1 The function f is defined by $f(x) = \ln\frac{\sqrt{x^2+5}}{x}$ for $x > 0$.

Show that f is a decreasing function for all values of $x > 0$. [3]

2 (i) Sketch, on the same diagram, the graphs of $y = -\frac{5}{x^{\frac{1}{2}}}$ and $y^2 = 4x$. [2]

(ii) State the value of k for which the x-coordinate of the point of intersection of these two graphs satisfies the equation $x^2 = k$. [2]

3 You just bought a brand new car. The value, V dollars, of the car depreciates over time. It is given that $V = 84000e^{kt} + 8500$, where t is the time in years since it was bought and k is a constant.

(i) What is the initial value of the vehicle? [1]

(ii) Calculate the value of k if, after 3 years, the value of the car is halved. [2]

(iii) After having driven the car for 25 years, you decided to change to a new car. A second-hand car dealer offers to buy the old car from you for $8000. Without using a calculator, justify whether you should accept the offer. [2]

4 (a) Find the values of k for which $3x(x+2)+k^2$ is never negative for all real values of x. [3]

(b) Given that $3x^2 + px + 84 < 0$ only when $4 < x < k$, find the value of p and of k. [3]

5 (a) Simplify $\log_3 2 \times \log_4 3 \times \log_5 4 \times .... \times \log_{n+1} n$. [2]

(b) Using the substitution $u = 6^x$, solve the equation $6^{x+1} - 6^{1-x} = 5$. [4]

[Turn over

6 (i) Prove that $\sec\theta - \dfrac{\cos\theta}{1+\sin\theta} = \tan\theta$. [3]

(ii) Find, in radians, for $0 < \theta < \pi$, the exact values of θ for which

$\sec\theta - \dfrac{\cos\theta}{1+\sin\theta} = \dfrac{1}{3}\cot\theta$. [3]

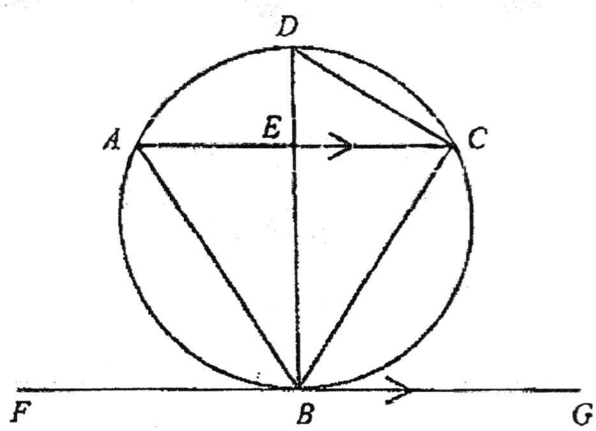
7 The point $P(-1,0)$ is a point on the graph of $y = |kx-2|$.

(i) Show that $k = -2$. [2]

(ii) Sketch the graph of $y = |kx-2|$, indicating the points of intersection with the axes. [3]

(iii) Hence, write down the range(s) of values of x for which $y > 2$. [1]

8 The diagram shows triangles ABC and BCD whose vertices lie on the circumference of a circle. The chords BD and AC intersect at E and AC is parallel to FG. FG is a tangent to the circle at B.



Show that

(i) triangle BCD is similar to triangle BEC, [3]

(ii) $BC^2 = BD \times BE$, [2]

(iii) triangle ABC is an isosceles triangle. [2]

9 A curve has the equation $y=6\sqrt{(1+2x)^3}$

(i) A point P moves along the curve in such a way that the x-coordinate of P is decreasing at a constant rate of 0.04 units per second. Find the x-coordinate of P at the instant the y-coordinate is decreasing at a rate of 0.045 units per second. [4]

(ii) Find the x-coordinate of the curve that splits the area bounded by the curve, the x-axis and the lines $x=2$ and $x=5$, into 2 halves of equal area. [4]

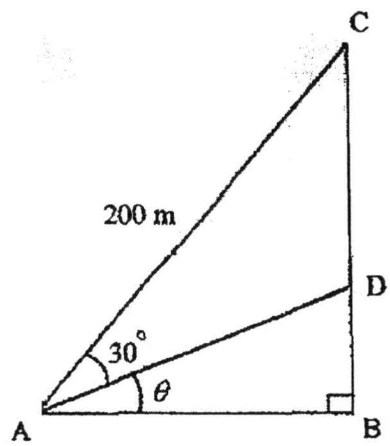
10 The function f is such that $f(x)=2\sin^2 x-\cos^2 x$.

(i) By expressing $f(x)$ in the form of $a+b\cos 2x$, show that $a=\frac{1}{2}$ and $b=-\frac{3}{2}$. [3]

(ii) Sketch the graph of $f(x)$ for $0\leq x\leq 2\pi$. [3]

(iii) By drawing a suitable line on the same axes, state the number of solutions to the equation $4\pi\sin^2 x-2\pi\cos^2 x-x=2\pi$. [3]

11



The Urban Redevelopment Authority (URA) in Singapore is gazetting a piece of right-angled triangular-shaped land ABC in Hougang Avenue 8. URA plans to build a public skate arena shown in the diagram.
An area ABD is to be built with ramps. $AC=200$ m and $\angle BAD=\theta$, where $0°<\theta<90°$

(i) Show that the area, A m^2, of the triangle ABC is given by $A=5000(\sin 2\theta+\sqrt{3}\cos 2\theta)$. [4]

(ii) Find $\frac{dA}{d\theta}$. [1]

(iii) Find the value of θ for which the area of the triangle ABC is maximum. [4]

[Turn over

12 A particle P travels in a straight line so that its velocity, v m/s, at time t seconds is given by $v = t^2 - 5t + 6$. The particle first crosses the fixed point O at $t = 1.5$ s.

(i) Find the acceleration of the particle at $t = 4$ s. [2]

(ii) Find the time interval during which the particle's velocity is decreasing. [2]

(iii) Find the displacement of the particle from O when it is first instantaneously at rest. [4]

(iv) Find the average speed of the particle for the first three seconds. [3]

End of Paper

1 The area of a right-angled triangle is $(4+6\sqrt{6})\text{ cm}^2$. The base of the triangle is $(6\sqrt{3}+\sqrt{8})\text{ cm}$.

(i) Show that the perpendicular height of the triangle, h, can be expressed as $a\sqrt{b}$ cm, where a and b are integers. [4]

(ii) The longest length of the right-angled triangle is H cm. Express H^2 in the form $p+q\sqrt{6}$, where p and q are integers. [3]

2 (i) Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan x \sin^2 x\right) = 2\sin^2 x + \sec^2 x - 1$. [3]

(ii) Hence find $\int_{\pi/[illegible]}^{\pi} \sin^2 x \, \mathrm{d}x$, leaving your answer in exact form. [4]

3 The roots of the quadratic equation $2x^2 - 3x + 4 = 0$ are α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [3]

(ii) Show that the value of $\alpha^3 + \beta^3$ is $-\frac{45}{8}$. [2]

(iii) Find the quadratic equation, with integer coefficients, whose roots are $\frac{\alpha}{\beta^2+1}$ and $\frac{\beta}{\alpha^2+1}$. [4]

4 The positive y-axis and the line $y = 3$ are tangents to a circle C. It is given that the x-coordinate of the centre of C is a, where $a > 0$.

