Name: MARKING SCHEME	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2025
SECONDARY 4

MATHEMATICS

4052/01

Paper 1

Friday 29 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use	
Total	**/ 90**

This document consists of **21** printed pages and **1** blank page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Calculate $\dfrac{\sqrt[3]{0.5-(-1.5)^2\times(3.5)}}{3\times 8}$.

Answer ....-0.0811.......... [1]

---

2 Simplify $\dfrac{6xy-3y-4x+8x^2}{2x^2+3x-2}$.

$$\frac{6xy-3y-4x+8x^2}{2x^2+3x-2}$$
$$=\frac{3y(2x-1)+4x(2x-1)}{(2x-1)(x+2)}$$
$$=\frac{(3y+4x)(2x-1)}{(2x-1)(x+2)}$$
$$=\frac{3y+4x}{x+2}$$

Note $(3y+4x)(2x-1)$
Note: $(2x-1)(x+2)$

Answer ....$\dfrac{3y+4x}{x+2}$.......... [3]

---

3 James deposited some money in a bank.
The bank pays a compound interest of 1.52% per annum, compounded half yearly.
After 4 years, James withdrew a total of $26561.05.

Calculate the interest earned. Give your answer to the nearest dollar.

$$26561.05=P\left(1+\frac{1.52\div 2}{100}\right)^8$$
$$P=\frac{26561.05}{(1.0076)^8}$$
$$=24999.9977$$

Interest
$$=26561.05-24999.9977$$
$$=\$1561$$

Time period for compounding should match: **hence 0.76 and 8**
You need to find the rate when the interest is charged so 1.52 need to be divided by 2.

P is principal (original amt), **not interest.**

Answer $......1561.......... [3]

4 **(a)** Solve $64x^2 = 8x$.

$64x^2 = 8x$

$64x^2 - 8x = 0$

$8x(8x-1) = 0$

$x = 0$ or $x = \frac{1}{8}$

Quad eqn, should equate to 0 to get 2 possible ans.
Factorise, not divide variable from both sides.

Answer $x =$ ......0........ or ......$\frac{1}{8}$.......... [2]

(b) Solve the equation $\frac{1}{2^{2x}} \div 4^{x+3} = 0.25$.

$\frac{1}{2^{2x}} \div 4^{x+3} = 0.25$

$2^{-2x} \div 2^{2x+6} = 2^{-2}$

$2^{-4x-6} = 2^{-2}$

$-4x - 6 = -2$

$x = -1$

Aim to change to common base

Answer $x =$ ....-1.............. [3]

---

5 Janice bought some chicken burgers and fish burgers for an outing.
The ratio of the number of chicken burgers to the number of fish burgers bought was 3 : 2.
At the end of the outing, there were 3 burgers of each type left.
The ratio of the number of chicken burgers consumed to the number of fish burgers consumed was 11 : 7.

Calculate the total number of burgers that Janice bought.

Let $3x$ be the number of chicken burgers bought.
Let $2x$ be the number of fish burgers bought.

Define!

$\frac{3x-3}{2x-3} = \frac{11}{7}$

Use fraction method or simul eqn method

$21x - 21 = 22x - 33$

$x = 12$

Total number of burger bought

$= 12 \times 5$

$= 60$

Answer ....60...................... [4]

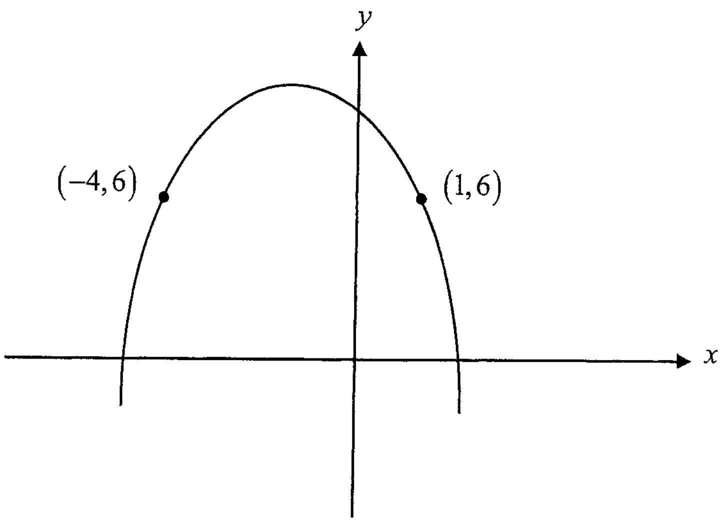
OR <u>Before</u>	**<u>After</u>**
Chicken: Fish	Chicken: Fish
3: 2 = 12: 8	11:7

This is not a good method. Unclear working.

Since the difference is 1 unit fir both chicken and fish so 1 unit : 3 burgers
Total number of burgers $= 20 \times 3$

$= 60$

6



The figure shows the graph of a curve $y=-(x-p)^2+q$, where p and q are constants.
Given that the curve passes through points (–4, 6) and (1, 6), find the values of p and q.

$p=\dfrac{1+(-4)}{2}$

$=-1.5$

Or solve using simul eqn

At (1, 6),

$6=-(1+1.5)^2+q$

$6=-6.25+q$

$q=12.25$

Answer $p=$ ....−1.5..........

$q=$ ....12.25.................... [3]

---

7 Solve the inequality $\dfrac{x}{3}<\dfrac{1}{4}(x+3)<5$.

And, not or

$\dfrac{x}{3}<\dfrac{1}{4}(x+3)$ and $\dfrac{1}{4}(x+3)<5$

$4x<3x+9$ $\quad$ $x+3<20$

$x<9$ $\quad$ $x<17$

$x<9$

Final ans must fulfil both inequality conditions.
That's why it's "**And**".
Can draw number line to see.

Answer ....$x<9$.......................... [3]

8 **(a)** Written as a product of its primes factors, $480 = 2^5 \times 3 \times 5$.

The number $480 \times 75 \times \frac{a}{b}$ is a perfect square where a and b are prime numbers.

Find the smallest value of $\frac{a}{b}$.

$$480 \times 75 \times \frac{a}{b} = 2^5 \times 3 \times 5 \times 3 \times 5^2 \times \frac{a}{b}$$

$$= 2^5 \times 3^2 \times 5^3 \times \frac{a}{b}$$

$75 = 3 \times 5^2$

$$\frac{a}{b} = \frac{2}{5}$$

Answer $\frac{a}{b}$ = ...$\frac{2}{5}$................. [2]

(b) Rectangle tiles, 60 cm by 35 cm each, are laid to form a square wall. Find the minimum area of the square wall.

$60 = 2^2 \times 3 \times 5$

$35 = 5 \times 7$

LCM of 60 and 35 $= 2^2 \times 3 \times 5 \times 7$

$= 420$

Minimum area $= 420 \times 420$

$= 176400 \text{ cm}^2$

Square wall has same length on all sides, hence find LCM which will be the length

Answer ....176400...........cm^2 [2]

---

9 Kate bought a dress during her holiday trip in UK at a discounted price of 30% using a credit card. The exchange rate of the credit card company was SGD \$1 = £0.53.

Given that the total amount she paid to the credit card company in Singapore dollars (SGD) is \$349.81, find the price of the dress in pounds (£) before the discount, correct to the nearest pound.

Some students misinterpreted and thought 30% → 185.39 pounds instead of 70% → 185.39. Both answers are accepted.

Amount paid in pounds

$= 349.81 \times 0.53$

$= 185.3993$ pounds

Amount of dress before discount

$= 185.3993 \times \frac{100}{70}$

$= 265$(correct to nearerst pounds)

OR

Amount of dress before discount

$= 185.3993 \times \frac{100}{30}$

$= 618$(correct to nearerst pounds)

Answer £......265.../618..................... [3]

10

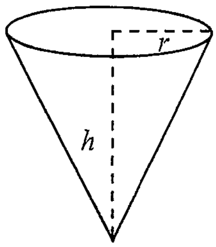


Figure 1

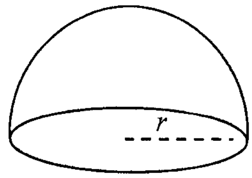


Figure 2

A conical candle in Figure 1 is melted to form a hemispherical candle in Figure 2.
The radius of the base of the cone and the hemisphere are both r cm. The height of the cone is h cm.

Find, in terms of r, the total surface area of the conical candle.

$$\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$$

$$h = 2r$$

Total surface area

$$= \pi r\left(\sqrt{(2r)^2 + r^2}\right) + \pi r^2$$

$$= \pi r\left(\sqrt{5r^2}\right) + \pi r^2$$

$$= \sqrt{5}\pi r^2 + \pi r^2$$

$$or \pi r^2\left(\sqrt{5}+1\right)$$

Equating correct formula for **volume for hemisphere not sphere.**
Need to find **an expression for h in terms of r** so that surface area can be in terms of r.

