中正中学 义顺

CHUNG CHENG HIGH SCHOOL YISHUN

Mid-Year Examination (2017)
Secondary 4 Express/ 5 Normal (Academic)

Candidate			
	Name	Register No	Class

MATHEMATICS
4048/01

For examiner's use
/ 80

Date: 2 May 2017
Duration: 2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential workings and units will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
Give your answer in the simplest form. Leave your answer in fraction where applicable or correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
For π, use your calculator value, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This paper consists of **19** printed pages, INCLUDING the cover page.

[TURN OVER

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved Surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere of radius} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 (a) Calculate $\frac{-1.4^2+2\pi^3}{4-\sqrt{17}}$, giving your answer to 2 significant figures.

(b) Express 0.35% as a fraction in its simplest form.

Answer: (a) ____________________ [1]

(b) ____________________ [1]

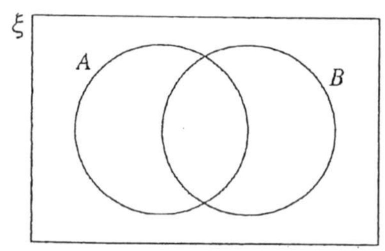
2 The universal set ξ is the set of natural numbers less than 14.

Given that $B = \{x: 2x - 7 < 19 \leq x + 10\}$.

(a) List all elements in B'.

Answer: (a) B'= ____________________ [2]

(b) On the Venn diagram, shade the region which represents $A' \cap B'$. [1]



3 A metal rod A has a length of 47 m, correct to the nearest m.

A metal rod B has a length of 63 m, correct to the nearest m.

Find

(a) the least possible length of metal rod A,

(b) the greatest possible difference in their lengths.

Answer: (a) ____________________ m [1]

(b) ____________________ m[1]

4 A charity carnival sells tickets to adults and children. The total cost of 3 adult tickets and 1 child ticket is \$29. The total cost of 1 adult ticket and 3 child tickets is \$19. Find the cost of one adult ticket and the cost of one child ticket.

Answer: *Cost of one adult ticket* = \$__________

Cost of one child ticket = \$__________ [3]

5 The mean, median, and modal height of 4 men are 178 cm, 176 cm, and 173 cm respectively. Write down the heights of each man in ascending order.

Answer: ______cm, ______cm, ______cm, ______cm [2]

6 Benjamin wants to invest $30000 in a savings account for 5 years. He finds information about two savings plans. Which of these savings plans should he choose in order to have more money at the end of 5 years? Show your working clearly.

Plan A

Simple interest of 4% per year

Plan B

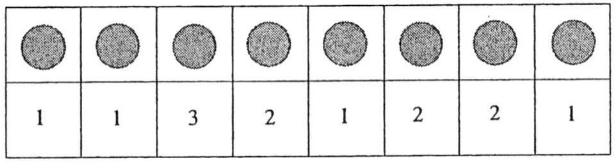
Compound interest of 1.5% per year

Answer: Plan_____ because_______________________________________ [3]

7 It is given that y is directly proportional to the cube root of x. Find the percentage increase in y when x is increased by 400%.

Answer: ______________________% [3]

8 A ball is dropped at random into one of the eight holes, numbered as shown in the diagram below. The number under each hole gives the score obtained when the ball drops into that hole. Each hole can only occupy one ball.



(a) State the probability of getting a score of 1.

Answer: (a) ______________________[1]

(b) Given that two balls are dropped at the same time, find the probability of scoring
(i) a total of 2,
(ii) a total of 6.

Answer: (b)(i) ______________________ [1]

(b)(ii) ______________________ [1]

9 (a) Express 315 as a product of its prime factors.

(b) On Youth Day, the teacher distributed 315 bookmarks and 90 pens equally among the students for her classes. Given that each student received the same number of bookmarks and pens with no leftover,

(i) find the largest possible number of students the teacher distributed for her classes,

(ii) find the number of bookmarks each student received.

Answer: (a) ______________________ [1]

(b)(i) ______________________ [1]

(b)(ii) ______________________ [1]

10 (a) Simplify $\frac{2a^4}{5bc} \div \frac{14a}{15c}$.

Answer: (a) ______________________ [2]

(b) The distance from Earth to the Moon is 384 400 km while the distance from Earth to Uranus is 2.72394×10^{12} m. By how many times is the distance of Earth from Uranus more than the distance of Earth from the Moon? Leave your answer in standard form.

