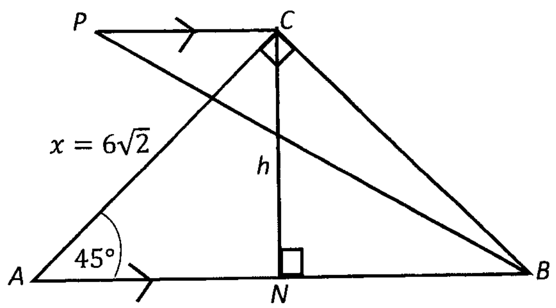
BMSS Maths Preliminary 4E Paper 1
2025 Marking Scheme
Total = 90 marks

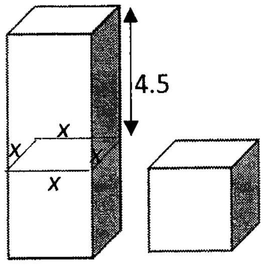
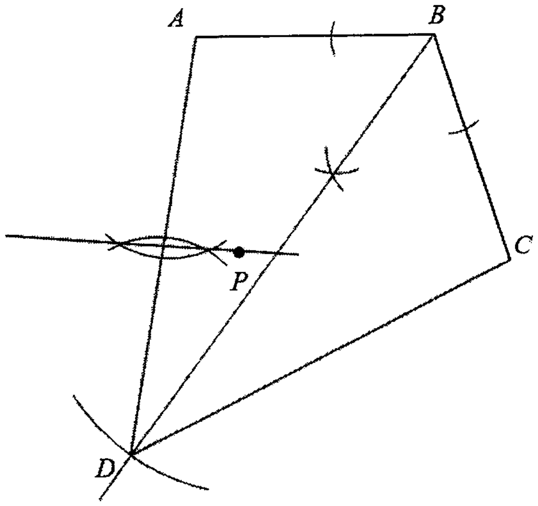
1		$\frac{21xy}{12} \div \frac{5x}{4y}$ $= \frac{21xy}{12} \times \frac{4y}{5x}$ $= \frac{7y^2}{5}$
2		$\left(\frac{5}{6}\right)^2$, 0.75 , $\frac{4}{5}$, $\frac{5}{6}$
3		$16x^2 - 9y^2$ $= (4x)^2 - (3y)^2$ $= (4x + 3y)(4x - 3y)$
4		$\frac{15.7^3}{8.64 - 2.76}$ = 658.145068 = 658.145 (first six digits)
5		The use of pictograms of different heights and sizes is misleading. Based on the height of the pictograms, some readers may interpret the annual cost of water in 2021 as \$400, half of the cost in 2024. Based on the size of the pictograms, some readers may interpret the annual cost of water in 2021 as \$200, a quarter of the cost in 2024. As such, this may lead to a misinterpretation of the graph.
6		$40 \times 7^4 + 9 \times 49^2 = 7^k$ $40 \times 7^4 + 9 \times \left(7^2\right)^2 = 7^k$ $40 \times 7^4 + 9 \times 7^4 = 7^k$ $49 \times 7^4 = 7^k$ $7^2 \times 7^4 = 7^k$ $7^{2+4} = 7^k$ $7^6 = 7^k$ $k = 6$
7	**(a)**	384×10^6 $= 3.84 \times 10^8$ m
	(b)	$\frac{3.84 \times 10^8}{102.75 \times 60 \times 60} = 1038.11841$ =1040 m/s (3sf)

8		Total amount after 6 years $= 15000\left(1+\frac{2.45}{100}\right)^6 = \$17344.54995$ Interest after 6 years $= 17344.54995 - 15000$ $= 2344.549953$ $= \$2344.55$
9	**(a)**	35000 cm = $35000 \div 100 \div 1000 = 0.35$ km Hence $n = 0.35$
	(b)	1 cm = 0.35 km 1 $cm^2 = 0.35^2 = 0.1225$ km^2 Area of lake on the map $= \frac{2.5}{0.1225}$ $= 20\frac{20}{49}$ cm^2
10	**(a)**	Area of parallelogram = $20 \times 27 = 540$ cm^2
	(b)	Perpendicular height of trapezium = $540 \div 30 = 18$ cm Area of trapezium $= \frac{1}{2} \times (20 + 30) \times 18$ $= \frac{1}{2} \times 50 \times 18$ $= 450$ cm^2
11		Original ratio = $2:5 = 10:25$ New ratio = $12:25$. 2 parts of lime cordial = 50 litres 1 part of lime cordial = 25 litres 5 parts of water were added to restore to the original ratio of $2:5$ or $12:30$, Amount of water that was added = $5 \times 25 = 125$ litres
12		$x^2 - 12x + 25 = 0$ $(x-6)^2 - 36 + 25 = 0$ $(x-6)^2 - 11 = 0$ $(x-6)^2 = 11$ $x - 6 = \sqrt{11}$ or $x - 6 = -\sqrt{11}$ $x = 6 + \sqrt{11}$ or $x = 6 - \sqrt{11}$ $x = 9.31662479$ or $x = 2.68337521$ $x = 9.32$ or $x = 2.68$