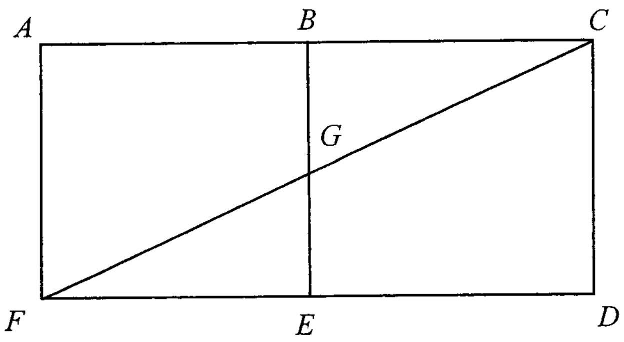
Use Pythagoras Thm $\sqrt{\left(2r^2\right)^2 + r^2}$ to find the slant length (l) of cone

Curved surface area of cone $= \pi r l$

Answer ....$\pi r^2\left(\sqrt{5}+1\right)$....or......cm^2 [3]

$10.2r^2$ or $3.24\pi r^2$

11 The diagram shows 2 squares *ABEF* and *BCDE*. *CF* is the diagonal of the rectangle *ABCDEF*. The lines *BE* and *CF* intersect at point *G*.



Prove that triangle *BGC* is congruent to triangle *EGF*. [3]

$\angle BGC = \angle EGF$ (vertical opposite angle)
$BC = EF$ (sides of square of ABEF and BCDE)
$\angle GEF = \angle GBC$ (alternate angle)
Triangle $BGC \equiv$ Triangle EGF (AAS)

Don't use "given" as reason unless the info is stated clearly in the qn.

Must state test name, like AAS, for case of congruency.
Must be careful if RHS is used. Must have 1hypotenuse mention. ASA used when side between 2 angles.

OR

$GC = GF$ (B and E are the midpoint of AC and DF/ G is midpoint of FC/BE/ BE bisects FC)
$BC = EF$ (sides of square of ABEF and BCDE)
$\angle BCF = \angle EFC$ (alternate angle)
Triangle $BGC \equiv$ Triangle EGF (SAS)

SAS used only if angle is in between 2 sides.

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12 It is given that *y* is inversely proportional to x^2, and that $y = 540$ for a particular value of *x*. Find the value of *y* when this value of *x* is tripled.

$y = \frac{k}{x^2}$

$k = 540x^2$

When *x* is tripled,

New $y = \frac{540x^2}{(3x)^2}$

$= \frac{540x^2}{9x^2}$

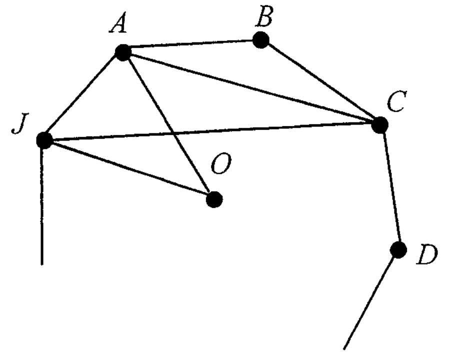
$= 60$

Be clear new *y* vs old *y*.

NOTE: $(3x)^2$, not $3x^2$

Answer $y =$ ....60....................... [3]

13 The diagram shows part of a regular decagon $ABC....J$. centre O.
AC and JC are lines joining the vertices A, C and J.
OA and OJ are lines that join from O to A and J respectively.



(a) Find

(i) angle ABC,

$\angle ABC$

$= \frac{(10-2)\times 180°}{10}$

$= 144°$

Decagon is 10 sides.
Like decade is 10 years.

Answer ....144.......................... ° [2]

(ii) angle AOJ.

$\angle AOJ$

$= \frac{360°}{10}$

$= 36°$

Answer ....36....................... ° [1]

(b) Given that O is the centre of a circle that passes through the vertices of the decagon, find angle ACJ,

$\angle ACJ$,

$\angle ACJ$

$= \frac{36°}{2}$

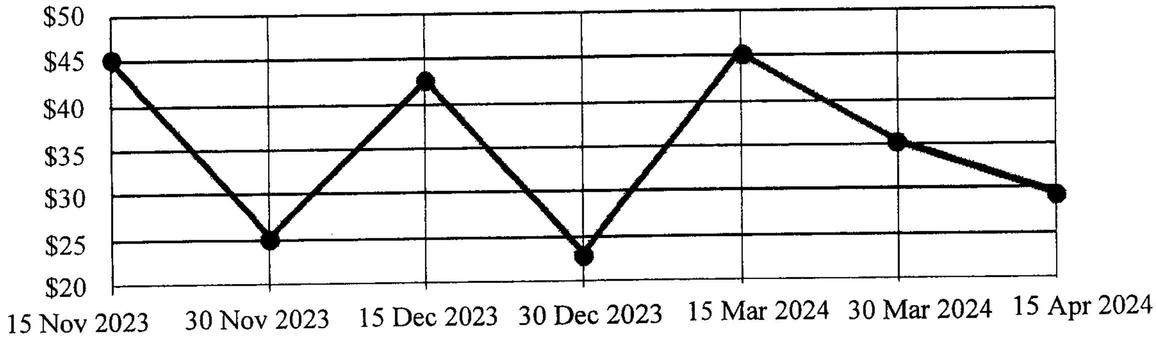
$= 18°$

Angle at centre is twice that of angle at circumference.
NOT angle in the same segment

Answer ....18...................... ° [1]

14 A company presented their 2023 financial report in this graph.

Instant Coffee Powder Price Trends



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Focus should be on price trend.
30 vs 31 days, not quite significant.

Answer
The vertical axis does not start from zero giving impression of extremely large price fluctuations. The price of coffee on 15 Nov 2023 **appears to be** **5 times** that of 30 Nov 2023 but is **actually** $\frac{45}{25} = 1.8$ **times** that of 30 Nov 2023

Give example with specific numbers/dates/days

OR
Not all months are shown between 30 Dec 2023 and 15 Mar 2024 giving impression of sharp increasing price trend. But price might have decreased between 30 Dec 2023 and 15 Mar 2024/ some may think that there is a sharp increase in the price to $45 within a short period of time of 15 days instead 3 months

[2]

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15 In 2005, the number of mobile phone users was 22.2×10^8.

(a) 22.2×10^8 can be written as k billion. Find k.

Billion is 10^9

Answer $k =$ ....2.22....................... [1]

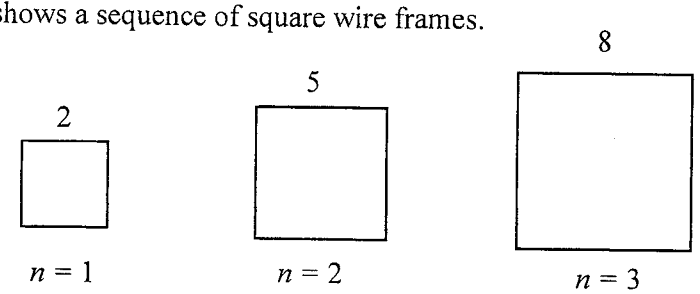
(b) By 2025, the number of mobile phone users has increased by 250%. Calculate the number of mobile phone users in 2025, giving your answer in standard form.

Number of mibile users
$= \frac{350}{100} \times 22.2 \times 10^8$
$= 7.77 \times 10^9$

Increased by 250% means 250% more, hence use 350% for total,

Answer ....7.77×10^9........ [2]

16 The diagram shows a sequence of square wire frames.



The lengths of a side of the first three frames are 2 cm, 5 cm, 8 cm respectively.

(a) Express the length of a side of the nth frame in terms of n.

Do not accept $3(n-1)+2$

Answer .... $3n-1$ ....cm [1]

(b) A piece of wire is 160 cm long.
Hence, explain why the piece of wire cannot be bent to form a square wire frame in the sequence.

Answer

Since

$4(3n-1)=160$

$-4+12n=160$

$n=13\frac{2}{3}$

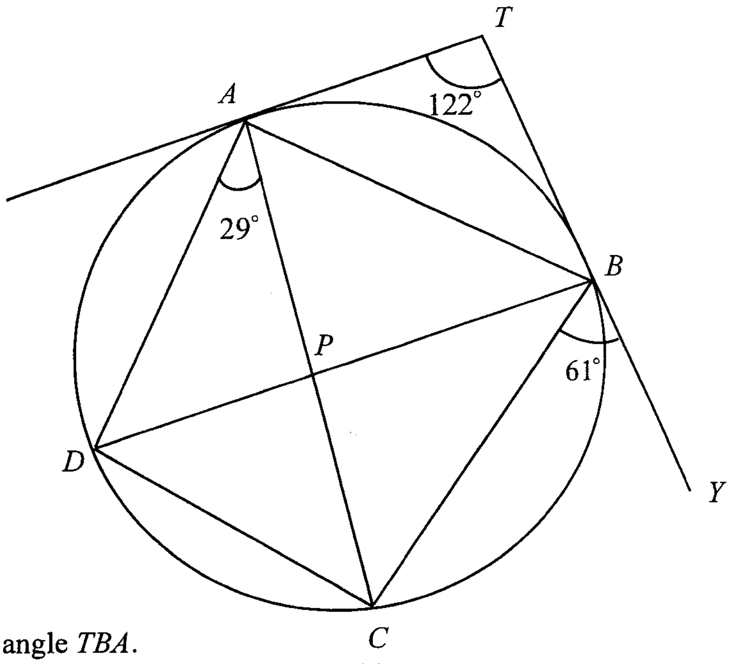
4 x part (a) = 160
Explain using eqn, step by step.