(i) Write down the larger possible y-coordinate of the centre of C, in terms of a. [1]

The line L is a tangent to C at the point $(8, 12)$ on the circle. The centre of C lies below and to the left of $(8, 12)$.

(ii) Show that $a = 5$ and write down the centre of C. [3]

(iii) Find

(a) the equation of C, [1]

(b) the equation of L, [3]

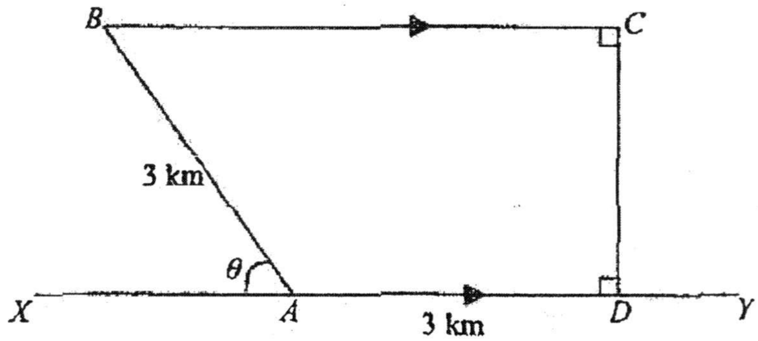
(c) the equation of the circle which is a reflection of C in the y-axis. [1]

5 (a) (i) Write down, in terms of n and y, the first 3 terms in the expansion of $(1+y)^n$. [2]

(ii) Hence or otherwise, find the value of n in the expansion of $(1+x+2x^2)^n$, given that the coefficient of x^2 is 44. [3]

(b) In the expansion of $\left(2x^2-\frac{1}{x}\right)^{12}$, find the ratio of the term independent of x to that of the coefficient of the middle term. [5]

6 Billy signed up for a race and was given a brochure showing the race route.



XY is a straight road. Participants would start running from point A to D, then from D to C, followed by C to B and finally from B back to A. BC is parallel to XY. CD is perpendicular to both BC and XY. $AB = AD = 3$ km and angle XAB is θ°. The total distance of the route is L km.

(i) Show that L can be expressed as $p\cos\theta + q\sin\theta + r$, where p, q and r are constants. [3]

(ii) Express L in the form $R\cos(\theta-\alpha)+r$, where $R > 0$ and α is an acute angle. [2]

(iii) The total length of the route is found to be 13 km. Find the values of θ. [3]

(iv) Billy claims that he can finish the race in under 49 minutes if he maintains his speed of 16 km/h throughout the race regardless of the value of θ. Is Billy's claim true? Explain your answer. [2]

7 **Answer the whole of this question on a sheet of graph paper.**

A particle moving in a certain medium, with speed v m/s, experiences a resistance to motion of R newtons. R and v are related by the equation $R = av^2 + bv$, where a and b are constants.

v	5	10	15	20	25
R	17	44	81	138	185

The table shows the experimental values of the variables v and R, but an error has been made in recording one of the values of R.

(i) Using graph paper, draw the graph of $\frac{R}{v}$ against v. [3]

Use your graph to

(ii) write down the value of v for which its recorded R value was incorrect and find the correct value of R, [2]

(iii) estimate the value of a and of b. [3]

In a different medium, R is directly proportional to v and $R = 30$ when $v = 5$.

(iv) Draw a suitable line on your graph to illustrate the second situation and use it to determine the value of v for which the resistance is the same in both mediums. [3]

8 The function $f(x) = 3x^3 + ax^2 + bx + 2$, where a and b are constants. $x - 1$ is a factor of $f(x)$. The remainder when $f(x)$ is divided by $x - 2$ is 2.5 times the remainder when $f(x)$ is divided by $x + 1$.

(i) Show that $a = 2$ and $b = -7$. [4]

(ii) Without using a calculator, solve $f(x) = 0$. [3]

(iii) Hence solve $3\sin^2 y - 2\sec y - 2\cos y + 4 = 0$ for $0 \le y \le 360°$. [4]

[TURN OVER

9 The equation of the curve is $y=\dfrac{4x-12}{x+3}$. The point P lies on the curve and has a positive x-coordinate. The normal to the curve at P makes an angle θ with the x-axis such that $\tan\theta=-6$.

(a) Show that the coordinates of P is (9 , 2). [4]

The point Q also lies on the curve and has a positive x-coordinate. The tangent to the curve at Q is parallel to the line $3y-2x=6$.

(b) Find the coordinates of Q. [3]

It is given further that the coordinates of R and S are (5 , −4) and (13 , −1) respectively.

(c) Determine whether $PQRS$ is a kite or not. Justify your answer. [2]

(d) Calculate the area of $PQRS$. [2]

10 (a) A curve is such that $\dfrac{d^2y}{dx^2}=4e^{-2x+1}$ and the gradient at $(2,e^{-3})$ is $-\dfrac{2}{e^3}-4$.

(i) Find $\dfrac{dy}{dx}$ in terms of x. [3]

(ii) Explain why the curve has no stationary points. [2]

(iii) Find the equation of the curve. [3]

(b) (i) Express $\dfrac{17+6x-5x^2}{(2x-1)(3-x)^2}$ in partial fractions. [4]

(ii) Hence find $\displaystyle\int\frac{17+6x-5x^2}{(2x-1)(3-x)^2}dx$. [3]

End Of Paper

Paper 1 Answer

Q1

Q. 1 Prelim 2017 AMath P1.

Final

$$f(x) = \ln \frac{\sqrt{x^2+5}}{x}$$

$$= \frac{1}{2}\ln(x^2+5) - \ln x$$

$$f'(x) = \frac{1}{2}\left(\frac{2x}{x^2+5}\right) - \frac{1}{x}$$

$$= \frac{x^2 - x^2 - 5}{x(x^2+5)}$$

$$= -\frac{5}{x(x^2+5)}$$ — (M1)

if $x > 0$, then $x(x^2+5) > 0$ — (M1)

$f'(x) < 0$ for $x > 0$

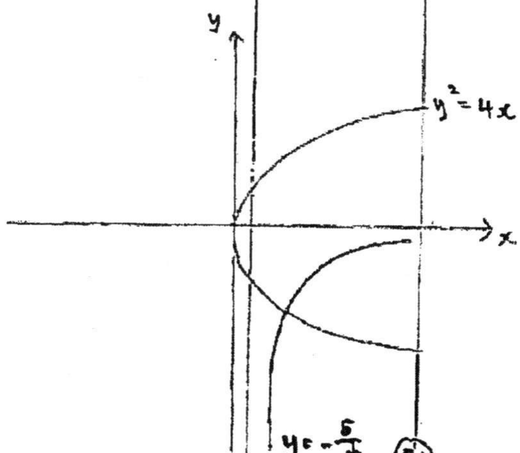
Hence, f in a decreasing function for $x > 0$ } both: (A1)

/3

Q2

Q.2

(i)



(ii) $y = -\frac{5}{x^{\frac{3}{2}}}$ — (1)

$y^2 = 4x$ — (2)

sub (1) into (2) : } either : (M1)

$\frac{25}{x^3} = 4x$

$x^4 = \frac{25}{4}$

$\therefore k = \frac{25}{4}$ (A1)

/(4)

Q3

Q.3

(i) $V = 84000e^{kt} + 8500$

when $t=0$, $V = 84000 + 8500 = 92,500$ — (B1)

$\therefore$ initial value of car $= \$92,500$.