13	**(a)**	$(A \cap B') \cup (B \cap A')$
	(b)(i)	$A = \{1, 2, 5, 10\}$ $B = \{2, 3, 5, 7\}$ $A \cap B = \{2, 5\}$
	(b)(ii)	$B' = \{1, 4, 6, 8, 9, 10\}$ $A \cup B' = \{1, 2, 4, 5, 6, 8, 9, 10\}$ $\text{n}(A \cup B') = 8$
14	**(a)**	$\frac{25+21+18+p+17+12+15}{7} = 18$ $\frac{108+p}{7} = 18$ $108 + p = 126$ $p = 18$
	(b)	Standard deviation $= \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2}$ $= \sqrt{\frac{25^2 + 21^2 + 18^2 + 18^2 + 17^2 + 12^2 + 15^2}{7} - (18)^2}$ $= \sqrt{\frac{2372}{7} - (18)^2} = \sqrt{\frac{104}{7}} = 3.85$ (3sf)
	(c)	After three years, the mean age of the staff will increase by 3 years and reach 21 years. However, the standard deviation will remain unchanged.
15		$36x + 20y = 5760 --(1)$ $36x - 20y = 1800 --(2)$ (1) + (2), $36x + 20y + 36x - 20y = 5760 + 1800$ $72x = 7560$ $x = 105$ From (1), $36(105) + 20y = 5760$ $3780 + 20y = 5760$ $20y = 1980$ $y = 99$ The total number of dog walks = 105 + 99 = 204
16	**(a)**	The modal marks is 38.
	(b)	Middle position $= \frac{14+1}{2} = 7.5$ th Median $= \frac{34+36}{2} = 35$
	(c)	Lower quartile = 26 Upper quartile = 41 Interquartile range $= 41 - 26 = 15$

17	**(a)**	$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \begin{pmatrix}-2\\3\end{pmatrix} - \begin{pmatrix}4\\5\end{pmatrix} = \begin{pmatrix}-6\\-2\end{pmatrix}$
	(b)	Magnitude of $\overrightarrow{PQ}$ $= \sqrt{(-6)^2 + (-2)^2} = \sqrt{40}$ $= 6.32455532$ $= 6.32$ (3sf)
	(c)	Gradient $= \frac{5-3}{4-(-2)} = \frac{1}{3}$ $y = \frac{1}{3}x + c$ When $x = 4, y = 5$, $5 = \frac{1}{3}(4) + c$ and $c = \frac{11}{3}$ $y = \frac{1}{3}x + \frac{11}{3}$
18	**(a)**	For right-angled triangle ABC, $AB^2 = AC^2 + BC^2$ (Pythagoras' Theorem) $AB^2 = x^2 + x^2$ $AB^2 = 2x^2$ $AB = \sqrt{2}x$ (shown)
	(b)	 Let h be the shortest distance from C to AB in cm. $\sin 45° = \frac{h}{6\sqrt{2}}$ $h = \sin 45° \times 6\sqrt{2} = 6$ cm

19	**(a)**	Acceleration $= \frac{80}{10} = 8\ m/s^2$
	(b)	Let t be the time when both cars were travelling at the same speed. $\frac{t}{10} = \frac{60}{80} \Rightarrow t = 7.5s$
	(c)	Let T be the time when Car B overtook Car A. Distance travelled $= \frac{1}{2} \times 10 \times 80 + (T-10)(80) = 60T$ $80T - 400 = 60T$ $T = 20s$
20	**(a)**	$h = ax(b-x)$ To find x-intercepts, let $h = 0$. $ax(b-x) = 0$ $ax = 0$ or $b - x = 0$ $x = 0$ or $x = b$ From the graph, the x-intercepts are $x = 0$ or $x = 6$. Hence $b = 6$. $h = ax(6-x)$ When $x = 0.3$, $h = 1.5$, $1.5 = a(0.3)(6-0.3)$ $1.5 = a(1.71)$ $a = \frac{50}{57}$
	(b)	$h = \frac{50}{57}x(6-x)$ When x=3, $h = \frac{50}{57}(3)(6-3) = \frac{150}{19} = 7\frac{17}{19}$ = 7.9 m (1dp)