So n is not a **positive integer**, cannot be bent to form a square wire frame in the sequence.

....

.... [2]

17 In the diagram below, A, B, C and D are points on a circle.
P is the intersection of lines of AC and BD. AT and YBT are tangents to the circle.
Angle $DAC = 29^\circ$, angle $ATB = 122^\circ$ and angle $CBY = 61^\circ$.



(a) Find the angle TBA.
Give a reason for each step of the working.
$TB = TA$ (tangent from external point) ← Isosceles because of this
$\angle TBA$
$= \dfrac{180^\circ - 122^\circ}{2}$ (base angles of isosceles triangle)
$= 29^\circ$

Answer ....29...................° [1]

(b) Find the angle DBC.
Give a reason for each step of the working.

Cannot use tangent perpendicular to radius as reason as P is not given as centre here.

Answer ......29 (angle in the same segment)...° [1]

(c) Show that point P is the centre of the circle. [2]

Explain either AC or BD is a diameter.

Answer
$\angle ABC = 180^\circ - 29^\circ - 61^\circ$(adjacent angles on a straight line) $= 90^\circ$
so by converse of the property of right angle in semicircle, ***AC* is the diameter of the circle**

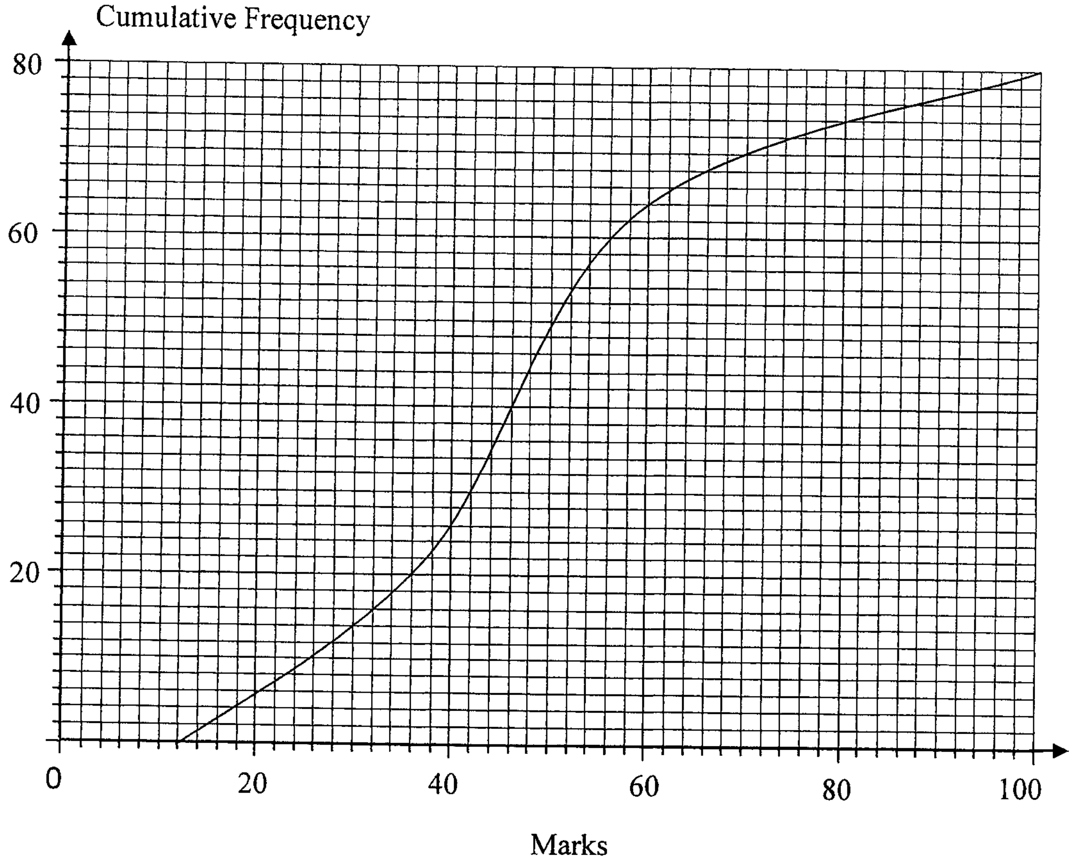
$\angle TBP = 180^\circ - 61^\circ - 29^\circ$(adjacent angles on a straight line) $= 90^\circ$, so by the converse of property of tangent perpendicular to radius. So ***DB* is also a diameter**. Therefor P is the centre of the circle as **it is the intersection of 2 diameters.**

OR
$\angle BDC = 61^\circ$(alternate segment theorem)
$\angle BDC = 180^\circ - 61^\circ - 29^\circ = 90^\circ$(sum of angle in a triangle)
By the property of right angle in semicircle, **BD is a diameter**.

State intersection of 2 diameters is centre.

Therefor P is the centre of the circle as **it is the intersection of 2 diameters.**

18 The marks obtained by a group of 80 students in a Mathematics Test are represented in the cumulative frequency curve below.



(a) Use the diagram to find the median mark.

Answer ....46....................... [1]

(b) 20% of the pupils who scored more than y marks are given a grade A.

Find the value of y.

Answer $y =$ ....60....................... [1]

(c) Two pupils are randomly selected from the class.

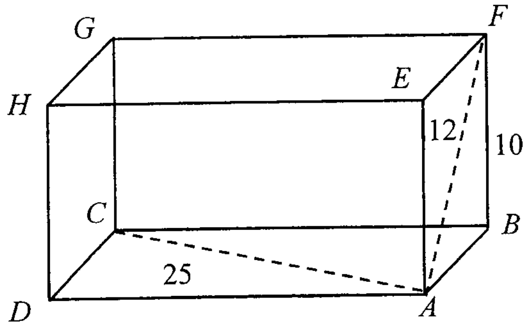
Find the probability that only one of the students obtained less than 40 marks.

$$\frac{26}{80}\times\frac{54}{79}+\frac{54}{80}\times\frac{26}{79}$$
$$=\frac{351}{790}$$

2 cases of outcome acceptable, so add the 2 cases up - First more than 40 & Second less than 40 - or First less than 40 & Second more than 40

Answer .... $\frac{351}{790}$ .... [2]

19 The diagram shows a cuboid *ABCDEFGH*.
$AC = 25$ cm, $FB = 10$ cm and $FA = 12$ cm.



Calculate the angle *GAF*.

$$AG = \sqrt{25^2 + 10^2}$$
$$= \sqrt{725}$$
$$\cos \angle GAF = \frac{12}{\sqrt{725}}$$
$$\angle GAF = 63.5°$$

Draw out the triangle GFA to see better.
AG is the hypotenuse of that triangle.
Note: triangle CAF is not a right angle triangle.

Answer ……………63.5……………… ° [2]

20 Given that $\varepsilon = \{x: x \text{ is an integer}, 1 \leq x \leq 12\}$, and

$A = \{x : x \text{ is a factor of } 6\}$
$B = \{x : \sqrt{x} < 3\}$

Find the value of $n(A' \cap B)'$.

$A' = \{4,5,7,8,9,10,11,12\}$
$B = \{1,2,3,4,5,6,7,8\}$
$(A' \cap B)' = \{1,2,3,6,9,10,11,12\}$
$n(A' \cap B)' = 8$

Can consider drawing a Venn diagram with all the elements inside to see better.

This is a 2 marks question so workings need to be clearly written in the answer space.

Answer ....8..................... [2]

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21 The stem and leaf diagram shows the points scored by a basketball team for 12 matches. The interquartile range of the data is 23 points.

Stem	Leaf
3	0
4	2 3 4
5	0 1 5 7
6	x 8 8 9

Find the value of x.

upper quartile
$= 43.5 + 23$
$= 66.5$
$\frac{\text{value} + 68}{2} = 66.5$
value $= 65$
$x = 5$

Lower Quartile $= \frac{42+43}{2} = 43.5$
Discrete data set, and there's six data in lower half, so the LQ is between 3rd & 4th piece of data.