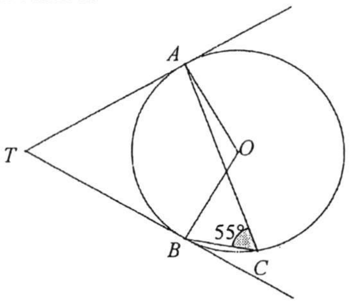
Answer: (b) ______________________ [2]

11 An area of 8 cm^2 on a map represents an actual area of 0.03 km^2. Calculate
(a) the area, in cm^2, on the map , which represents an actual area of 6000 m^2,
(b) the actual distance, in km, represented by a length of 7.9 cm on the map.

Answer: (a) ______________________ cm^2 [2]

(b) ______________________ km [2]

12 The diagram shows a circle, ABC with centre O.
TA and *TB* are tangents to the circle.
$\angle ACB = 55°$.



Find

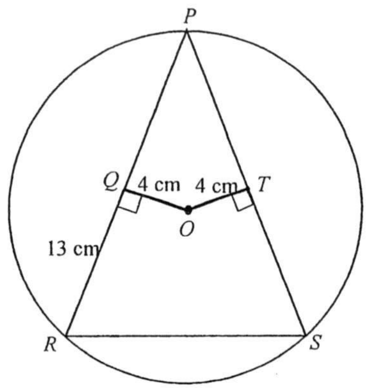
(a) $\angle AOB$,

Answer (a) $\angle AOB =$ ______________________ ° [1]

(b) $\angle ATB$,

Answer (b) $\angle ATB =$ ______________________ ° [2]

13 In the figure, O is the centre of the circle, $OQ = OT = 4$ cm and $QR = 13$ cm. P, S and R are points on the circle.



(a) State the length of PS.

Answer: (a) ____________________ cm [1]

(b) Find the length of the shortest chord.

Answer: (b) ____________________ cm [3]

14 The coordinates of A is $(3,-5)$. Another point B is such that $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$.

(a) Find $|\overrightarrow{AB}|$.

Answer: (a) ____________________ units [1]

(b) Find the coordinates of B.

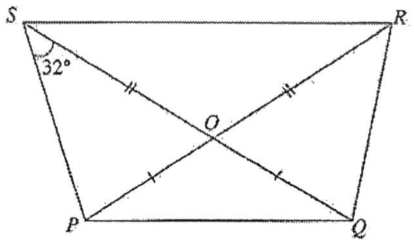
Answer: (b) (__________ , __________) [1]

(c) Given the coordinates of C is $(-1, m)$ and A, B and C are collinear.
Find (i) $\overrightarrow{AC}$,
(ii) m.

Answer: (c)(i) ____________________ [1]

(ii) ____________________ [2]

15 In the diagram, $OR = OS$, $QO = PO$ and $\angle PSQ = 32°$.



(a) Show that triangles POS and QOR are congruent, stating your reasons clearly.
Answer (a)

[2]

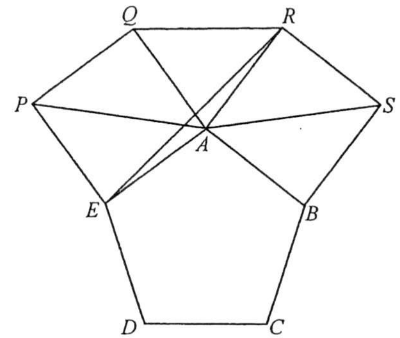
(b) Name another pair of congruent triangles.

Answer: (b) ______________________ [1]

16 (a) Calculate the size of each interior angle of a regular pentagon.

Answer: (a) ______________________ ° [1]

(b)



In the diagram above, $ABCDE$ is a regular pentagon. $EPQA$ and $ARSB$ are two squares.

Calculate

(i) reflex $\angle PED$,

Answer: (b)(i) ______________________ ° [1]

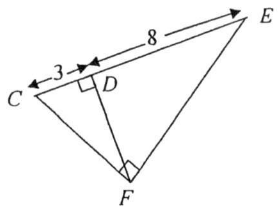
(ii) obtuse $\angle PAR$,

Answer: (b)(ii) ______________________ ° [1]

(iii) acute $\angle ERA$.

Answer: (b)(iii) ______________________ ° [1]

17 In the diagram, CFE is a right-angled triangle. D is a point on CE such that $\angle CFE = \angle CDF = 90°$.
All the measurements are in cm.