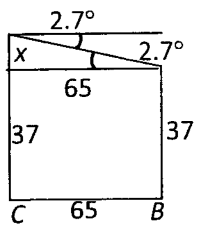
21	(a)	Each exterior angle $= \frac{2}{15} \times 180° = 24°$ Sum of exterior angles = 360° Number of sides of the polygon $= \frac{360°}{24°} = 15$
	(b)	Let the length of the cube be x cm. The surface area is reduced by $4x \times 4.5 = 18x$ cm² $18x = 162$ $x = 9$  Volume of the volume of the original cuboid $= 9 \times 9 \times 13.5$ = 1093.5 cm³
22	(a)	Construct the angle bisector of angle ABC, **showing arcs.** Then construct the Kite $ABCD$ with BD = 8 cm 
	(b)	AD = 6.5 cm
	(c)	Construct the perpendicular bisector of AD, **showing two pairs of arcs.**
	(d)	Mark a possible point P, which is inside the kite, equidistant from A and D, and is nearer to AB than BC. Label this point P.

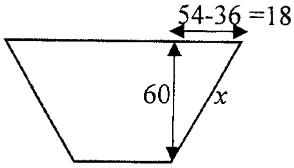
23	**(a)**	Angle $BGA = 40°$ (isos. triangle BAG) Angle $DGA = 90°$ (angle between tangent and radius of a circle is a right angle) Angle $BGC = 90° - 40° = 50°$ Angle $GBC = 40° + 40° = 80°$ (ext. angle of Triangle BAG) Angle $BCG = 180° - 50° - 80° = 50°$ (angle sum of triangle) Since Angle $BGC =$ Angle $BCG = 50°$, triangle BCG is isosceles.
	(b)(i)	Angle $GFE = 180 - 80 = 100°$ (angles in opposite segments are supplementary)
	(b)(ii)	Angle $ODE = 80°$ (angles in the same segment are equal) Angle $DOE = 180 - 80 - 80 = 20°$ (OD=OE=radius and triangle ODE is isos.)
24	**(a)**	Volume of cylinder $= \pi \times 5^2 \times 8 = 628.3185307\ \text{cm}^3$ $= 628\ \text{cm}^3$ (3sf)
	(b)	Volume of cylinder $= \pi \times 5^2 \times 3 = 235.619449$ Volume of hemisphere $= \frac{1}{2}\left(\frac{4}{3}\right)\pi \times 5^3 = 261.7993878$ Total Volume $= 235.619449 + 261.7993878$ $= 497.4188368\ \text{cm}^3 = 497\ \text{cm}^3$ (3sf)
	(c)	The lotion in Figure I because 1 cm^3 of lotion in Figure I costs $\frac{\$40}{628.318530} =$ \$0.0637. This is cheaper than the cost of 1cm^3 of lotion in Figure II , which is $\frac{\$36.99}{497.4188368} = \$0.0744$.