(ii) When $t=3$

$V = 84000e^{k(3)} + 8500 = \frac{92,500}{2}$ — (M1)

$e^{3k} = 0.4494$

$3k = \ln 0.4494$

$\therefore k = -0.2666$

$= -0.267$ (3 sf) — (A1)

(iii) $V = 84000e^{-0.267t} + 8500$

Since $84000e^{-0.267t} > 0$ (OR) As $t \to \infty$, $e^{-0.267t} \to 0$

$\therefore V > 8500$ either: (M1) $V \to 8500$

Since value of car is at least \$8,500, you should not accept the offer

(M1): to mention at least \$8,500 or > \$8,500.

5

Q4

Q.4

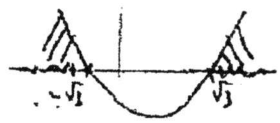
(i) let $y = 3x(x+2) + k^2$

$= 3x^2 + 6x + k^2$

$\therefore\ 6^2 - 4(3)(k^2) \le 0$ — (M1)

$12k^2 - 36 \ge 0$

$(k+\sqrt{3})(k-\sqrt{3}) \ge 0$ — (M1)



$\therefore\ k \le -\sqrt{3}$ or $k \ge \sqrt{3}$ — (A1)

(ii)

$(x-4)(x-k) = 0$

$x^2 - 4x - kx + 4k = 0$

$3x^2 + 3(-4-k)x + 12k = 0$

(OR) $3x^2 - 12x - 3kx + 12k = 0$ } either: (M1)

(OR) $3(x-4)(x-k) < 0$

$3x^2 + 3(-4-k)x + 12k < 0$

By comparison,

$12k = 84$

$\therefore\ k = 7$ — (A1)

and $3(-4-k) = p$

$\therefore\ p = -33$

(OR) $-12 - 3k = p$

$\therefore\ p = -33$ } — either: (A1)

(6)

Q5

Q.5

(i) $\log_3 2 \times \log_4 3 \times \log_5 4 \times \ldots \times \log_{(n+1)} n$

$= \frac{\lg 2}{\lg 3} \times \frac{\lg 3}{\lg 4} \times \frac{\lg 4}{\lg 5} \times \ldots \frac{\lg n}{\lg(n+1)}$ — (M1) or using ln

$= \frac{\lg 2}{\lg(n+1)}$ (OR) $\frac{1}{\log_2 (n+1)}$ — (A1)

(OR) $\log_{n+1} 2$

(ii) $6^{x+1} - 6^{1-x} = 5$

$6(6^x) - \frac{6}{6^x} = 5$ — (M1)

Let $u = 6^x$

$\therefore 6u - \frac{6}{u} - 5 = 0$

$6u^2 - 5u - 6 = 0$ — (M1)

$(3u+2)(2u-3) = 0$

$\therefore u = \frac{3}{2}$ $\left(u = -\frac{2}{3} \text{ is rejected} \right)$ } — (M1)

Hence, $6^x = \frac{3}{2}$

$x = \frac{\lg \frac{3}{2}}{\lg 6}$

$= 0.226$ (3 sf) — (A1)

(6)

Q6

Q.6

(i) $\sec\theta - \frac{\cos\theta}{1+\sin\theta} = \tan\theta$

$$LHS = \frac{1}{\cos\theta} - \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{1+\sin\theta - \cos^2\theta}{\cos\theta(1+\sin\theta)} \quad \text{— (M1)}$$

$$= \frac{1+\sin\theta - (1-\sin^2\theta)}{\cos\theta(1+\sin\theta)} \quad \text{(OR)} \quad \frac{\sin^2\theta+\cos^2\theta+\sin\theta-\cos^2\theta}{\cos\theta(1-\sin\theta)}$$

$$= \frac{\sin^2\theta+\sin\theta}{\cos\theta(1+\sin\theta)} \quad \text{— (M1)}$$

$$= \frac{\sin\theta(\sin\theta+1)}{\cos\theta(1+\sin\theta)}$$

$$= \tan\theta \ \text{(proven)} \quad \text{— (A1)}$$

(ii) For $\sec\theta - \frac{\cos\theta}{1+\sin\theta} = \frac{1}{3}\cot\theta$

$\therefore \tan\theta = \frac{1}{3}\cot\theta$

$\tan^2\theta = \frac{1}{3}$ — (M1)

$\tan\theta = \pm\sqrt{\frac{1}{3}}$

$\theta = \frac{\pi}{6}$ or $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$ — (A1)

(A1)

/ (6)

Q7

Q. 7.

(i) subst. P(−1, 0) into

$y = |kx - 2|$

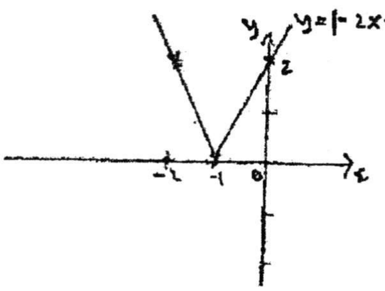
$0 = |-k - 2|$ — (M1)

$\therefore k = -2$ (shown) — (A1)

(ii) $y = |-2x - 2|$

when $x = 0$, $y = 2$

when $x = -2$, $y = 2$ — Any pt for $x < -1$ calculated — (M1)

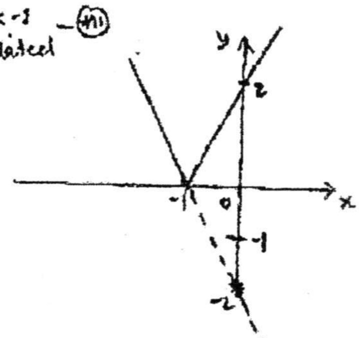


Graph: intercepts shown on axes — (M1)

correct graph — (A1)

(OR) for $y = -2x - 2$

when $x = 0$, $y = -2$



$y = -2x - 2$ drawn — (M1)

(iii) If not shown above:

when $y = 2$, $-2x - 2 = 2$ or $-2x - 2 = -2$

$x = -2$ or $x = 0$

$\therefore x < -2$ or $x > 0$ — (B1)