Show that triangles CFE and CDF are similar.
Answer (a)

[2]

(b) Calculate CF.

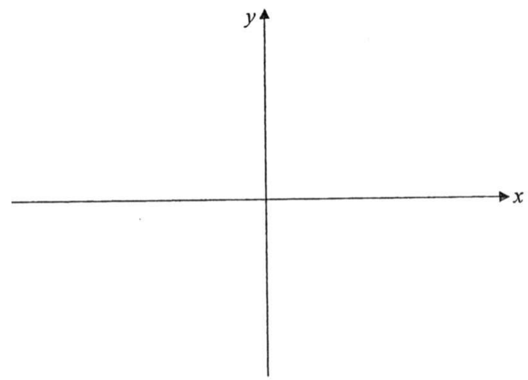
Answer: (b) $CF =$ ______________ cm [2]

18 (a) Express $-x^2 + 3x - 5$ in the form $-(x-a)^2 - b$.

Answer: (a) ______________ [2]

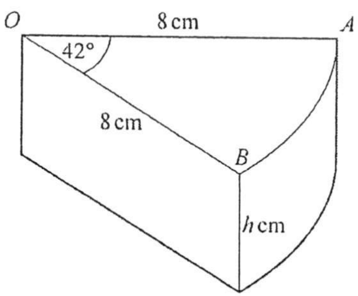
(b) Hence, sketch the graph of $y = -x^2 + 3x - 5$, showing clearly the turning point and the intercepts, if any.

Answer (b)(ii)



[2]

19 A wedge of cheese in the shape of a prism is cut from a cylinder of cheese of height h cm. The radius of the cylinder, OA, is 8 cm and the $\angle AOB = 42°$.

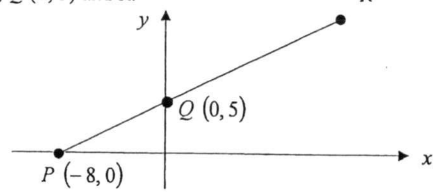


(a) The volume of the wedge of cheese is 100 cm^3.
Show that the value of h is 4.26 cm correct to 2 decimal places.
Answer: (a) [2]

(b) Calculate the total surface area of the wedge of cheese.

Answer: (b) ____________________ cm^2 [3]

20 The diagram below shows the sketch of a straight line passing through three points P (-8, 0), Q (0, 5) and R.



(a) Write down the equation of the line.

Answer: (a) ____________________ [1]

(b) Given that another line that is parallel to the line PQR, passes through the point (4, 5), find the coordinates of the y-intercept of the line.

Answer: (b) (__________ , __________) [2]

(c) The point R is such that PQ: QR is 1 : 2. Find the coordinates of R.

Answer: (c) ____________________ [2]

21 A team of 4 NPCC cadets took part in an orienteering competition.

The diagram below shows a scale drawing representing 3 checkpoints A, B and C.
B is due south of A.

In the drawing, 1 cm represents 0.5 km.

(a) Using ruler and compasses only, construct on the diagram,

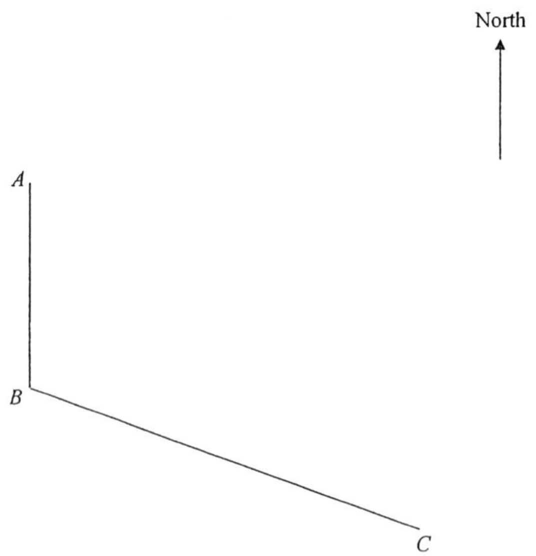
(i) the bisector of angle ABC, [1]

(ii) the perpendicular bisector of BC. [1]