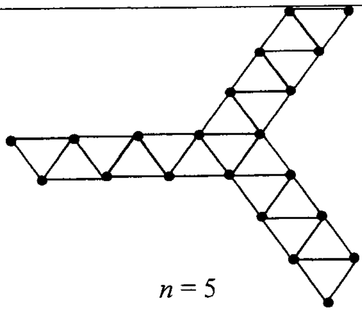
25	(a)	$PC = AC$ (sides of square $ACPQ$) $\angle PCA = \angle BCR = 90°$ (angle of squares $ACPQ$ and $BCRS$) $\angle PCB = \angle ACB + 90°$ $\angle ACR = \angle ACB + 90°$ $\therefore \angle PCB = \angle ACR$ $BC = RC$ (sides of square $BCRS$) $\therefore \Delta BPC \equiv \Delta RAC$ (SAS) (shown)
	(b)(i)	$\frac{\text{Length of X}}{\text{Length of Y}} = \sqrt[3]{\frac{216}{512}} = \frac{3}{4}$ $\frac{\text{Area of X}}{\text{Area of Y}} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$
	(b)(ii)	Volume of jug $X = \pi r^2 h = 216$ Volume of jug Z $= \pi(2r)^2 h$ $= 4\pi r^2 h$ $= 4(216)$ $= 864 \text{ cm}^3$
26	(a)	2600 → 2, 1300 1300 → 2, 650 650 → 2, 325 325 → 5, 65 65 → 5, 13 $2600 = 2^3 \times 5^2 \times 13$
	(b)(i)	$A = p \times q^{r+2} \times 13$ $B = p^2 \times q^r \times 13$ LCM of A and B $= p^2 \times q^{r+2} \times 13 = 5^2 \times 2^3 \times 13$ $p = 5, q = 2$ and $r = 1$
	(b)(ii)	$A = 5 \times 2^3 \times 13$ $B = 5^2 \times 2^1 \times 13$ HCF of A and B $= 5 \times 2 \times 13$ Number of packets of rice $= 2^2 = 4$ Number of bottles of oil $= 5$

BMSS Maths Preliminary 4E Paper 2
2025 Marking Scheme
Total = 90 marks

1	**(a)**	$7-8x=6(1-2x)$ $7-8x=6-12x$ $-8x+12x=6-7$ $4x=-1$ $x=-\frac{1}{4}$
	(b)	$-3y>25$ $y<-\frac{25}{3}$ Largest integer is -9
	(c)	$Aq=\pi p(2p+q)$ $Aq=2\pi p^2+\pi pq$ $Aq-\pi pq=2\pi p^2$ $q(A-\pi p)=2\pi p^2$ $q=\frac{2\pi p^2}{A-\pi p}$
	(d)	$\frac{x}{x+3}-\frac{4x+1}{x+1}=2$ $\frac{x^2+x-(4x+1)(x+3)}{(x+3)(x+1)}=2$ $\frac{x^2+x-(4x^2+13x+3)}{(x+3)(x+1)}=2$ $\frac{-3x^2-12x-3}{x^2+4x+3}=2$ $-3x^2-12x-3=2(x^2+4x+3)$ $-5x^2-20x-9=0$ $5x^2+20x+9=0$ $x=\frac{-20\pm\sqrt{20^2-4(5)(9)}}{2(5)}$ $x=\frac{-20\pm\sqrt{220}}{10}$ $x=-0.5167603026$ or $x=-3.483239697$ $x=-0.517$ or $x=-3.48$ (3sf)

2	**(a)(i)**	$270° - 50° = 220°$ or $180° + 40° = 220°$
	(a)(ii)	$\frac{1}{2} \times 73.5 \times 65 \times \sin 70^0$ = 2244.690748 = 2240 m^2 (3sf)
	(a)(iii)	Using cosine rule, $\cos \angle CBD = \frac{65^2 + 90^2 - 55^2}{2(65)(90)}$ $= \frac{31}{39} = 0.7948717949$ $\angle CBD = \cos^{-1}\left(\frac{31}{39}\right)$ = 37.35685197 = 37.4° (1 dp)
	(b)	Let d be the shortest distance from C to AB. $d = 65 \sin 60°$ = 56.29165125 =56.3 m
	(c)	 $\tan 2.7 = \frac{x}{65}$ h= 65 × tan2.7 = 3.065 322187 Height of building at C = 37 + 3.065322187 = 40.1 m (3sf)