/6

Q8

Q.8

(i) $\angle CBG = \angle BDC$ (alt. segment theorem) — (M1)

$\angle CBG = \angle BCE$ (alt. $\angle$s, AC // FG)

$\therefore \angle BDC = \angle BCE$

$\angle CBD = \angle EBC$ (common $\angle$) — (M1)

$\therefore$ triangle BCD is similar to triangle BEC — (A1)
(AAA similarity) (shown)

(ii) $\frac{BC}{BD} = \frac{BE}{BC}$ — (M1)

$\therefore BC^2 = BD \times BE$ (shown) — (A1)

(iii) $\angle BAC = \angle BDC$ ($\angle$s in same segment) — (M1)

$\angle BDC = \angle BCE$ (above) — (M1)

$\therefore \angle BAC = \angle BCE$

Hence $\triangle ABC$ is isosceles (shown)

(OR) (OR)

$\angle CBG = \angle BCA$ (above) — (M1)

$\angle CBG = \angle CAB$ (alt. segment theorem) — (M1)

$\therefore \angle BCA = \angle CAB$

Hence, $\triangle ABC$ is isosceles (shown)

/ (7)

Q9

Q.9

(i) $y = 6\sqrt{(1+2x)^3} = 6(1+2x)^{\frac{3}{2}}$

when $\frac{dx}{dt} = -0.04$ and $\frac{dy}{dt} = -0.045$ } both: (M1) (or shown below)

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$ — (M1) for $\frac{dy}{dx}$

$-0.045 = 6\left(\frac{3}{2}\right)(1+2x)^{\frac{1}{2}}(2) \cdot (-0.04)$ } (M1) for $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{d}{d}$

$(1+2x)^{\frac{1}{2}} = \frac{0.045}{0.04} \times \frac{1}{18}$

$(1+2x) = 0.003906$

$\therefore x = -0.4980$

$= -0.498$ (3 s.f.) — (A1)

(ii) Let the x-coordinate $= a$

$\therefore \int_2^a 6(1+2x)^{\frac{3}{2}}\,dx = \int_a^5 6(1+2x)^{\frac{3}{2}}\,dx$ — (M1)

$6\left[\frac{2}{5}(1+2x)^{\frac{5}{2}}\left(\frac{1}{2}\right)\right]_2^a = 6\left[\frac{2}{5}(1+2x)^{\frac{5}{2}}\left(\frac{1}{2}\right)\right]_a^5$ — (M1)

$\left[(1+2x)^{\frac{5}{2}}\right]_2^a = \left[(1+2x)^{\frac{5}{2}}\right]_a^5$

$(1+2a)^{\frac{5}{2}} - (1+2(2))^{\frac{5}{2}} = (1+2(5))^{\frac{5}{2}} - (1+2a)^{\frac{5}{2}}$ } either (M1)

$2(1+2a)^{\frac{5}{2}} = (11)^{\frac{5}{2}} + (5)^{\frac{5}{2}}$

$(1+2a)^{\frac{5}{2}} = \frac{457.2}{2}$

$1+2a = 8.782$

$a = 3.891$

$= 3.89$ (3 s.f.) — (A1)

/8

Q10

Q10.

(i) $f(x) = 2\sin^2 x - \cos^2 x$

$= 2(1-\cos^2 x) - \cos^2 x$ — (M1)

$= 2 - 3\cos^2 x$

$= \frac{1}{2} - \frac{3}{2}(2\cos^2 x - 1)$ — (M1)

$= \frac{1}{2} - \frac{3}{2}\cos 2x$ — (A1)

$\therefore a = \frac{1}{2}$ and $b = -\frac{3}{2}$ (shown)

(OR) $f(x) = 2\sin^2 x - \cos^2 x$

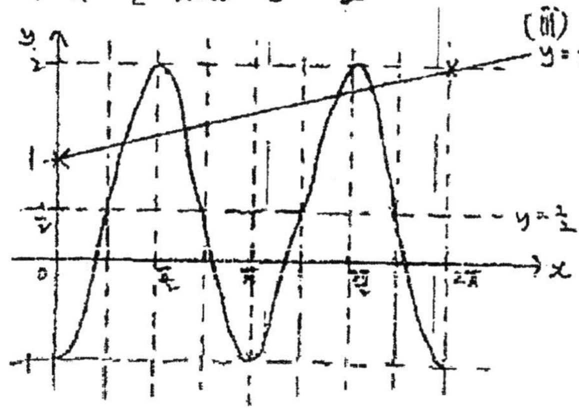
$= 1 - \cos 2x - \frac{\cos 2x + 1}{2}$ — (M1)

(M1) — $= 1 - \cos 2x - \frac{1}{2}\cos 2x - \frac{1}{2}$

$= \frac{1}{2} - \frac{3}{2}\cos 2x$ — (A1)

$\therefore a = \frac{1}{2}$ and $b = -\frac{3}{2}$

(ii)



(iii) $y = \frac{1}{2\pi}x + 1$ — correct line drawn (M1)

(M1) Graph to correctly intersect $y = \frac{1}{2}$

(M1) max = $y = 2$, min = $y = -1$, amplitude = $\frac{3}{2}$, 2 cycles. ($\pi \times 2$)

(A1) correct shape

(iii) $4\pi\sin^2 x - 2\pi\cos^2 x - x = 2\pi$

$2\pi(2\sin^2 x - \cos^2 x) = x + 2\pi$

$2\sin^2 x - \cos^2 x = \frac{1}{2\pi}x + 1$ — (M1)

Draw $y = \frac{1}{2\pi}x + 1$

when $x = 2\pi$, $y = 2$

$\therefore$ no. of solutions $= 4$ — (A1)

9

Q11

Q.11

(i) $A = \frac{1}{2}[200\sin(\theta+30^\circ)][200\cos(\theta+30^\circ)]$ — (M1)

$= 20000(\sin\theta\cos30^\circ + \cos\theta\sin30^\circ)$
$\times(\cos\theta\cos30^\circ - \sin\theta\sin30^\circ)$

$= 20000(\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta)(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta)$ — (M1)

$= 5000(3\sin\theta\cos\theta - \sqrt{3}\sin^2\theta + \sqrt{3}\cos^2\theta - \sin\theta\cos\theta)$ (M1)

$= 5000(\sin2\theta + \sqrt{3}\cos2\theta)$ — (A1)

(ii) $\frac{dA}{d\theta} = 5000(2\cos2\theta - 2\sqrt{3}\sin2\theta)$
$= 10000(\cos2\theta - \sqrt{3}\sin2\theta)$ } either: — (B1)

(iii) Let $\frac{dA}{d\theta} = 10000(\cos2\theta - \sqrt{3}\sin2\theta) = 0$

$\therefore \tan2\theta = \frac{1}{\sqrt{3}}$ — (M1)

