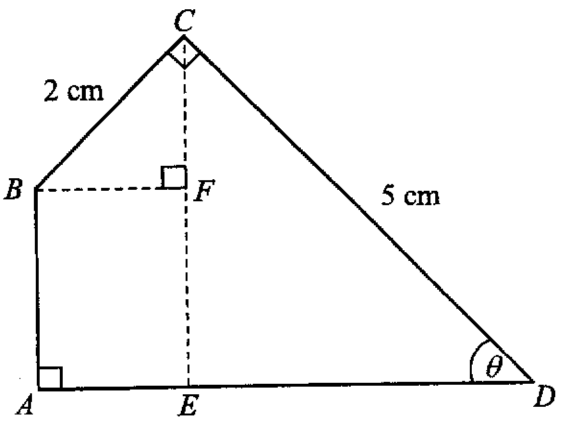
(b) Find the initial displacement of the particle. [3]

(c) Explain clearly why the total distance travelled by the particle in the interval $t = 0$ to $t = 7$ is **not** obtained by finding the value of s when $t = 7$. [2]

(d) Find the total distance travelled by the particle in the interval $t = 0$ to $t = 7$. [3]

8 **(a)** On the same axes, sketch, for $0 \le x \le 2\pi$, the graphs of $y = \sin 2x$ and $y = 1 - 3\cos x$. [3]

(b)



The diagram shows a quadrilateral in which CFE and AED are straight lines. $BC = 2$ cm and $CD = 5$ cm. Angle BCD = angle BFC = angle $BAE = 90°$. Angle $CDE = \theta$, where θ is an angle in degree.

(i) Show that the perimeter of the quadrilateral is $3\cos\theta + 7\sin\theta + 7$. [3]

(ii) Express $3\cos\theta + 7\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is acute. [3]

The perimeter of the quadrilateral is 13 cm.

(iii) Find the angle θ and hence find the total area of the quadrilateral. [3]

9 **(a)** Express $\frac{3x}{3x+1}$ in the form $a+\frac{b}{3x+1}$ where a and b are constants, and hence find $\int \frac{3x}{3x+1}\,\mathrm{d}x$. [4]

(b) Given that $y = x\ln(3x+1)$, find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(c) Using the results from parts **(a)** and **(b)**, show that $\int_0^4 \ln(3x+1)\,\mathrm{d}x = a\ln 13 + b$, where a and b are constants to be found. [5]

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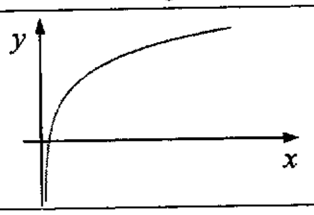
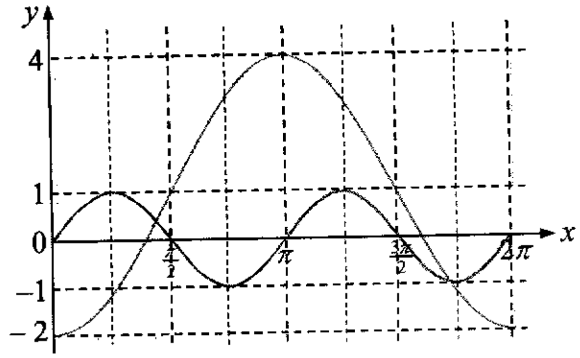
Mark Scheme for 2024 S4E5N Add Math Prelim **Paper 2**				
1	$\frac{1-8x-3x^2}{(x-1)(2x^2+3)} = \frac{A}{x-1} + \frac{Bx+C}{2x^2+3}$	M1		Realising the form of the partial fractions
	$1-8x-3x^2 = A(2x^2+3)+(x-1)(Bx+C)$	M1		Realising the need to eliminate denominator
	When $x=1$, $1-8(1)-3(1)^2 = A[2(1)^2+3]$			
	$-10=5A \rightarrow A=-2$			
	When $x=0$, $1=-2[2(0)^2+3]+(0-1)(C)$			
	$1=-6-C \rightarrow C=-7$			
	When $x=2$, $1-8(2)-3(2)^2=-2[2(2)^2+3]+[2B-7]$			
	$-27=-22+2B-7 \rightarrow B=1$			
	$\frac{-2}{x-1}+\frac{x-7}{2x^2+3}$	A3, 2, 1		-1 for each error inc. final answer
			[5]	
2a	$2x^2-7x-4-c=0$	M1		Eliminates y or x
	$b^2-4ac=(-7)^2-4(2)(-4-c)$	M1		Uses the discriminant
	$49+32+8c>0$			
	$c>-10.125$	A1		Accept $c>-\frac{81}{8}$
	smallest integer $c=-10$	A1		
2b	$y=kx^2+2(2k-5)x+9k$			
	$b^2-4ac=(4k-10)^2-4(k)(9k)$	M1		Uses the discriminant
	$16k^2-80k+100-36k^2<0$			
	$-20k^2-80k+100<0 \rightarrow k^2+4k-5>0$			
	$(k+5)(k-1)>0$			
	$k<-5$ or $k>1$	A2		A1 for each
	$k<0 \rightarrow k<-5$	B1		s.o.i. when $k>1$ rejected
			[8]	
3a	$\frac{1-\cos x}{\sin x-\operatorname{cosec} x+\cot x}$			
	$\frac{1-c}{s-\frac{1}{s}+\frac{1}{t}}$	B1		cosec and cot all correct
	$\frac{1-c}{\frac{s^2-1+c}{s}} \rightarrow \frac{s(1-c)}{s^2-1+c}$	M1		Correct algebra
	$\frac{s(1-c)}{-c^2+c}$	M1		Uses $s^2+c^2=1$
	$\frac{s(1-c)}{c(1-c)}=t$	A1		All correct
3b	$\tan 2x=-3$	M1		Uses part (a) and replaces x with $2x$
	$\alpha=\tan^{-1}(3)$	M1		For finding basic angle
	$2x=180°-\alpha,\ 360°-\alpha$			
	$x=54.2°, 144.2°$	A1		Accept 54.21… and 144.21…
3c	$t=\frac{2t}{1-t^2}$			