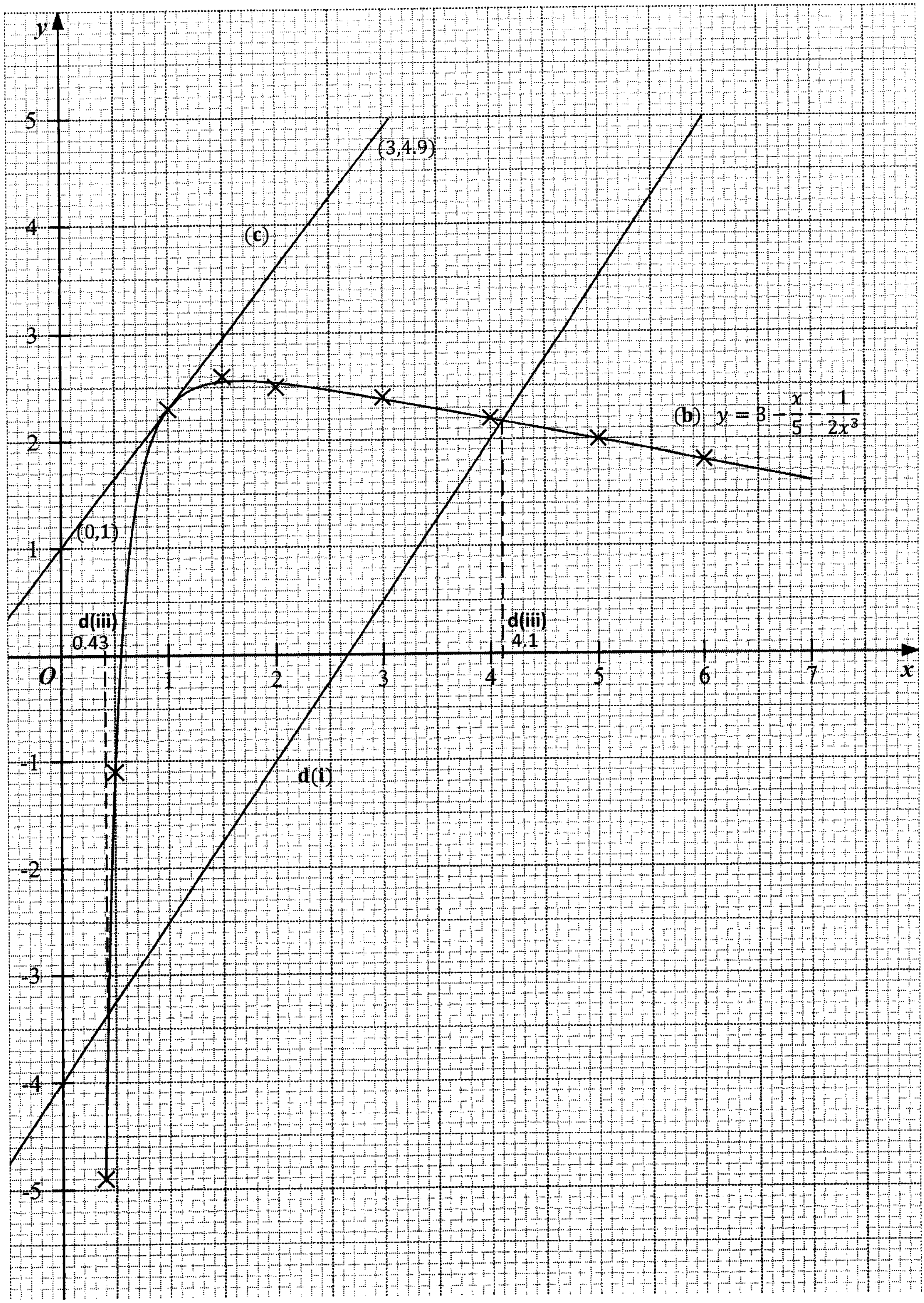
3	**(a)**	Let V be the quantity of paint and d be the depth of container. $V = kd^2$, where k is a constant $180 = k(60)^2$ $k = \frac{1}{20}$ $V = \frac{d^2}{20}$ When $d = 90, V = \frac{(90)^2}{20} = 405$ ml
	(b)(i)	Using Pythagoras' Theorem, $x^2 = 60^2 + 18^2$ $x^2 = 3924$ $x = \sqrt{3924} = 62.64183905$ $x = 62.642$ (5sf) 
	(b)(ii)	Base area of small circle $= \pi(36)^2 = 4071.504079$ Slant height of smaller cone $= \sqrt{36^2 + 120^2} = 125.2836781$ Curved surface area of smaller cone $= \pi \times 36 \times 125.2836781 = 14169.25018$ Curved surface area of larger cone $= \pi \times 54 \times (125.2836781 + 62.642) = 31880.84021$ Difference in curved surface area $= 31880.84021 - 14169.25018 = 17711.59003$ Total surface area of the outside of the pot $= 4071.504079 + 17711.59003 = 21783.09411$ $= 21800$ cm^2 (3sf)
	(b)(iii)	$\frac{V_L}{V_S} = \left(\frac{h_L}{h_S}\right)^3$ $\frac{2}{1} = \left(\frac{h_L}{60}\right)^3$ $\frac{h_L}{60} = \sqrt[3]{2}$ $h_L = \sqrt[3]{2} \times 60 = 75.59526299 = 75.6$ (3sf) The height of the larger pot is 75.6 cm.