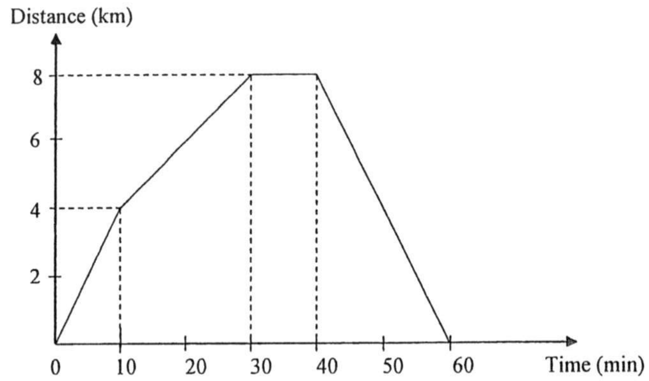
(b) A referee, R, is to be stationed equidistant from B and C and equidistant from AB and BC. Find and label the position of R on the diagram. [1]

(c) The cadets are instructed to find a building D which is on a bearing of 210° and 2.5 km from B. Find and label the position of D on the diagram. [1]

Answer



22 The diagram below shows the distance-time graph of a cyclist's journey.



The cyclist started from rest.

(a) Find the average speed for the whole journey.

Answer: (a) ____________ km/min [1]

(b) Find the distance travelled in the first 24 minutes.

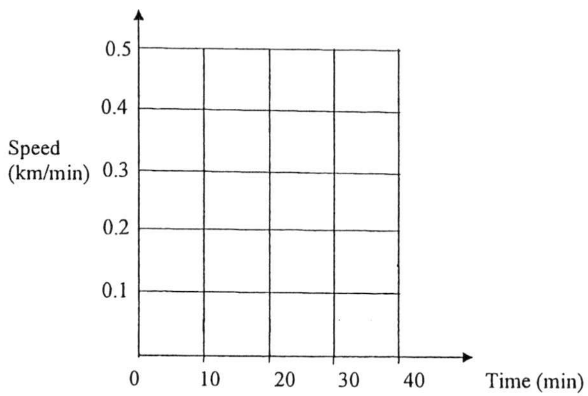
Answer: (b) ____________ km [2]

(c) Find the acceleration during the last 5 minutes.

Answer: (c) ________________ km/min² [1]

(d) On the grid given below, draw the speed-time graph for the first 40 minutes of the journey.

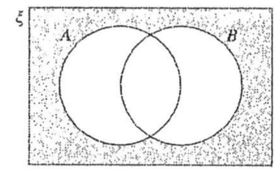
Answer (b)(ii)



[3]

~*End of Paper*~

2017 CCHY Mid-year Exam 4E5N EMP1 Marking Scheme

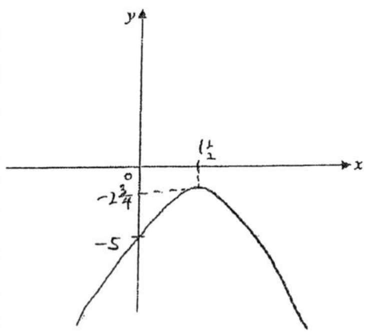
Qn	Solution	Marks
1(a)	-490	B1
1(b)	$\frac{7}{2000}$	B1
2(a)	$2x-7<19$ $\quad 19\leq x+10$ $2x<26$ $\quad 9\leq x$ $x<13$ $\quad x\geq 9$ $B' = \{1, 2, 3, 4, 5, 6, 7, 8, 13\}$	 M1 A1
2(b)	 $A' \cap B'$	B1
3(a)	46.5	B1
3(b)	$46.5 \leq$ length A < 47.5 $62.5 \leq$ length B < 63.5 Greatest difference in length = 63.5 – 46.5 m = 17 m	 B1
4	Let x be the no. of adult tickets. Let y be the no. of child tickets. $3x+y=29$--------------(1) $x+3y=19$--------------(2) (1)×3: $9x+3y=87$-----(3) (3)−(2): $8x=68$ $x=\frac{68}{8}$ $=8.50$ subst $x=8.50$ into (1), $3(8.50)+y=29$ $y=29-3(8.50)$ $=3.50$ ∴ Cost of adult ticket is \$8.50 and cost of child ticket is \$3.50.	M1 A1 A1
5	Third height : 176 × 2 – 173 = 179 Fourth height : 4(178) – 179 – 2(173) = 187 Height in order : 173, 173, 179, 187	M1 A1