	$t-t^3=2t$			
	$t^3+t=0 \to t(t^2+1)=0$			
	$t=0$ (not valid for $0°<x<180°$)	B1		Must include and then reject $t=0$
	or $t^2=-1$ not possible. No solution	B1		Realises there is no solution to $t^2=-1$
	Alternative Answer:			
	Correct sketch of $y=\tan x$ and $y=\tan 2x$	B1		
	No point of intersection for $0°<x<180°$ and so no solution to the equation $\tan x=\tan 2x$	B1		
			[9]	
4a	$\log_2 x+\dfrac{\log_2 x}{\log_2 8}=\dfrac{\log_2 2}{\log_2 4}$	M1		Change of base
	$\log_2 x+\dfrac{\log_2 x}{3}=\dfrac{1}{2}$	B1		For '$=\frac{1}{2}$'
	$\log_2 x+\dfrac{1}{3}\log_2 x=\dfrac{1}{2}\to\log_2 x=\dfrac{3}{8}$	M1		Making $\log_2 x$ the subject
	$x=2^{\frac{3}{8}}\to m=\dfrac{3}{8}$	A1		
4b		B2		B1 for curvature of decreasing gradient observed B1 for graph close to asymptote observed with x-intercept 1
4c	$\log_5(\frac{2x-11}{x-4})=1$	M1		Combine to single log
	$\frac{2x-11}{x-4}=5$			
	$2x-11=5(x-4)\to x=3$	A1		
	If $x=3$, $\log_5(2x-11)=\log_5(-5)$ or $\log_5(x-4)=\log_5(-1)$	M1		Substitute into log expression
	Does not exist $\to$ No solutions	A1		Correct argument and conclusion
			[10]	
5a	$f'(x)=e^{x^2+x}(2x+1)$	B1		
	Since $x>0$, $2x+1>0$ and $e^{x^2+x}>0$	M1		Argues correctly
	$f'(x)>0 \to$ increasing	A1		$f'(x)>0$ or $e^{x^2+x}(2x+1)>0$ must be seen
5bi	$\dfrac{(x+3)(2x)-x^2(1)}{(x+3)^2}=\dfrac{x^2+6x}{(x+3)^2}$	B2, 1		B1 for unsimplified
	$\dfrac{x^2+6x}{(x+3)^2}=0\to x=0,\ x=-6$	M1		Sets to 0 and solves
	(0, 0) and (−6, −12)	A1 A1		SR1 answers not in coordinate form
5bii	$\dfrac{18}{(x+3)^3}$	B1		
	$x=0\to\dfrac{d^2y}{dx^2}=\dfrac{2}{3}>0 \to$ minimum point	DB1		Correct $\frac{d^2y}{dx^2}$ value and conclusion