4	(a)	 $n = 5$
	(b)(i)	$a = 39$, $b = 21$
	(b)(ii)	$T_n = 7+3(n-1) = 3n + 4$
	(b)(iii)	$3n + 4 = 159$ $3n = 155$ $n = \frac{155}{3}$ Since n is not a positive integer, it is not possible to have a diagram with 159 triangles.
	(b) (iv)	$S_n = 15+6(n-1) = 6n + 9$ $S_n = 6n + 8 + 1 = 2(3n+4) + 1$ Since S_n can be expressed in the form $2m + 1$, where $m = 3n + 4$ is integer, S_n is odd for every integer n.
	(c)	$S = 2T + 1 = 2(P - 2) + 1 = 2P - 3$ Hence, $S = 2P - 3$

5	(a)	$E = \begin{pmatrix} 0.83 \\ 0.77 \\ 1.28 \end{pmatrix}$
	(b)	**D=ME** $= \begin{pmatrix} 100000 & 150000 & 200000 \\ 120000 & 180000 & 150000 \end{pmatrix} \begin{pmatrix} 0.83 \\ 0.77 \\ 1.28 \end{pmatrix}$ $= \begin{pmatrix} 454500 \\ 430200 \end{pmatrix}$
	(c)	The elements in **D** represent the total value in SGD of the deposits that Henry and John each owned.
	d(i)	$A = \begin{pmatrix} 100000 & 150000 & 200000 \\ 120000 & 180000 & 150000 \end{pmatrix} \begin{pmatrix} 1.18\% & 0 & 0 \\ 0 & 2.74\% & 0 \\ 0 & 0 & 0.375\% \end{pmatrix}$ $= \begin{pmatrix} 1180 & 4110 & 750 \\ 1416 & 4932 & 562.5 \end{pmatrix}$
	d(ii)	The elements in **A** represent the annual interest for each of the three time deposits in AUD, NZD and USD that that Henry and John each earned.
	d(iii)	$\begin{pmatrix} 1180 & 4110 & 750 \\ 1416 & 4932 & 562.5 \end{pmatrix} \begin{pmatrix} 0.83 \\ 0.77 \\ 1.28 \end{pmatrix}$ $= \begin{pmatrix} 5104.1 \\ 5692.92 \end{pmatrix}$ Henry: \$5104.10 John: \$5692.92

6	(a)	$\frac{80}{100} \times 250 = 200.$ From the graph, 80th percentile is 54
	(b)	$\frac{3}{10} \times 250 = 75$ students failed the test. From graph, pass mark is 34.
	c(i)	$p = 100 - 30 = 70$ $q = 230 - 100 = 130$
	c(ii)	$\Sigma f = 250$ $\Sigma fx = (30 \times 10) + (70 \times 30) + (130 \times 50) + (20 \times 70)$ $= 10300$ $\Sigma fx^2 = (30 \times 10^2) + (70 \times 30^2) + (130 \times 50^2) + (20 \times 70^2)$ $= 489000$ Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{10300}{250} = 41.2$ Standard deviation $= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\text{mean})^2} = \sqrt{\frac{489000}{250} - (41.2)^2}$ $= 16.07980099 = 16.1$
	c(iii)	Case 1: First student scored under 20 marks and the second student scored 60 marks or more $\frac{30}{250} \times \frac{20}{249}$ Case 2: First student scored 60 marks or more and the second student scored under 20 marks $\frac{20}{250} \times \frac{30}{249}$ Required probability $= \left(\frac{30}{250} \times \frac{20}{249}\right) + \left(\frac{20}{250} \times \frac{30}{249}\right) = \frac{8}{415}$
	c(iv)	The pupils in school B performed better on average in the Science test than the pupils in school A. This is because they obtained a higher mean score (44.8 > 41.2). The pupils in school B were more consistent in their performance in the Science test than the pupils in school A. This is because they obtained a lower standard deviation (12.1 < 16.1).