6	At the end of 5 years, Plan A: $30000(1+(0.04\times 5)) = 36000$ Plan B: $30000(1+0.015)^5 = 32318.52$ Benjamin should choose Plan A as it gives him more money at the end of the five years or Benjamin should choose Plan A because \$36000 >\$32318.52	M1 M1 A1
7	$y = k\sqrt[3]{x}$ $x_1 = 5x,\ y_1 = k\sqrt[3]{5x}$ $= \sqrt[3]{5}\left(k\sqrt[3]{x}\right)$ $= \sqrt[3]{5}y$ % increase in $y = \frac{y_1 - y}{y}\times 100\%$ $= \frac{\sqrt[3]{5}y - y}{y}\times 100\%$ $= \frac{y\left(\sqrt[3]{5}-1\right)}{y}\times 100\%$ $= \left(\sqrt[3]{5}-1\right)\times 100\%$ $= 70.9976\%$ $= 71.0\%$ (3sf)	M1 M1 A1
8(a)	$\frac{1}{2}$	B1
8(b)(i)	P(total is 2) $= \frac{4}{8}\times\frac{3}{7}$ $= \frac{3}{14}$	B1
8(b)(ii)	P(total is 6) = 0	B1
9(a)	$315 = 3^2\times 5\times 7$	B1
9(b)(i)	$315 = 3^2 \times 5 \times 7$ $90 = 2 \times 3^2 \times 5$ HCF $= 3^2 \times 5$ $= 45$ ∴ greatest possible number of students the teacher distributed the bookmarks and pens = 45	B1

9(b)(ii)	Number of books each student gets $= \frac{3^2\times 5\times 7}{3^2\times 5}$ $= 7$	B1
10(a)	$\frac{2a^4}{5bc}\div\frac{14a}{15c}$ $= \frac{2a^4}{5bc}\times\frac{15c}{14a}$ $= \frac{3a^3}{7b}$	M1 A1
10(b)	Number of times Earth is further from Uranus compared to the Moon $= \frac{2.72394\times 10^{12}}{384\,400\times 1000}$ $= 7.0862 \times 10^3$ $= 7.09\times 10^3$ (3 s.f.)	M1 A1
11(a)	0.03 km² = 0.03 x $(1000\text{m})^2$ = 30000 m² 30000 m² : 8 cm² ÷5, 6000 m² : 1.6 cm² ∴ area on map = 1.6 cm²	M1 A1
11(b)	8 cm² : 0.03 km² 8 cm² : $\frac{3}{100}$ km² ÷8, 1 cm² : $\frac{3}{800}$ km² take sq root, 1 cm : $\sqrt{\frac{3}{800}}$ km actual distance on ground = 7.9 cm x $\sqrt{\frac{3}{800}}$ km = 0.48377 km ~ 0.484 km	M1 A1
12(a)	∠AOB = 2 × 55° (∠at centre = 2 × ∠ at circumference) = 110°	B1
12(b)	∠OAT = 90° (tangent ⊥ radius) ∠ATB = 360° − 90° − 90° − 110° (∠ sum of quadrilateral) = 70°	M1 A1

13(a)	$PS = 26cm$	B1
13(b)	In right-angled ΔQPO, $\tan \angle QPO = \frac{4}{13}$ $\angle QPO = 17.1027°$ Let M = foot of the perpendicular from P to RS In right-angled ΔPMR, $\sin \angle RPM = \frac{RM}{RP}$ $\sin 17.1027° = \frac{RM}{26}$ RM = 26 sin 17.1027° $\therefore$ RS = 2 x 26 sin 17.1027° cm = 15.2924 cm ~ 15.3 cm (3sf) $\therefore$ the shortest chord, RS = 15.3 cm	M1 M1 A1
14(a)	$\overrightarrow{AB} = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$ $\lvert\overrightarrow{AB}\rvert = \sqrt{1^2 + 9^2}$ units = 9.05539 units = 9.06 units (3sf)	B1
14(b)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $\overrightarrow{OB} = \overrightarrow{AB} + \overrightarrow{OA}$ $= \begin{pmatrix} 1 \\ 9 \end{pmatrix} + \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ $= \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ $\therefore$ coordinates of B = (4, 4)	B1
14c(i)	$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$ $= \begin{pmatrix} -1 \\ m \end{pmatrix} - \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ $= \begin{pmatrix} -4 \\ m+5 \end{pmatrix}$	B1
14c(ii)	$\begin{pmatrix} -4 \\ m+5 \end{pmatrix} = k\begin{pmatrix} 1 \\ 9 \end{pmatrix}$	M1