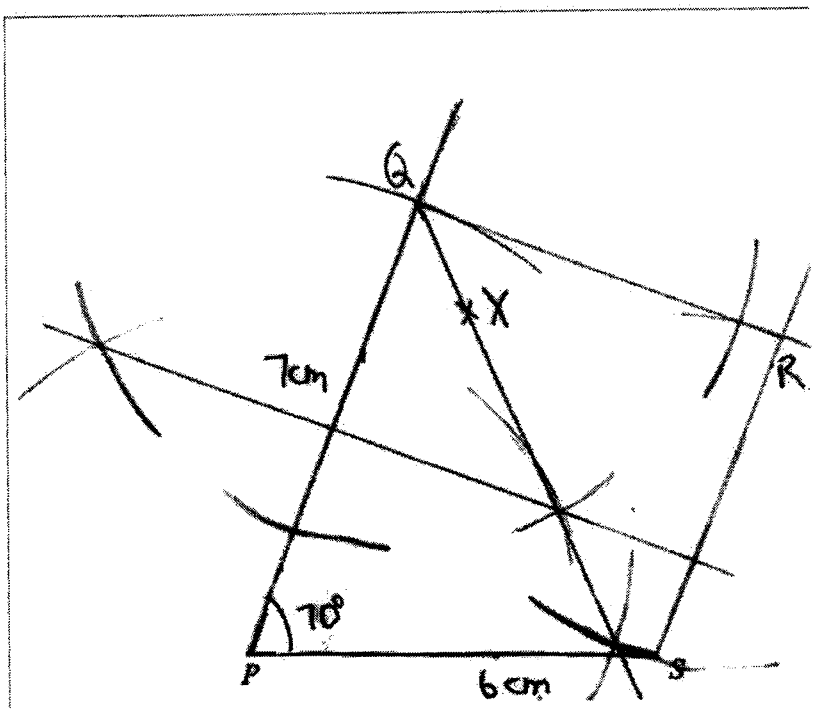
Answer $x =$ ....5.......................... [3]

22 Mrs Lim has a garden in the shape of a trapezium $PQRS$ where PQ is parallel to SR. The side PQ = 42 m, PS = 36 m SR = 30 m and angle $QPS = 70°$.
On a scale drawing where the scale used is 1: n, PS is represented by a line of 6 cm below.

(a) Find the value of n.

Answer n = ...600.................... [1]

(b) Construct a scale drawing of the garden using the value of n found in **(a)**. [2]



Proper arcs used for the construction need to be seen.
Especially for Q and R.

There need to be arcs above and below the line PQ for perpendicular bisector.
No fake arcs.
X cannot be at the intersection as X needs to be nearer to Q.

(c) On your diagram,

(i) Construct perpendicular bisector of PQ. [1]

(ii) Construct the angle bisector of angle PQR. [1]

(d) Mrs Lim wants to build a fountain X, nearer to Q and equidistant from QR and QP, mark and label clearly a possible position of X. [1]

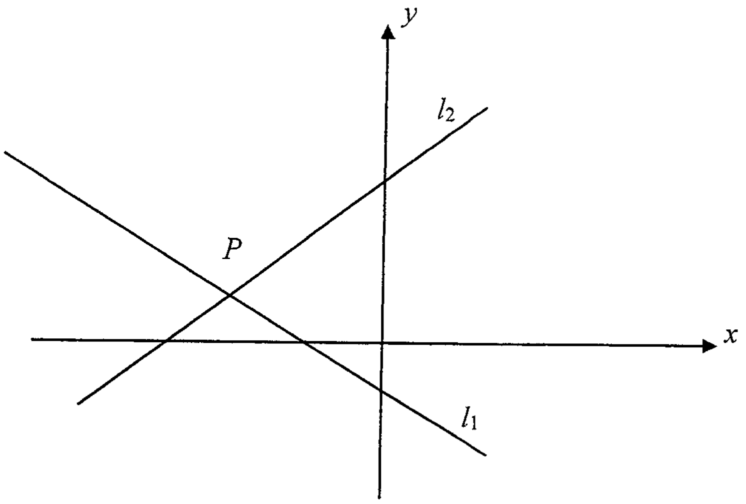
(e) Using the value of n, calculate the actual area of a small pond in the garden if the area of the pond on the scale drawing is 4 cm^2.

1cm^2 represents 6^2=36 m^2
4cm^2 represents 144 m^2

Area scale is square of length scale.

Answer ....144...........m^2 [2]

23



(a) The line l_1 an equation $x+y+2=0$ and l_2 has an equation $x-3y+4=0$. P is the point intersection of the lines l_1 and l_2. Find the coordinates of the point P

$x+y+2=0 \quad y=-x-2$ -------(1)

$x-3y+4=0 \quad y=\frac{1}{3}(x+4)$ -------(2)

$\frac{1}{3}(x+4)=-x-2$

$x+4=-3x-6$

$4x=-10$

$x=-2.5$

Subt. x = -2.5 into (1):

$y=-(-2.5)-2$

$=0.5$

Intersection of two lines is solving simul eqn in algebra form

A1

Answer (.......−2.5......., ...0.5.........) [3]

(b) The equation of the line l_3 is $y=\frac{1}{3}(x-2)$. Explain why the lines l_2 and l_3 do not meet

Answer

Equation of l_3 $y=\frac{1}{3}(x+4)$ has the **same gradient of** $\mathbf{\frac{1}{3}}$ as l_3 and the y **intercept of the lines are different** so they do not meet.

Same gradient does not mean two different lines, so need explain more.

[2]

(c) Points A and B lies on the line l_1 such that the y-coordinate of A is 5.5 and B is below the x-axis. Given that AP: PB = 2: 1, find the equation of the horizontal line that passes through the point B.

2 units represent $5.5-0.5=5$

1 unit represent 2.5

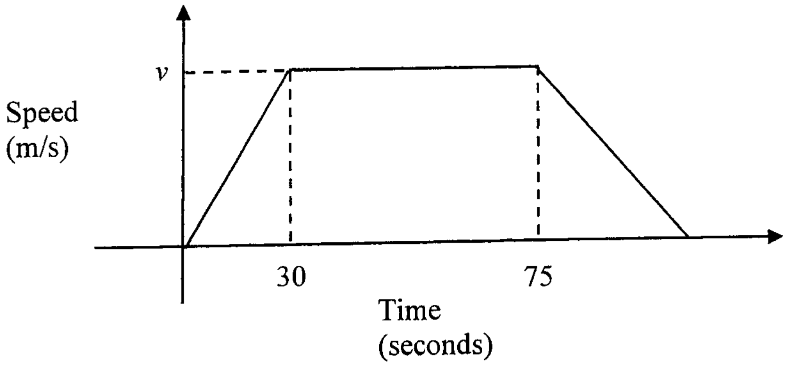
$y=0.5-2.5$

$=-2$

Look at similar triangles.
Ratio of corresponding sides is the same.

Answer .... $y = -2$ .... [1]

24 The diagram shows the speed-time graph for a lorry's journey. The lorry starts from rest and accelerates uniformly to a speed of v m/s in 30 seconds. The lorry then travels at a constant speed for 45 seconds before it decelerates and comes to a rest.
The distance travelled for the first 75 seconds is 1.2 km.



(a) The area beneath the speed-time graph represents the distance travelled by the lorry.

Calculate the greatest speed, v m/s of the lorry.

$$1200 = \frac{1}{2}(45+75)v$$
$$1200 = 60v$$
$$v = 20$$

> The units must be the same for both sides of eqn.

Answer $v =$ ....20...................... [2]

(a) Given that the lorry decelerates at 5 m/s^2, calculate the total time taken for the journey.

Time taken for deceleration

$= \frac{20}{5}$

$= 4s$

Total time taken for journey

= 75+4

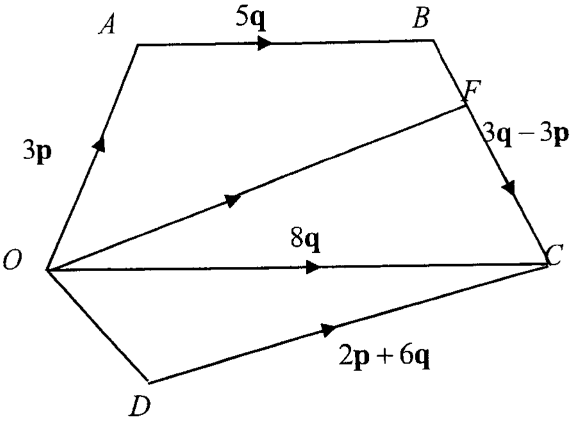
= 79 s

> Gradient of speed-time graph is acceleration/deceleration.
> $\frac{20-0}{\text{time taken}} = -5$

Answer ....79....................s [2]

Turn over for Question 25

25



In the diagram, $\overrightarrow{OA} = 3\mathbf{p}$, $\overrightarrow{OC} = 8\mathbf{q}$, $\overrightarrow{AB} = 5\mathbf{q}$, $\overrightarrow{BC} = 3\mathbf{q} - 3\mathbf{p}$ and $\overrightarrow{DC} = 2\mathbf{p} + 6\mathbf{q}$.

F is point on BC such that $BF : FC = 1 : 2$.

(a) Express, as simply as possible, in terms of **p** and **q**,

(i) $\overrightarrow{OF}$,

It is $\overrightarrow{BC} = 3\mathbf{q} - 3\mathbf{p}$, not $\overrightarrow{FC}$.

$\overrightarrow{OF} = \overrightarrow{OC} + \overrightarrow{CF}$

$= 8\mathbf{q} - 2\mathbf{q} + 2\mathbf{p}$

$= 6\mathbf{q} + 2\mathbf{p}$

Answer .... $6\mathbf{q} + 2\mathbf{p}$ .... [1]

(ii) $\overrightarrow{AD}$.