$\therefore 2\theta = 30^\circ$

$\theta = 15^\circ$ — (A1)

$\frac{d^2A}{d\theta^2} = 10000(-2\sin2\theta - 2\sqrt{3}\cos2\theta)$
$= -20000(\sin2\theta + \sqrt{3}\cos2\theta)$ } either: — (M1)

when $\theta = 15^\circ$

$\frac{d^2A}{d\theta^2} = -20000(\sin30^\circ + \sqrt{3}\cos30^\circ) < 0$

$\therefore$ at $\theta = 15^\circ$, area of triangle is maximum } both: (A1)

(9)

Q12

Q.12

(i) $v = t^2 - 5t + 6$

$a = \frac{dv}{dt} = 2t - 5$ — (M1)

at $t = 4$, acceleration $= 3\ m/s^2$ — (A1)

(ii) for velocity to be decreasing;

$a < 0$

$2t - 5 < 0$ — (M1)

$t < \frac{5}{2}$ (or 2.5)

$\therefore$ time interval is $0 < t < \frac{5}{2}$ s — (A1)

(iii) At rest, $v = 0$

$t^2 - 5t + 6 = 0$

$(t-2)(t-3) = 0$

$t = 2$ or 3 — (M1)

$s = \int (t^2 - 5t + 6)\, dt$

$= \frac{t^3}{3} - \frac{5t^2}{2} + 6t + c$ — (M1)

when $t = 1.5$, $s = \frac{(1.5)^3}{3} - \frac{5(1.5)^2}{2} + 6(1.5) + c = 0$

$c = -4.5$

$s = \frac{t^3}{3} - \frac{5t^2}{2} + 6t - 4.5$ (M1)

$\therefore$ at $t = 2$,

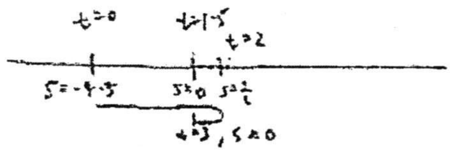
$s = \frac{2^3}{3} - \frac{5(2)^2}{2} + 6(2) - 4.5$

$= \frac{1}{6}$ m (or 0.167 m) — (A1)

Displacement required $= \frac{1}{6}$ m (or 0.167 m)

Q.12

(iv)



at $t=0$, $s = -4.5$ m

at $t=3$, $s = \frac{3^3}{3} - \frac{5(3)^2}{2} + 6(3) - 4.5$ — (M1)

$= 0$

$\therefore$ average speed $= \frac{4.5 + \frac{1}{6} + \frac{1}{6}}{3}$ } (M1) for distance travelled

$= 1.611$

≈ 1.61 (or $1\frac{11}{18}$) m/s — (A1)

/ (11)

Paper 2 Answer

Q1

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10a. $\frac{1}{2} \times h \times (6\sqrt{3}+\sqrt{8}) = 4+6\sqrt{6}$.

$h = \frac{8+12\sqrt{6}}{6\sqrt{3}+\sqrt{8}} \times \frac{6\sqrt{3}-\sqrt{8}}{6\sqrt{3}-\sqrt{8}}$ [M1]

$= \frac{48\sqrt{3}-8\sqrt{8}+72\sqrt{18}-12\sqrt{48}}{36(3)-8}$ [M1]

$= \frac{48\sqrt{3}-16\sqrt{2}+216\sqrt{2}-48\sqrt{3}}{100}$ [M1] Simplify: $\sqrt{8}$, $\sqrt{18}$, $\sqrt{48}$

$= \frac{200\sqrt{2}}{100} = 2\sqrt{2}$ cm. [A1]

b. $H^2 = (2\sqrt{2})^2 + (6\sqrt{3}+\sqrt{8})^2$ [M1]

$= 4(2) + 36(3) + 12\sqrt{24} + 8$ [M1]

$= 124 + 12\sqrt{24}$

$= 124 + 24\sqrt{6}$ [A1]

EX 255 (rev 2008)

Q2

2 i. $\frac{d}{dx}(\tan x \sin^2 x) = \sin^2 x \sec^2 x + \tan x (2\sin x)\cos x$ [M1]

$= \sin^2 x \left(\frac{1}{\cos^2 x}\right) + \frac{\sin x}{\cos x}(2\sin x)\cos x$ [M1] for either

$= \tan^2 x + 2\sin^2 x$

$= 2\sin^2 x + \sec^2 x - 1.$ [A1]

ii $\int_{\frac{\pi}{4}}^{\pi} \sin^2 x \, dx = \frac{1}{2}\int_{\frac{\pi}{4}}^{\pi} 2\sin^2 x + \sec^2 x - 1 - \sec^2 x + 1 \, dx$ [M1]

$= \frac{1}{2}\int_{\frac{\pi}{4}}^{\pi} 2\sin^2 x + \sec^2 x - 1 \, dx + \frac{1}{2}\int_{\frac{\pi}{4}}^{\pi} 1 - \sec^2 x \, dx$

[M1] [M1]

$= \frac{1}{2}\left[\tan x \sin^2 x\right]_{\frac{\pi}{4}}^{\pi} + \frac{1}{2}\left[x - \tan x\right]_{\frac{\pi}{4}}^{\pi}$

$= \frac{1}{2}\left[\tan\pi \sin^2\pi - \tan\frac{\pi}{4}\sin^2\frac{\pi}{4}\right] + \frac{1}{2}\left[\pi - \tan\pi - \frac{\pi}{4} + \tan\frac{\pi}{4}\right]$

$= \frac{1}{2}\left[-\left(\frac{\sqrt{2}}{2}\right)^2\right] + \frac{1}{2}\left[\pi - \frac{\pi}{4} + 1\right]$

$= \frac{1}{2}\left[-\frac{2}{4}\right] + \frac{1}{2}\left[\frac{3\pi}{4} + 1\right]$

$= -\frac{1}{4} + \frac{3\pi}{8} + \frac{1}{2}$

$= \frac{1}{4} + \frac{3\pi}{8}$ [A1]

OR $\int_{\frac{\pi}{4}}^{\pi} 2\sin^2 x + \sec^2 x - 1 \, dx = \left[\tan x \sin^2 x\right]_{\frac{\pi}{4}}^{\pi}$ [M1]

$2\int_{\pi/4}^{\pi} \sin^2 x \, dx + \left[\tan x - x\right]_{\pi/4}^{\pi} = \left[\tan x \sin^2 x\right]_{\frac{\pi}{4}}^{\pi}$ [M1]

$2\int_{\pi/4}^{\pi} \sin^2 x \, dx = \left[\tan x \sin^2 x - \tan x + x\right]_{\pi/4}^{\pi}$ [M1]

$= \pi - \left[\frac{1}{2} - 1 + \frac{\pi}{4}\right]$

$\int_{\pi/4}^{\pi} \sin^2 x \, dx = \frac{1}{4} + \frac{3\pi}{8}$ [A1]

Q3

FX 255 (rev 2008)