7	(a)	$p = 3 - \frac{6}{5} - \frac{1}{2(6)^3} = 1.8$
	(b)	Draw a smooth graph using the table of points See the graph overleaf.
	(c)	Draw a tangent to the curve at (1, 2.3). The tangent overleaf passes through the point (0, 1) and (3, 4.9) and the gradient is $\frac{4.9-1}{3-0} = 1.3$
	(d)(i)	Draw the line with gradient 1.5 that passes through the point $(2, -1)$. See overleaf
	(d)(ii)	The equation is $y = 1.5x - 4$
	(d)(iii)	x-coordinates of points where the line intersects the curve are $x = 0.43$ and $x = 4.1$. See the x-coordinates overleaf


y
x
O
5
4
3
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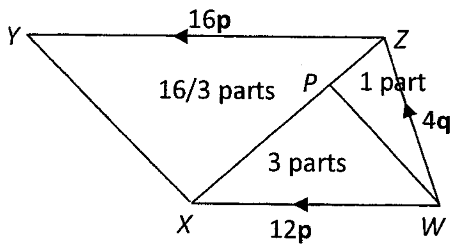
5
6
7
(3,4.9)
(c)
(0,1)
(b) $y = 3 - \frac{x}{5} - \frac{1}{2x^3}$
d(iii)
0.43
d(iii)
4.1
d(i)

8	(a)	$\overrightarrow{ZX} = \overrightarrow{ZW} + \overrightarrow{WX}$ $= -4\mathbf{q} + 12\mathbf{p}$
	(b)	$\overrightarrow{WP} = \overrightarrow{WZ} + \overrightarrow{ZP}$ $= 4\mathbf{q} + \frac{1}{4}\overrightarrow{ZX}$ $= 4\mathbf{q} + \frac{1}{4}(-4\mathbf{q} + 12\mathbf{p})$ $= 4\mathbf{q} - \mathbf{q} + 3\mathbf{p})$ $= 3\mathbf{p} + 3\mathbf{q}$
	(c)	$\overrightarrow{XY} = \overrightarrow{XZ} + \overrightarrow{ZY}$ $= -12\mathbf{p} + 4\mathbf{q} + \frac{4}{3}(12\mathbf{p})$ $= -12\mathbf{p} + 4\mathbf{q} + 16\mathbf{p} = 4\mathbf{p} + 4\mathbf{q}$ $\overrightarrow{XY} = \frac{4}{3}\overrightarrow{WP}$ Since $\overrightarrow{XY} = \frac{4}{3}\overrightarrow{WP}$ and $\overrightarrow{XY}$ is a scalar multiplication of $\overrightarrow{WP}$, the line *XY* is parallel to the line *WP*.
	d(i)	$\frac{Area\ of\ \Delta WPX}{Area\ of\ \Delta XYP} = \frac{3}{4}$
	d(ii)	$\frac{\text{Area of } \Delta WZP}{\text{Area of } \Delta WXZ} = \frac{1}{4}$
	d(iii)	 $\frac{\text{Area of } \Delta WZP}{\text{Area of } \Delta YXZ} = \frac{\text{area of } \Delta WZP}{\text{area of } \Delta WXZ} \times \frac{\text{area of } \Delta WXZ}{\text{area of } \Delta YXZ}$ $= \frac{1}{4} \times \frac{12}{16} = \frac{3}{16}$

9	(a)	$\cos\theta = \frac{ON}{1}$ $ON = \cos\theta$ $h = 1 - \cos\theta$
	(b)	Area of segment = Area of sector $OACB$ – Area of triangle OAB $= \frac{1}{2} \times 1 \times 1 \times 2\theta - \frac{1}{2} \times 1 \times 1 \times \sin 2\theta$ $= \theta - \frac{1}{2}\sin 2\theta$ Shaded segment area $= \theta - \frac{1}{2}\sin 2\theta$
	(c)	When $h = 0.80$, $0.8 = 1 - \cos\theta$ $\cos\theta = 0.2$ $\theta = \cos^{-1} 0.2 = 1.369438406$ rad $y = \frac{1.369438406 - \frac{1}{2}\sin(2 \times 1.369438406)}{\pi(1)^2} \times 100$ $= 37.3530039$ When $h = 0.81$, $0.81 = 1 - \cos\theta$ $\cos\theta = 0.19$ $\theta = \cos^{-1} 0.19 = 1.37963418$ rad $y = \frac{1.37963418 - \frac{1}{2}\sin(2 \times 1.37963418)}{\pi(1)^2} \times 100$ $= 37.97739981$ Since $37.353\% < 37.5\% < 37.977\%$, $0.8 < h < 0.81$. A suitable depth of petrol that indicates that the tank is $\frac{3}{8}$ full of petrol, or the tank is 37.5% full, is 0.802 m.