$\overrightarrow{OD} = 8\mathbf{q} - 2\mathbf{p} - 6\mathbf{q}$

$= 2\mathbf{q} - 2\mathbf{p}$

$\overrightarrow{AD} = -3\mathbf{p} - 2\mathbf{p} + 2\mathbf{q}$

$= 2\mathbf{q} - 5\mathbf{p}$

Alternative method:

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$

$= 5\mathbf{q} + 3\mathbf{q} - 3\mathbf{p} - 2\mathbf{p} - 6\mathbf{q}$

$= 2\mathbf{q} - 5\mathbf{p}$

Answer $2\mathbf{q} - 5\mathbf{p}$ .... [2]

(b) Write down two facts about $\overrightarrow{OD}$ and $\overrightarrow{BC}$.

Answer

Since $\frac{3}{2}\overline{OD} = \overline{BC}$, $\overline{OD}$ is parallel to $\overline{BC}$, and magnitude of $\overline{BC}$ is $\frac{3}{2}$ times that of $\overline{OD}$.

Vector is about direction and magnitude. Two facts should relate to them.

[2]

(c) Find the value of $\dfrac{\text{area of triangle } OBC}{\text{area of quadrilateral } OABC}$.

Triangle OBC and trapezium OABC share common height. Write the fraction of their areas using formula to find value.

Answer ......$\frac{8}{13}$...... [1]

Name:	Class:	MARKING SCHEME

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2025
SECONDARY 4

MATHEMATICS
4052/02

Paper 2

Monday 25 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer. Omission of essential working will result in loss of marks. The total number of marks for this paper is 90.

The use of an approved scientific calculator is expected where appropriate. If the degree of accuracy is not specified in the question, and if the answer is not exact give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use	
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total Marks	

This document consists of **19** printed pages and **1** blank page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{, where } \theta \text{ is in radian}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radian}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 40 students from Class A took the standing broad jump test.
The students' jump distances are shown in the table.

Jump Distance (x cm)	$120 < x \leq 140$	$140 < x \leq 160$	$160 < x \leq 180$	$180 < x \leq 200$
Number of students	6	13	14	7

(a) Use the table to estimate the mean jump distance of Class A.

$$\text{mean jump distance} = \frac{130(6)+150(13)+170(14)+190(7)}{40}$$

$$= \frac{6440}{40}$$

$$= 161 \text{ cm}$$

B1

Answer .... cm [1]

(b) The mean jump distance of 40 students from Class B is 155 cm. Use this information to make a comparison between the jump distances of the 2 classes.

Students from Class 1A **generally jumped further/longer distance** than Class 1B for the standing broad jump test as the **mean jumping distance** of Class 1A (**161 cm**) is **more** than that of Class 1B (**155 cm**).

B1

(c) The standard deviation of the jump distances of Class B is 16 cm.
A new student joined Class B and his jump distance is 196 cm.
Calculate the

(i) new mean jump distance for Class B,

$$\text{New mean jump distance} = \frac{155(40)+196}{41}$$

$$= 156 \text{ cm}$$

B1

Answer .... cm [1]

(ii) new standard deviation of the jump distances of Class B.

$$\sqrt{\frac{\Sigma x^2}{40} - 155^2} = 16$$

$$\frac{\Sigma x^2}{40} - 155^2 = 16^2$$

$$\Sigma x^2 = 40\left(16^2 + 155^2\right)$$

$$= 971240$$

M1- find Σx^2

$$\text{New standard deviation} = \sqrt{\frac{971240+196^2}{41} - 156^2}$$

M1- 41 and 156^2 in the formula

$$= 17.0222\ldots$$

$$= 17.0 \text{ cm (3sf)}$$

A1

Answer .... cm [3]

2 **(a)** Simplify $\frac{3x-4}{5x^2-x}\times\frac{15x^3}{9x-12}$.

$$\frac{3x-4}{5x^2-x}\times\frac{15x^3}{9x-12}=\frac{3x-4}{x(5x-1)}\times\frac{15x^3}{3(3x-4)}$$

M1 – any correct factorisation

$$=\frac{1}{5x-1}\times\frac{5x^2}{1}$$

$$=\frac{5x^2}{5x-1}$$

A1

Answer .... [2]

(b) Simplify $\left(\frac{4}{9x^2}\right)^{-\frac{3}{2}}\div 3x$.

$$\left(\frac{4}{9x^2}\right)^{-\frac{3}{2}}\div 3x=\left(\frac{9x^2}{4}\right)^{\frac{3}{2}}\times\frac{1}{3x}$$

M1 – change to multiply or show division by $3x$

$$=\frac{27x^3}{8}\times\frac{1}{3x}$$

M1 – correct indices law used

$$=\frac{9x^2}{8}$$

A1

Answer .... [3]

(c) The period, T seconds, of a pendulum swing is given by $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum in metres and g m/s^2 is the acceleration acting on the pendulum due to gravity.

(i) Find the value of T when $g = 10$ and $L = 1.2$.

$$T = 2\pi\sqrt{\frac{1.2}{10}}$$
$$= 2.1765...$$
$$= 2.18 \text{ (3sf)}$$

B1

Answer .... [1]

(ii) Make L the subject of the formula.

$$T = 2\pi\sqrt{\frac{L}{g}}$$
$$\frac{T}{2\pi} = \sqrt{\frac{L}{g}}$$
$$\frac{T^2}{4\pi^2} = \frac{L}{g}$$

M1 – square both sides correctly

$$L = \frac{T^2 g}{4\pi^2}$$

A1 (A0 if answer blank doesn't have "$L =$")

Answer .... [2]

(d) Express as a single fraction in its simplest form $\frac{3}{2x^2-8} - \frac{2}{x+2}$.

$$\frac{3}{2x^2-8} - \frac{2}{x+2} = \frac{3}{2(x^2-4)} - \frac{2}{x+2}$$
$$= \frac{3}{2(x-2)(x+2)} - \frac{4(x-2)}{2(x-2)(x+2)}$$

M1 – common denominator

$$= \frac{3-4(x-2)}{2(x-2)(x+2)}$$
$$= \frac{3-4x+8}{2(x-2)(x+2)}$$

M1 – simplify numerator

$$= \frac{11-4x}{2(x-2)(x+2)} \quad \text{or} \quad \frac{11-4x}{2x^2-8}$$

A1

Answer .... [3]

3 Apollo 11 was the first manned mission to land on the Moon. It was launched on July 16, 1969, at 9:32 am from Kennedy Space Center in Florida.

Apollo 11 travelled an estimated total distance of 384000 km at an average speed of x km/s to reach the Moon.

(a) Show that Apollo 11 took $\frac{320}{3x}$ hours to reach the Moon.

Average speed in km/h = $3600x$ km/h

$$\text{Time in hours} = \frac{384000}{3600x}\text{ h} \quad \boxed{\text{M1}}$$

$$= \frac{320}{3x}\text{ h} \quad \boxed{\text{AG}}$$

OR

$$\text{Time taken} = \frac{384000}{x}\text{ s}$$

$$= \frac{384000}{3600x}\text{ h}$$

(b) The average speed for the return trip back to Earth was 0.5 km/s faster than the average speed of Apollo 11 from Earth to the Moon.
Assuming that the estimated distance for the return trip is also 384000 km, write and simplify an expression, in terms of x, for the number of hours taken for return journey.

Average speed for return trip in km/h = $3600(x+0.5)$ km/h

$$\text{Time in hours} = \frac{384000}{3600(x+0.5)} \quad \boxed{\text{M1}}$$

$$= \frac{384000}{1800(2x+1)}$$

$$= \frac{640}{3(2x+1)} \text{ or } \frac{640}{6x+3}\text{ h} \quad \boxed{\text{A1 (A0 for } \frac{320}{3x+1.5})}$$

Answer .... h [2]

(c) The return trip for Apollo 11 to travel from the Moon back to Earth was shorter by 32 hours. Write down an equation to represent this information and show that it reduces to $6x^2+3x-10=0$.

Answer

$$\frac{320}{3x}-\frac{640}{6x+3}=32 \quad \boxed{\text{M1: part (a) – part (b) = 32}}$$

$$\frac{320(6x+3)-640(3x)}{3x(6x+3)}=32 \quad \boxed{\text{M1: combine to single fraction}}$$

$$\frac{10(6x+3)-20(3x)}{3x(6x+3)}=1$$

$$60x+30-60x=3x(6x+3)$$

$$30=18x^2+9x$$

$$18x^2+9x-30=0$$

$$6x^2+3x-10=0 \text{ (shown)}$$

$\boxed{\text{AG}}$

OR

$$\frac{1920x+960-1920x}{18x^2+9x}=32$$

$$960=32(18x^2+9x)$$

$$960=576x^2+288x$$

$$576x^2+288x-960=0 \quad \boxed{\text{M1: simplify correctly}}$$

[3]

(d) Solve the equation $6x^2+3x-10=0$, giving your answers correct to 5 decimal places.

$$6x^2+3x-10=0$$

$$x=\frac{-3\pm\sqrt{3^2-4(6)(-10)}}{2(6)}$$ B1 – quadratic formula

$$=\frac{-3\pm\sqrt{249}}{12}$$

$x=1.064977...$ or $x=-1.564977...$ M1 – correct values for both ans

$=1.06498$ (5dp) $\quad =-1.56498$ (5dp)

A1 for both ans in 5dp

Answer $x=$ .... or .... [3]

(e) Hence, find the estimated date and time that Apollo 11 reached the Moon.