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3 i. $2x^2 - 3x + 4 = 0$

$\alpha + \beta = \frac{3}{2}$ [M1]

$\alpha\beta = 2$ [M1]

$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{3}{2}\right)^2 - 2(2)$

$= -\frac{7}{4}$ [A1]

ii. $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= \frac{3}{2}\left(-\frac{7}{4} - 2\right)$ [M1]

$= -\frac{45}{8}$ [A1]

iii $\frac{\alpha}{\beta^2+1} + \frac{\beta}{\alpha^2+1} = \frac{\alpha(\alpha^2+1) + \beta(\beta^2+1)}{(\alpha\beta)^2 + \beta^2 + \alpha^2 + 1}$

$= \frac{\alpha^3 + \alpha + \beta^3 + \beta}{(\alpha\beta)^2 + \alpha^2 + \beta^2 + 1}$

$= \frac{-\frac{45}{8} + \frac{3}{2}}{2^2 - \frac{7}{4} + 1}$ [M1]

$= -\frac{33}{26}$ [M1]

$\left(\frac{\alpha}{\beta^2+1}\right)\left(\frac{\beta}{\alpha^2+1}\right) = \frac{\alpha\beta}{(\alpha\beta)^2 + \beta^2 + \alpha^2 + 1}$

$= \frac{2}{2^2 - \frac{7}{4} + 1}$

$= \frac{8}{13}$ [M1]

$x^2 + \frac{33}{26}x + \frac{8}{13} = 0$

$26x^2 + 33x + 16 = 0$ [A1].

Q4

4 i.	y-coordinate = a+3	[B1]
ii	centre = (a, a+3)	
	$\sqrt{(a-8)^2+(a+3-12)^2} = a$	[M1]
	$(a-8)^2 + (a-9)^2 = a^2$	
	$a^2 - 16a + 64 + a^2 - 18a + 81 = a^2$	
	$2a^2 - 34a + 145 - a^2 = 0$	
	$a^2 - 34a + 145 = 0$	
	$a = \dfrac{34 \pm \sqrt{34^2 - 4(1)(145)}}{2(1)}$	
	$= \dfrac{34 \pm 24}{2}$	
	a = 29 (rejected) or a = 5	[M1]
	∴ centre = (5, 8)	[A1]
iii a	Eq^{n} of C : $(x-5)^2 + (y-8)^2 = 25$	[B1]
b.	$M_{radius} = \dfrac{12-8}{8-5}$	
	$= \dfrac{4}{3}$	
	$M_L \left(\dfrac{4}{3}\right) = -1$	
	$M_L = -\dfrac{3}{4}$	[M1]
	∴ Eq^{n} of L : $y = -\dfrac{3}{4}x + c$; (8, 12)	
	$12 = -\dfrac{3}{4}(8) + c$	[M1]
	c = 18	
	∴ $y = -\dfrac{3}{4}x + 18$	Either [A1]
	$4y = -3x + 72$	
iv.	centre = (−5, 8)	
	∴ Eq^{n} of C : $(x+5)^2 + (y-8)^2 = 25$	[B1]

Q5

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5ai $(1+y)^n = 1 + \binom{n}{1}y + \binom{n}{2}y^2 + \ldots$ [M1]

$= 1 + ny + \frac{n(n-1)}{2}y^2 + \ldots$ [M1]

ii $(1+x+2x^2)^n = 1 + n(x+2x^2) + \frac{n(n-1)}{2}(x+2x^2)^2 + \ldots$ [M1]

$= 1 + nx + 2nx^2 + \frac{n(n-1)}{2}(x^2 + \ldots) + \ldots$

$\therefore$ coefficient of $x^2 = 2n + \frac{n(n-1)}{2}$

$2n + \frac{n(n-1)}{2} = 44$ [M1]

$4n + n(n-1) = 88$

$n^2 + n(n-1) = 88$

$n^2 + 3n - 88 = 0$

$(n+11)(n-8) = 0$

(rejected) $n = -11$ or $n = 8$ [A1]

5b $(2x^2 - \frac{1}{x})^{12}$

$T_{r+1} = \binom{12}{r}(2x^2)^{12-r}(-\frac{1}{x})^r$

$= \binom{12}{r}(2)^{12-r}x^{24-2r}(-1)^r(x)^{-r}$

$= \binom{12}{r}(2)^{12-r}(-1)^r x^{24-3r}$ [M1]

$24 - 3r = 0$

$3r = 24$

$r = 8$ [M1]

Term independent of $x = \binom{12}{8}(2)^{12-8}(-1)^8$ [M1]

$= 7920$

middle term is the 7th term

when $r = 6$, coefficient of $T_7 = \binom{12}{6}(2)^{12-6}(-1)^6$ [M1]

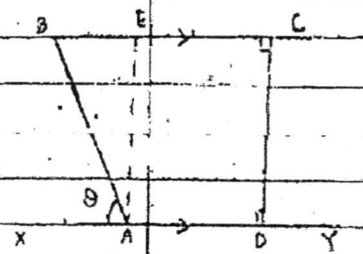
$= 59136$

Ratio $= \frac{7920}{59136}$

$= \frac{15}{112}$ OR Ratio $= 15 : 112$ [A1].

Q6

6i. $\angle ABC = \theta$ (alt. ∠, // lines)



$\therefore \sin\theta = \frac{AE}{3}$

$AE = 3\sin\theta$ [M1]

$\cos\theta = \frac{BE}{3}$

$BE = 3\cos\theta$ [M1]

$L = 3+3+3\cos\theta + 3+3\sin\theta$

$= 3\cos\theta + 3\sin\theta + 9$ [A1]

ii $L = R\cos(\theta - \alpha) + r$

$\therefore R = \sqrt{3^2 + 3^2}$

$= \sqrt{18} = 3\sqrt{2}$

$\tan\alpha = 1$ — Either [M1]

$\alpha = 45°$

$\therefore L = 3\sqrt{2}\cos(\theta - 45°) + 9$ or $L = \sqrt{18}\cos(\theta - 45°) + 9$ [A1].

iii when $L = 13$,

$3\sqrt{2}\cos(\theta - 45) + 9 = 13$ [M1]

$\cos(\theta - 45) = \frac{4}{3\sqrt{2}}$

$\theta - 45 = 19.47, -19.47$ → ~~[M1] for B-A~~

$\theta = 64.47, 25.53$

$\approx 64.5°, 25.5°$ [A1]

iv Distance ran by billy $= \frac{49}{60} \times 16$

$= 13\frac{1}{15}$ km or 13.1 km } (*)

max L occur when $\cos(\theta - 45) = 1$, $L = (3\sqrt{2} + 9)$ km. } or $= 13.24$ km

No, since the distance covered by Billy is less than the maximum distance, L. [A1]

OR max time $= \frac{3\sqrt{2}+9}{16} \times 60$ } (*) Either (*) [M1]

$= 49.6$ min

NO, the maximum time required to finish the race is more than the time taken by Billy. [A1].