	$x=-6 \to \frac{d^2y}{dx^2}=-\frac{2}{3}<0 \;\to$ maximum point	DB1		Correct $\frac{d^2y}{dx^2}$ value and conclusion
			[11]	
6a	$m_{PQ}=\frac{7-1}{2-(-6)}=\frac{3}{4}$	B1		
	$M_{PQ}=(\frac{2+(-6)}{2},\frac{7+1}{2})=(-2,\ 4)$	B1		
	Gradient of Perpendicular $=-\frac{4}{3}$	M1		Uses $m_1m_2=-1$
	$y-4=-\frac{4}{3}(x-(-2))$			
	$y=-\frac{4}{3}x+\frac{4}{3}$	A1		
6b	$-\frac{4}{3}x+\frac{4}{3}=-2x-4$	M1		Realises the need to use sim eqns
	$x=-8$			
	$(-8,12)$	A1		
	$\sqrt{(-8-2)^2+(12-7)^2}$	M1		Uses distance formula correctly with P or Q
	$\sqrt{125}$	A1		
	$(x+8)^2+(y-12)^2=125$	B1√		√ for their radius and centre
6c	$(-8-\sqrt{125},12)$	B2√		B1 for each, √ for their radius and centre
			[11]	
7a	$v=-0.3(4-t)^2+1.2$			
	$a=0.6(4-t)$	B1		
	When $v=0 \to (4-t)^2=4$	M1		Sets to 0
	$t=2,\ t=6$	A1		
	$a=0.6(4-2)=1.2\text{ m/s}^2$	A1		
7b	$s=\int -0.3(4-t)^2+1.2\ dt$	M1		Realises need to integrate
	$s=0.1(4-t)^3+1.2t\quad(+c)$	A1		
	When $t=1,\ s=2.5,\ c=-1.4 \to$ $s=0.1(4-t)^3+1.2t-1.4$			Use $t=1$ and $s=3.9$ to find c
	When $t=0, s=5\text{m}$	A1		
7c	Particle turned during $t=0$ to $t=7$	B1		
	When $t=7$, it only gives distance from O	B1		
7d	When $t=2,\ s=1.8$ or When $t=6,\ s=5$ OR distance $=\left\|\int_0^2 v\ dt\right\|=3.2\text{ m}$ or $\int_2^6 v\ dt=3.2\text{ m}$	M1		For finding displacement at either time it turns
	When $t=7,\ s=4.3$ OR distance $=\left\|\int_6^7 v\ dt\right\|=0.7\text{ m}$	M1		For finding displacement at ending time
	distance $=(5-1.8)+(5-1.8)+(5-4.3)=7.1\text{ m}$	A1		
			[12]	

8a		B3		B1 for two complete sine cycles with correct amplitude and intersections at *x*-axis B1 for one negative cosine cycle B1 for cosine cycle with correct amplitude and intersections at axis of rotation
8bi	$AB = CE - CF = 5\sin\theta - 2\cos\theta$	B1		
	$AD = ED + BF = 5\cos\theta + 2\sin\theta$	B1		
	$\text{Perimeter} = 5\sin\theta - 2\cos\theta + 5\cos\theta + 2\sin\theta + 7$ $\rightarrow 3\cos\theta + 7\sin\theta + 7$	B1		Answer was given – so all working must be correct
8bii	$R = \sqrt{3^2 + 7^2} = \sqrt{58}$	B1		
	$\tan\alpha = \frac{7}{3} \rightarrow \alpha = 66.8°$	M1		For finding α
	$\sqrt{58}\cos(\theta - 66.8°)$	A1		
8biii	$\sqrt{58}\cos(\theta - 66.8°) + 7 = 13$			
	$\cos(\theta - 66.8°) = \frac{6}{\sqrt{58}}$			
	$\alpha = 38.0°$	M1		For finding α
	$\theta - 66.8° = -\alpha,\ \alpha$			
	$\theta = 28.8°$, $104.8°$ (rejected)	A1		Accept 28.78…
	$6.75\ \text{cm}^2$	A1		
			[12]	
9a	$\frac{3x}{3x+1} = 1 - \frac{1}{3x+1}$	B1		
	$\int \frac{3x}{3x+1}\,dx = x - \frac{1}{3}\ln(3x+1) + c$	B3, 2, 1		−1 for each error in the integration – needs +c
9b	$\frac{dy}{dx} = \ln(3x+1) + x \times \frac{1}{3x+1} \times 3$	M1 B1 A1		Uses product formula. B1 '$\frac{1}{3x+1}$'. A1 '×3'
	$= \ln(3x+1) + \frac{3x}{3x+1}$			
9c	$\int \ln(3x+1) + \frac{3x}{3x+1}\,dx - \int \frac{3x}{3x+1}\,dx$ $\rightarrow x\ln(3x+1) - \int \frac{3x}{3x+1}\,dx$ OR $\ln(3x+1) = \frac{dy}{dx} - \frac{3x}{3x+1}$ $\rightarrow x\ln(3x+1) - \int \frac{3x}{3x+1}\,dx$	M1		Attempts to use the result of part (b)

	$x\ln(3x+1)-\int\frac{3x}{3x+1}\,dx$ $\rightarrow x\ln(3x+1)-\int 1-\frac{1}{3x+1}\,dx$	M1		Realises the need to use part (a)
	$[x\ln(3x+1)]_0^4-\left[x-\frac{1}{3}\ln(3x+1)\right]_0^4$	A1		All correct
	$[4\ln 13-0]-\left[4-\frac{1}{3}\ln 13\right]$	M1		Use definite integral formula on $x\ln(3x+1)$ or antiderivative from part (a)
	$\frac{13}{3}\ln 13-4$	A1		For both values
			[12]	