Hence, $x=1.064977...$

Time taken $=\frac{320}{3(1.064977...)}$ M1

$=100.1585...$ hours

$=4$ days $4.1585..$ hours M1 – conversion of units

$=4$ days 4 hours 10 min (nearest minute)

Apollo 11 reached the Moon on **July 20, 1969**, at **1:42 pm**.

A1 – correct date, A1 – correct time

Answer **Date:** ...., **Time:** .... [4]

4 (a) $\mathbf{P}=\begin{pmatrix}2 & 1\\ 3 & -2\end{pmatrix}$ and $\mathbf{Q}=\begin{pmatrix}3 & -2\\ -6 & 11\end{pmatrix}$. Given that $\mathbf{P}^2-\mathbf{Q}=k\,\mathbf{P}$, find the value of k.

$$\begin{pmatrix}2 & 1\\ 3 & -2\end{pmatrix}\begin{pmatrix}2 & 1\\ 3 & -2\end{pmatrix}-\begin{pmatrix}3 & -2\\ -6 & 11\end{pmatrix}=\begin{pmatrix}4+3 & 2-2\\ 6-6 & 3+4\end{pmatrix}-\begin{pmatrix}3 & -2\\ -6 & 11\end{pmatrix}$$

B1 – correct multiplication

$$=\begin{pmatrix}7 & 0\\ 0 & 7\end{pmatrix}-\begin{pmatrix}3 & -2\\ -6 & 11\end{pmatrix}$$

$$=\begin{pmatrix}4 & 2\\ 6 & -4\end{pmatrix}$$

M1 – correct subtraction

$$=2\begin{pmatrix}2 & 1\\ 3 & -2\end{pmatrix}$$

$k=2$

A1

Answer $k=$ ....[3]

(b) A donut shop sells 3 flavours of donuts: cinnamon, chocolate and strawberry.

The table below shows the number of donuts sold by the shop at different times of a day (before 7pm and from 7pm to 9pm).

	Cinnamon	**Chocolate**	**Strawberry**
Before 7pm	150	240	165
7pm to 9pm	25	38	21

(i) Represent the data in the above table by a 2×3 matrix **A**.

$$\mathbf{A}=\begin{pmatrix}150 & 240 & 165\\ 25 & 38 & 21\end{pmatrix}$$

B1

Answer **A** = ... [1]

(ii) Given $\mathbf{B}=\begin{pmatrix}1\\1\\1\end{pmatrix}$, find **AB**.

$$\mathbf{AB}=\begin{pmatrix}150 & 240 & 165\\ 25 & 38 & 21\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$=\begin{pmatrix}150+240+165\\ 25+38+21\end{pmatrix}$$

$$=\begin{pmatrix}555\\ 84\end{pmatrix}$$

B1

Answer **AB** = ... [1]

(iii) Explain clearly what the elements in **AB** represent.

The elements represent the **total number of donuts sold** before 7pm and from 7pm to 9 pm of the day respectively. OR

555 donuts were sold before 7pm of the day and 84 donuts were sold from 7pm to 9pm of the day.

Before 7pm of the day, the price for each flavour of donuts is \$2.80.
From 7pm to 9pm, all donuts are sold at a 25% discount.

(iv) Write down a matrix **C** such that **CAB** gives the total amount of money collected from the sales of donuts for the day.

$$\mathbf{C} = \begin{pmatrix} 2.8 & 2.1 \end{pmatrix}$$ B1

Answer **C** = ... [1]

(v) Hence, find the total amount of money collected from the sales of donuts for the day.

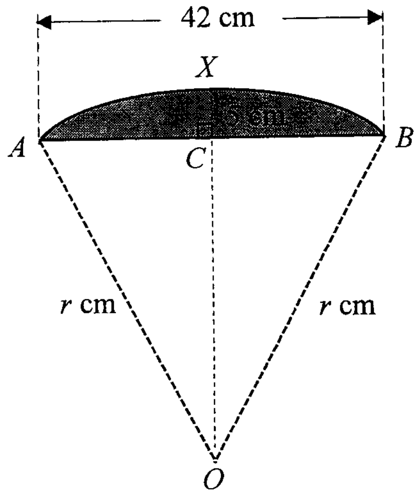
$$\begin{aligned}\mathbf{CAB} &= \begin{pmatrix} 2.8 & 2.1 \end{pmatrix}\begin{pmatrix} 555 \\ 84 \end{pmatrix} \\ &= \begin{pmatrix} 2.8(555) + 2.1(84) \end{pmatrix} \\ &= \begin{pmatrix} 1730.4 \end{pmatrix}\end{aligned}$$ M1

Total Amount of money collected = \$1730.40 A1 (A0 if **(1730.4)** not found)

Answer \$ .... [2]

5 The cross section of a speed bump, AXB, is formed by removing triangle AOB from sector $OAXB$. The sector, centre O, has a radius of r cm.

The width of the speed bump, AB, is 42 cm and the height CX is 5 cm.



(a) Show that $r = 46.6$.

Answer

OC is the height of triangle AOB.

$OC = (r - 5)$ cm — M1 – finding height in terms of r

$AC = 21$ cm ($\perp$ bisector of chord)

By Pythagoras' Theorem,

$r^2 = 21^2 + (r-5)^2$ — M1 – Pythagoras' Theorem

$r^2 = 441 + r^2 - 10r + 25$

$0 = 466 - 10r$

$10r = 466$ — M1 – Simplify to linear equation

$r = \dfrac{466}{10}$

$= 46.6$ (shown) — AG

[3]

(b) Calculate

(i) angle AOB in radians,

B1 – Cosine Rule or TOA CAH SOH for Angle AOC

$42^2 = 46.6^2 + 46.6^2 - 2(46.6)(46.6)\cos\angle AOB$

$\cos\angle AOB = \dfrac{46.6^2 + 46.6^2 - 42^2}{2(46.6)(46.6)}$ — M1 – Cosine Rule ($\cos\angle AOB$ the subject) or Multiply Angle AOC by 2

$= 0.59384...$

$\angle AOB = \cos^{-1}(0.59384...)$

$= 0.93497...$

$= 0.935$ rad (3sf) — A1

Answer .... [3]

(ii) the cross-sectional area of the speed bump.

M2ft – Area of Sector, Area of Triangle

$$\text{Area of cross section} = \frac{1}{2}(46.6)^2(0.93497...) - \frac{1}{2}(46.6)^2\sin(0.93497...)$$
$$= 1015.1746... - 873.6$$
$$= 141.574...$$
$$= 142 \text{ cm}^2 \text{ (3sf)}$$

A1

Answer .... [3]

According to safety regulations, the height of a speed bump cannot exceed 10 cm.
A new geometrically similar speed bump is designed such that the cross-sectional area is 225% of the cross-sectional area of the speed bump shown in the diagram.

(c) Does the new speed bump meet the safety regulations?
Show clear mathematical calculations to support your answer.
Answer

$$\frac{\text{Area}_{new}}{\text{Area}_{original}} = \frac{225}{100}$$
$$= \frac{9}{4} \quad \text{or} \quad 2.25$$
$$\frac{\text{Height}_{new}}{\text{Height}_{original}} = \sqrt{\frac{9}{4}} \quad \text{or} \quad \sqrt{2.25}$$

M1 – finding height ratio

$$\text{Height}_{new} = \frac{3}{2}(5) \quad \text{or} \quad 1.5(5)$$
$$= 7.5 \text{ cm}$$