EX 285 (rev 2005)

-minus one mark on the Q2 if no label of the y-axis.

[B1]: points

[B1]: best fitting line

[B1]: scale of the axes.

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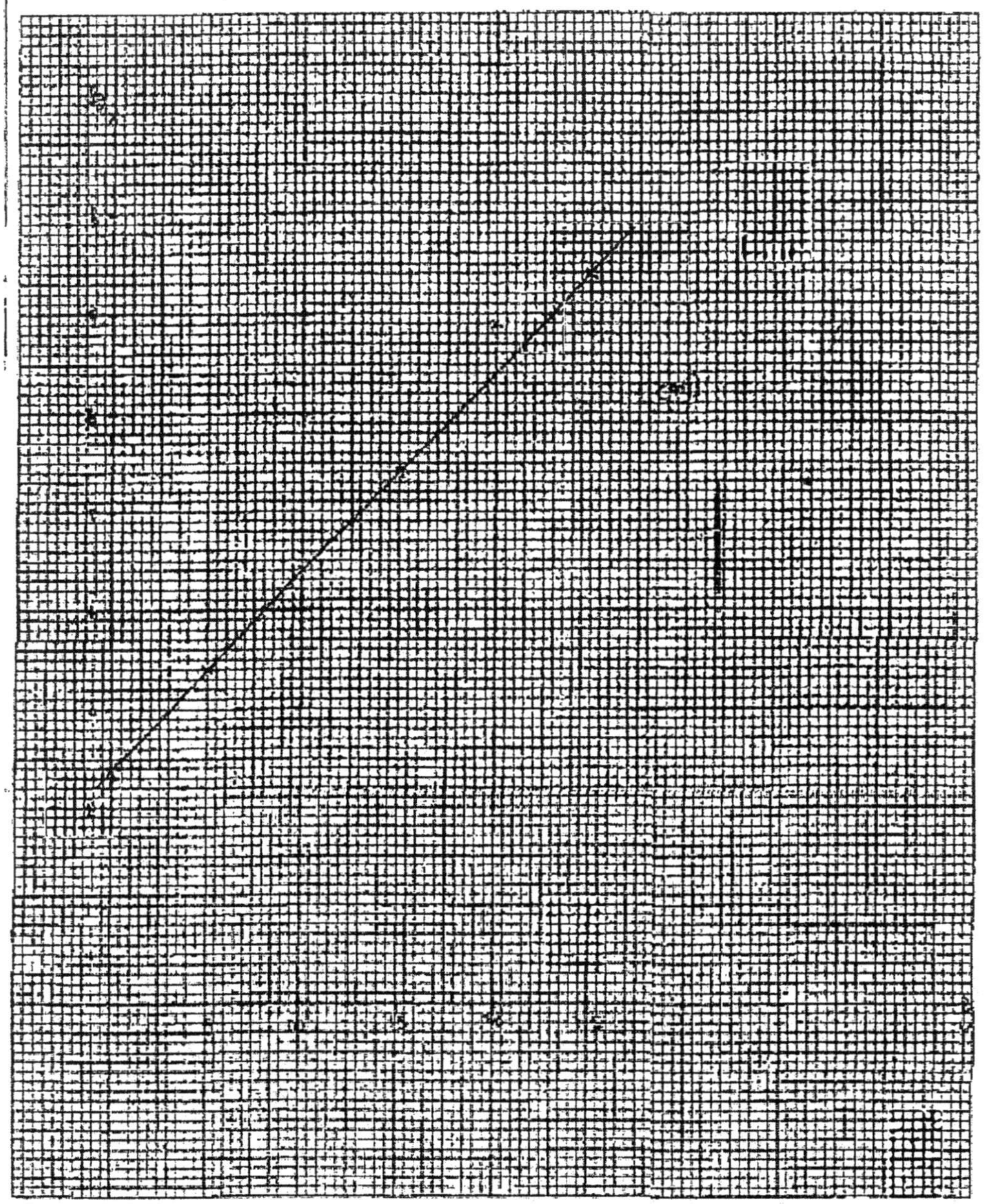

Q7

C

v	5	10	15	20	25
$\frac{R}{v}$	3.4	4.4	5.4	6.9	7.4

+bv

$\frac{R}{v} = av + b$. [M1]

(ii) incorrect $v = 20$. [A1].

when $v = 20$, $\frac{R}{v} = 6.4$.

$\frac{R}{20} = 6.4$

$R = 128$ (±2) [A1].

(iii) $b = 2.4$ (±0.1) [A1]

$R = av^2 + bv$

$\frac{R}{v} = av + b$ [M1]

$a = \frac{7-4}{23-8}$

$= \frac{3}{15}$

$= 0.2$ [A1].

$\frac{6.9-4.1}{23.5-7.5} \leq \text{grad} \leq \frac{7.1-3.9}{22.5-8.5}$

$0.175 \leq \text{grad} \leq 0.2285$

(iv). $R = kv$, where k is a constant.

when $R = 30$, $v = 5$,

$30 = k(5)$

$k = 6$. } [M1].

$\therefore R = 6v$

$\frac{R}{v} = 6$.

$\therefore v = 18$ (±0.5) [A1].

Q8

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C

8i. $f(x) = 3x^3 + ax^2 + bx + 2$

$f(1) = 0$

$3 + a + b + 2 = 0$

$a + b = -5$ — ① (M1)

$f(2) = 2.5\, f(-1)$

$3(2)^3 + 4a + 2b + 2 = [-3 + a - b + 2](2.5)$ (M1)

$26 + 4a + 2b = [-1 + a - b]2.5$

$26 + 4a + 2b = -2.5 + 2.5a - 2.5b$

$1.5a + 4.5b = -28.5$ — ②

from ① $a = -5 - b$ — ③

Sub ③ into ②

$1.5(-5 - b) + 4.5b = -28.5$ (M1)

$-7.5 - 1.5b + 4.5b = -28.5$

$3b - 7.5 = -28.5$

$3b = -21$

$b = -7$; $a = 2$ (A1)

ii $f(x) = 3x^3 + 2x^2 - 7x + 2$

$= (x-1)(3x^2 + 5x - 2)$ (M1)

$= (x-1)(3x-1)(x+2)$ (M1)

$f(x) = 0$

$(x-1)(3x-1)(x+2) = 0$

$x - 1 = 0$ or $3x - 1 = 0$ or $x + 2 = 0$

$x = 1$ $x = \frac{1}{3}$ $x = -2$ (A1)

Long division:

$$
\begin{array}{r}
3x^2 + 5x - 2 \\
x - 1 \;\big)\; 3x^3 + 2x^2 - 7x + 2 \\
-(3x^3 - 3x^2) \\
5x^2 - 7x + 2 \\
-(5x^2 - 5x) \\
-2x + 2 \\
-(-2x + 2) \\
0
\end{array}
$$

EX 255 (rev 2005)