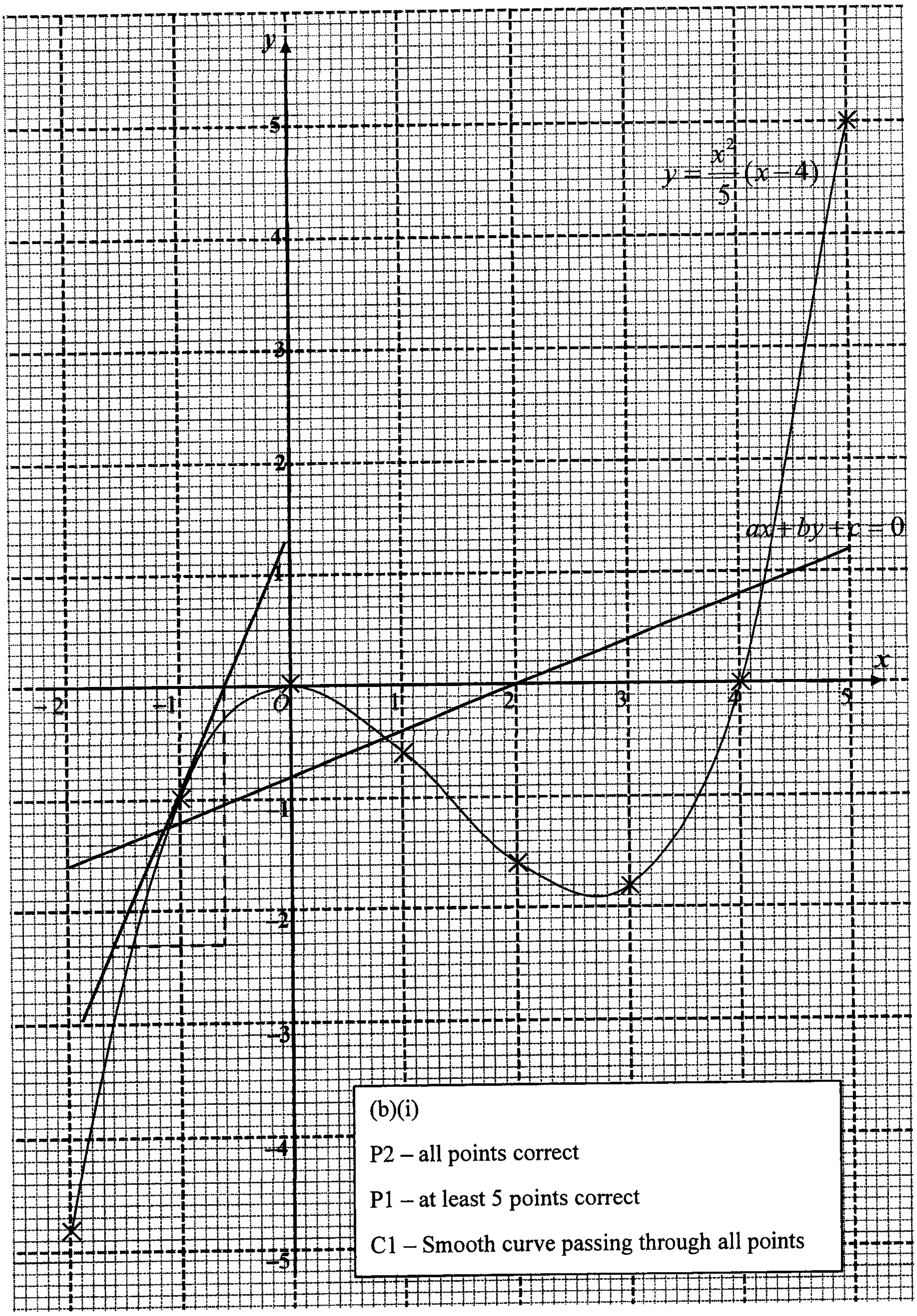
Since the height of the new speed bump **(7.5 cm) is less than 10 cm**, the new speed bump meets the safety regulations

A1 (height can be 3sf)

....

.... [2]

6 The graph of $ax + by + c = 0$, where a, b and c are integers, is drawn for $-2 \le x \le 5$ on the grid below. The line $ax + by + c = 0$ intersects the x-axis at (2, 0) and the y-axis at $(0, -0.8)$.



(b)(i)

P2 – all points correct

P1 – at least 5 points correct

C1 – Smooth curve passing through all points

(a) Given that a is a prime number, find the values of a, b and c.

y-intercept $= -0.8$, Gradient $= \dfrac{0-(-0.8)}{2-0}$ M1 – finding gradient

$= 0.4$

Equation of line: $y = 0.4x - 0.8$

Multiplying both sides by 5: $5y = 2x - 4$ M1 – equation of line (integer coefficients)

Equation of line in the form of $ax + by + c = 0$: $2x - 5y - 4 = 0$

$\therefore a = 2, b = -5, c = -4$ A1

Answer $a =$ .... , $b =$........ , c = ... [3]

(b) The table below shows the corresponding x and y values for the graph of $y = \dfrac{x^2}{5}(x-4)$.

x	−2	−1	0	1	2	3	4	5
y	−4.8	−1	0	−0.6	−1.6	−1.8	0	5

(i) On the same grid, plot the points given in the table and join them with a smooth curve. [3]

(ii) By drawing a tangent, find the gradient of the curve at $(-1,-1)$.

Gradient $= \dfrac{0-(-2.3)}{-0.6-(-1.6)}$ M1 – tangent drawn at $(-1,-1)$

$= 2.3$ Exact value = 2.2 (acceptable range: 1.7 to 2.7)

A1

Answer .... [2]

(iii) Write down the x-coordinates of the points where the line intersects the curve.

$x =$ **−1.1, 0.85** and **4.25** (exact to 2dp : −1.10, 0.85 and 4.25)

Accept +/−0.1 : B2 – all 3 answers correct, B1 – at least 2 correct .... [2]

(iv) These values of x are solutions of the equation $Px^3 + Qx^2 + ax + c = 0$.

Find the value of P and of Q.

$2x - 5y - 4 = 0$ ---- (1)

$y = \dfrac{x^2}{5}(x-4)$

$5y = x^2(x-4)$

$5y = x^3 - 4x^2$ ---- (2)

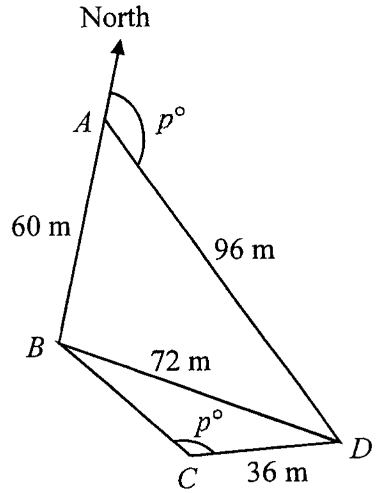
Substitute (2) into (1): $2x - (x^3 - 4x^2) - 4 = 0$ M1 – simultaneous eqns

$2x - x^3 + 4x^2 - 4 = 0$

$-x^3 + 4x^2 + 2x - 4 = 0$

$\therefore P = -1,\ Q = 4$ A2 – both answers

7 The diagram shows a field $ABCD$ on horizontal ground. BD is a path across the field.
$AB = 60$ m, $BD = 72$ m, $AD = 96$ m and $CD = 36$ m.
B is due south of A. Angle $BCD = p°$. The bearing of D from A is also $p°$.



(a) Without solving for any angles, show that the exact value of $\cos p°$ is -0.6625.

Answer

$$\cos \angle BAD = \frac{60^2 + 96^2 - 72^2}{2(60)(96)}$$

M1 – cosine rule

$$= \frac{7632}{11520}$$

$$= 0.6625 \quad \text{or} \quad \frac{53}{80}$$

Since $p° = 180° - \angle BAD$ (adjacent $\angle$s on a straight line),

$$\cos p° = -\cos \angle BAD$$
$$= -0.6625 \quad \text{(shown)}$$

M1 – supplementary angle rule or indicating p as obtuse angle

AG

[2]

(b) Hence, find angle CBD.

$$p° = \cos^{-1}(-0.6625)$$
$$= 131.4908...°$$

$$\frac{\sin \angle CBD}{36} = \frac{\sin 131.4908...°}{72}$$

M1 – sine rule

$$\sin \angle CBD = 0.37453...$$
$$\angle CBD = \sin^{-1}(0.37453...)$$
$$= 21.9953...°$$
$$= 22.0° \text{ (1dp)}$$

A1

Answer ....° [2]

(c) Find the area of the field, *ABCD*.

M1 – find angle *CDB*

$$\angle CDB = 180° - 21.9953...° - 131.4908...° \text{ (}\angle\text{ sum of triangle)}$$
$$= 26.5138...°$$

$$\angle BAD = 180° - 131.4908...° \text{ (adjacent }\angle\text{s on a straight line)}$$
$$= 48.5091...°$$

$$\text{Area of } ABCD = \frac{1}{2}(60)(96)(\sin 48.5091...°) + \frac{1}{2}(36)(72)(\sin 26.5138...°)$$
$$= 2157.2983... + 578.5528...$$
$$= 2735.8511...$$
$$= 2740 \text{ m}^2 \text{ (3sf)}$$

M1 – area of one triangle

A1

Answer .... m^2 [3]

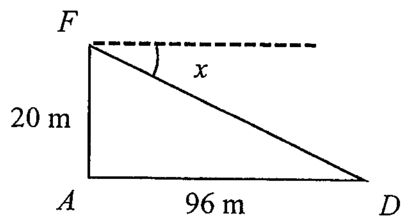
(d) A bird is at *F*, which is 20 m vertically above *A*.

Find the angle of depression of *D* from *F*.

Let x be the angle of depression.

$$\tan x = \frac{20}{96}$$ M1

$$x = \tan^{-1}\left(\frac{20}{96}\right)$$
$$= 11.768...$$
$$= 11.8° \text{ (1dp)}$$ A1



If find wrong angle (Angle *AFD*), M0, A0

Answer .... [2]

The town council would like to build a circular track passing through *A*, *B*, *C* and *D*.