			C
iii	$3\sin^2 y - 2\sec y - 2\cos y + 4 = 0$		
	$3\sin^2 y - \frac{2}{\cos y} - 2\cos y + 4 = 0$		
	$3(1-\cos^2 y)\cos y - 2 - 2\cos^2 y + 4\cos y = 0$	either underlined: [M1]	
	$3\cos y - 3\cos^3 y - 2 - 2\cos^2 y + 4\cos y = 0$		
	$3\cos^3 y + 2\cos^2 y - 7\cos y + 2 = 0$		
	let $y = \cos y$		
	$\cos y = 1$ or $\cos y = \frac{1}{3}$ or $\cos y = -2$	(N.A) [M1]	
	$y = 0°, 360°$ [M1] Basic ∡ $= 70.52$		
	$y = 70.52,\ 360 - 70.52$		
	$= 70.52,\ 289.48$		
	$\approx 70.5°,\ 289.5°$	[A1].	
	$y \approx 0°, 70.5°, 289.5°, 360°$		

Q9

EX 255 (rev 2005)

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9a. $y = \frac{4x-12}{x+3}$

$\frac{dy}{dx} = \frac{(x+3)(4)-(4x-12)}{(x+3)^2}$ [M1]

$= \frac{4x+12-4x+12}{(x+3)^2}$

$= \frac{24}{(x+3)^2}$

$m_{normal} = -6$

$m_{tangent} = \frac{1}{6}$

When $\frac{dy}{dx} = \frac{1}{6}$, $\frac{24}{(x+3)^2} = \frac{1}{6}$ [M1]

$(x+3)^2 = 144$

$x+3 = 12$ or $x+3 = -12$

$x = 9$ $\quad x = -15$ (rejected) [M1]

When $x = 9$, $y = 2$ [A1].

$\therefore P(9,2)$ (shown) [no [A1] if $x=-15$ is not rejected]

b. $3y - 2x = 6$

$y = \frac{2}{3}x + 2$

when $\frac{dy}{dx} = \frac{2}{3}$

$\frac{2}{3} = \frac{24}{(x+3)^2}$ [M1].

$(x+3)^2 = 36$

$x+3 = 6$ or $x+3 = -6$. [M1]

$x = 3$ $\quad x = -9$ (rejected)

$\therefore$ when $x = 3$, $y = 0$.

Q (3,0) [A1].

C

c. $m_{PR} = \frac{2-(-4)}{9-5}$

$= \frac{6}{4} = \frac{3}{2}$

$m_{QS} = \frac{0-(-1)}{3-13}$ — Either [M1]

$= -\frac{1}{10}$

$m_{PR} \times m_{QS} = -\frac{1}{10}\left(\frac{3}{2}\right)$

$= -\frac{3}{20} \neq -1.$ [M1].

Since $m_{PR} \times m_{QS} \neq -1$, ∴ PQRS is not a kite

d. Area $= \frac{1}{2}\begin{vmatrix} 9 & 3 & 5 & 13 & 9 \\ 2 & 0 & -4 & -1 & 2 \end{vmatrix}$. [M1]

$= \frac{1}{2}\left|(0-12-5+26)-(6+0-52-9)\right|$

$= \frac{1}{2}(64)$

$= 32 \text{ units}^2$ [M1].

OR

9c. $PQ = \sqrt{(9-3)^2+(2-0)^2}$

$= \sqrt{40} = 6.3245$

$QR = \sqrt{(5-3)^2+(-4-0)^2}$

$= \sqrt{20} \approx 4.4721$

$PS = \sqrt{(9-13)^2+(2+1)^2}$

$= \sqrt{25} = 5$

$SR = \sqrt{(13-5)^2+(-1+4)^2}$

$= \sqrt{73} = 8.5440$

[M1] allow ecf.

Q10

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10a. $\frac{d^2y}{dx^2} = 4e^{-2x+1}$

$\frac{dy}{dx} = \frac{4e^{-2x+1}}{-2} + C$ [M1]

$= -2e^{-2x+1} + C$

when $x=2$, $\frac{dy}{dx} = -\frac{2}{e^3} - 4$

$-\frac{2}{e^3} - 4 = -2e^{-3} + C$ [M1]

$C = -4$

$\frac{dy}{dx} = -2e^{-2x+1} - 4$, [A1]

ii. Since $e^{-2x+1} > 0$, [M1] $-2e^{-2x+1} < 0$, $-2e^{-2x+1} - 4 < 0$,

$\frac{dy}{dx} \neq 0$ [A1].

$\therefore$ The curve has no stationary point

OR when $\frac{dy}{dx} = 0$, $e^{-2x+1} = -2$ [M1]

Since $e^{-2x+1} > 0$ for all x, there is no solution for x. [A1]

iii. $y = \int -2e^{-2x+1} - 4\,dx$

$= \frac{-2e^{-2x+1}}{-2} - 4x + C$ [M1].

$= e^{-2x+1} - 4x + C$

$y = e^{-2x+1} - 4x + C$ $(2, e^{-3})$

$e^{-3} = e^{-3} - 8 + C$ [M1]

$C = 8$

$y = e^{-2x+1} - 4x + 8$ [A1].

EX 255 (rev 2005)

bi $\frac{17+6x-5x^2}{(2x-1)(3-x)^2} = \frac{A}{2x-1} + \frac{B}{3-x} + \frac{C}{(3-x)^2}$

$= \frac{A(3-x)^2 + B(2x-1)(3-x) + C(2x-1)}{(2x-1)(3-x)^2}$

$17+6x-5x^2 = A(3-x)^2 + B(2x-1)(3-x) + C(2x-1)$

let $x=3$, $17+6(3)-5(3)^2 = C(5)$

$5C = -10$

$C = -2$ [M1]

let $x=\frac{1}{2}$, $17+6(\frac{1}{2})-5(\frac{1}{2})^2 = A(3-\frac{1}{2})^2$

$\frac{25}{4}A = 18\frac{3}{4}$

$A = 3$ [M1]

let $x=0$, $17 = 3(3)^2 + B(-1)(3) + (-2)(-1)$

$17 = 27 - 3B + 2$

$-3B = -12$

$B = 4$ [M1]

$\therefore \frac{17+6x-5x^2}{(2x-1)(3-x)^2} = \frac{3}{2x-1} + \frac{4}{3-x} - \frac{2}{(3-x)^2}$ [A1]

ii. $\int \frac{17+6x-5x^2}{(2x-1)(6-2x)^2}\,dx$

$= \int \frac{3}{2x-1} + \frac{4}{3-x} - \frac{2}{(3-x)^2}\,dx$ [M1]

$= \frac{3}{2}\ln(2x-1) - 4\ln(3-x) - \left[\frac{2(3-x)^{-1}}{(-1)(-1)}\right] + C$

$= \frac{3}{2}\ln(2x-1) - 4\ln(3-x) - \frac{2}{3-x} + C$

[A1] [A1]

*Penalise 1 mark if no '+c' is seen.