(e) Determine whether this plan is feasible by explaining if it is possible to draw a circle passing through the 4 points.

Answer

$$\angle BAD = 180° - p° \text{ (adjacent }\angle\text{s on a straight line)}$$

$$\angle BCD = p° \text{ (given)}$$

M1 – state pair of supplementary angles

Since $\angle BAD + \angle BCD = 180°$, it satisfies the property "**angles in opposite segments are supplementary**". Therefore, *ABCD* is a cyclic quadrilateral and a circle can be drawn passing through the 4 points.

The plan is feasible.

A1 – conclude with angles in opposite segments

8 The position vectors of S and T are $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ k \end{pmatrix}$ respectively.

(a) Find the possible values of k if $|\overrightarrow{ST}| = \sqrt{45}$ units.

$$\overrightarrow{OS} = \begin{pmatrix} 5 \\ -4 \end{pmatrix},\ \overrightarrow{OT} = \begin{pmatrix} 2 \\ k \end{pmatrix}$$

$$\overrightarrow{ST} = \begin{pmatrix} 2 \\ k \end{pmatrix} - \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \\ k+4 \end{pmatrix}$$ B1

$$|\overrightarrow{ST}| = \sqrt{45}$$

$$\sqrt{(-3)^2 + (k+4)^2} = \sqrt{45}$$ M1

$$9 + (k+4)^2 = 45$$

$$(k+4)^2 = 36$$

$$k + 4 = \pm 6$$

$$k = -10 \quad \text{or} \quad k = 2$$ A1

Answer $k =$ .... or .... [3]

(b) $\overrightarrow{RS} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$. Find the equation of the line RS.

Gradient of $RS = \dfrac{6}{-2}$ OR Find coordinates of $R = (7, -10)$

$= -3$ M1

Equation of RS:

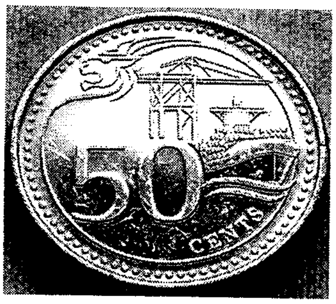
$$y - (-4) = -3(x - 5)$$

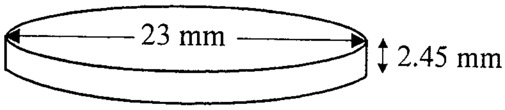
$$y + 4 = -3x + 15$$

$$y = -3x + 11$$ A1

Answer .... [2]

9 The Singapore 50-cent coin can be modelled by a cylinder with diameter of 23 mm and thickness of 2.45 mm.





(a) Show that the volume of the 50-cent coin is 1.018 cm^3 when correct to 4 significant figures.

Answer

23 mm = 2.3 cm
2.45 mm = 0.245 cm

$$\text{Volume of 50 cents coin} = \pi\left(\frac{2.3}{2}\right)^2(0.245)$$

$= 1.01791\ldots$
$= 1.018\ \text{cm}^3$ (4sf)

M1 – correct volume formula (can be in mm^3)

M1 – more dp before final ans

AG

[2]

(b) The Singapore 10-cent coin can also be modelled by a cylinder with diameter of 18.5 mm and thickness of 1.38 mm. Explain whether the 10-cent and 50-cent coins are geometrically similar.

Answer

$$\frac{\text{Diameter}_{10\text{-cent}}}{\text{Diameter}_{50\text{-cent}}} = \frac{18.5}{23} = \frac{37}{46} = 0.804 \text{ (3sf)}$$

$$\frac{\text{Thickness}_{10\text{-cent}}}{\text{Thickness}_{50\text{-cent}}} = \frac{1.38}{2.45} = \frac{138}{245} = 0.563 \text{ (3sf)}$$

M1 – finding ratio of diameter or thickness

Or finding volume ratio

Since $0.804 \neq 0.563$, the **ratio of the corresponding lengths (diameter and thickness) of the 2 coins are not equal**. Therefore, the 10-cent and 50-cent coins are not geometrically similar.

A1 – conclude using ratio of corresponding sides

....

.... [2]

You are asked to design a set of 3 coins. You can assume that all three coins are cylindrical in shape.

The first coin has a diameter of 16.5 mm, thickness of 1.4 mm and will be made using copper.

The second and third coins must meet the following requirements:

	Second Coin	**Third Coin**
Diameter (mm)	At least 25% larger than the diameter of First Coin	At least 15% larger than the diameter of Second Coin
Thickness (mm)	1.8 mm	2.4 mm
Mass (*M* g)	$M \geq 4.6$	$M < 7.8$
Type of metal to make coin	Steel	Zinc

The density of the metals are as follows:

Type of Metal	**Density (g/cm^3)**
Copper	8.96
Steel	7.86
Zinc	7.14

The minting machinery can only produce coins with diameters up to one decimal point of millimetres. For example, 17.1 mm is allowed but 17.13 mm is not.

(c) Find the mass of the first coin.

16.5 mm = 1.65 cm
1.4 mm = 0.14 cm

$$\text{Volume of } 1^{st} \text{ coin} = \pi\left(\frac{1.65}{2}\right)^2(0.14)$$

M1 – correct volume formula (can be in mm^3)

$$= 0.29935\ldots \text{ cm}^3$$

$$\text{Mass} = 0.29935\ldots \times 8.96$$

$$= 2.68 \text{ g (3sf)}$$

A1

Answer .... g [2]

(d) Showing your calculations clearly, suggest a possible diameter for the second coin and for the third coin.

M1: Understanding of **% larger** to find diameter of any coin.

M1: Correct **rounding up** of diameter for 2nd or 3rd coin (1dp in mm and satisfying at least).

M1: Volume for one coin with correct units (cm^3)

M1: Find mass with correct formula ($M = VD$) for any coin.

A1: Correct diameters for both coins.

2nd coin

Smallest diameter

$= \frac{125}{100} \times 16.5$

$= 20.625$

$= 20.7$ mm (**round up** to be at least 25% larger)

$\text{Volume} = \pi\left(\frac{2.07}{2}\right)^2 (0.18)$

$= 0.60576\ldots \text{ cm}^3$

$\text{Mass} = 0.60576\ldots \times 7.86$

$= 4.76$ g (3sf)

Since 4.76g > 4.6g, possible diameter for 2nd coin = 20.7 mm

3rd coin

$\text{Smallest diameter} = \frac{115}{100} \times 20.7$

$= 23.805$

$= 23.9$ mm (**round up** to be at least 15% larger)

$\text{Volume} = \pi\left(\frac{2.39}{2}\right)^2 (0.24)$

$= 1.0767\ldots \text{ cm}^3$

$\text{Mass} = 1.0767\ldots \times 7.14$

$= 7.69$ g (3sf)

Since 7.69g < 7.8g, possible diameter for 3rd coin = 23.9 mm

Therefore, possible diameter for the 2nd coin is 20.7mm and diameter of 3rd coin is 23.9 mm.

Other possible accepted diameters:

2nd coin: 20.8 mm (Mass = 4.81g), 3rd coin: 24 mm (Mass = 7.75g)

2nd coin: 20.7 mm (Mass = 4.76g), 3rd coin: 24 mm (Mass = 7.75g)

Incorrect answer: 2nd coin: 20.6 mm (Mass = 4.72g) with any 3rd coin diameter.

For students who started with 3rd coin to find maximum diameter, diameter of 3rd coin must be 24 mm or less (**For Mass = 7.8g, diameter = 24.07 mm**)

Steps:

3rd Coin

$$\text{Maximum volume for 3rd coin} = \frac{7.8}{7.14}$$

$$= 1.0924\ldots \text{ cm}^3$$

$$\text{Maximum radius} = \sqrt{\frac{1.0924\ldots}{\pi(0.24)}}$$

$$= 1.20369\ldots \text{ cm}$$

$$\text{Maximum diameter} = 2.40739\ldots$$

$$= 24.07 \text{ mm}$$

Since Mass of 3rd coin must be less than 7.8g, maximum diameter of 3rd coin = 24.0 mm

2nd Coin

Since 3rd coin is at least 15% larger than 2nd coin,

$$\text{Largest possible diameter} = \frac{100}{115} \times 24.0$$

$$= 20.869\ldots$$

$$= 20.8 \text{ mm (round down)}$$

Check Mass and diameter meets requirement:

$$\text{Mass} = \pi\left(\frac{2.08}{2}\right)^2 (0.18)(7.86)$$

$$= 4.81 \text{ g (3sf) (more than 4.6 g)}$$

$$\frac{\text{Diameter of 2nd coin}}{\text{Diameter of 1st coin}} = \frac{20.8}{16.5}$$

$$= 1.2606\ldots$$

Since Diameter of 2nd coin is 126% of diameter of 1st coin, it also satisfies the diameter requirement.

Therefore, diameter of 2nd coin = 20.8 mm, diameter of 3rd coin = 24.0 mm