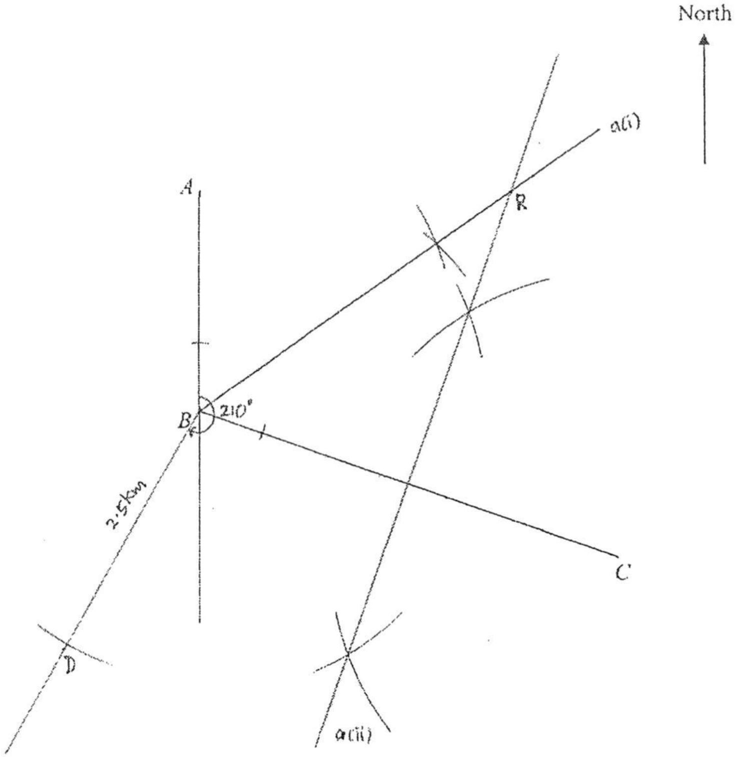
	comparing terms, $k = -4$ $m + 5 = 9k$ $m = 9(-4) - 5$ $= -41$	A1
15(a)	$QO = PO$ (given) (S) $RO = SO$ (given) (S) $\angle ROQ = \angle SOP$ (vertically opposite angles) (A) $\therefore \Delta POS$ and ΔQOR are congruent (SAS)	M1 R1
15(b)	triangle SPR and triangle RQS Or triangle PSQ and triangle QRP	B1
16(a)	Each interior angle = $\frac{(5-2)\times 180^0}{5}$ = 108°	B1
16(b)(i)	reflex $\angle$PED = 90° + 108° = 198°	B1
16(b)(ii)	In triangle QAR, $\angle$QAR = 360° – 90° – 90° – 108° = 72° $\therefore$ obtuse $P\hat{A}R$ = 72°+45° = 117°	B1
16(b)(iii)	$\angle ERA = \frac{180° - (72° + 90°)}{2}$ (EA = AR, triangle EAR is isosceles) = 9° $\therefore$ acute $\angle$ERA = 9°	B1
17(a)	$\angle FCE = \angle DCF$ (Common Angle) $\angle CFE = \angle CDF = 90°$ $\therefore$ Triangles CFE and CDF are similar since 2 pairs of corresponding angles are equal.	M1 R1
17(b)	ΔCFE and ΔCDF are similar, $\frac{CF}{CD} = \frac{FE}{DF} = \frac{CE}{CF}$ =scale factor $\frac{CF}{CD} = \frac{CE}{CF}$ $\frac{CF}{3} = \frac{11}{CF}$ $CF^2 = 33$	M1

	$CF = \sqrt{33}$ $= 5.74456$ ~ 5.74 $\therefore$ CF = 5.74 cm	A1
18(a)	$-x^2 + 3x - 5$ $= -\left(x^2 - 3x + 5\right)$ $= -\left[x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 5\right]$ $= -\left[\left(x - \frac{3}{2}\right)^2 + 2\frac{3}{4}\right]$ $= -\left(x - 1\frac{1}{2}\right)^2 - 2\frac{3}{4}$	M1 A1
18(b)	 B1 (Shape of curve) B1 (correct pts of turning pts & y-intercept)	
19(a)	Cross sectional area of prism $= \frac{42°}{360°} \times \pi \times 8^2$ Volume of prism $= \frac{42°}{360°} \times \pi \times 8^2 \times h$ $100 = \frac{42°}{360°} \times \pi \times 8^2 \times h$ $h = \frac{360° \times 100}{42° \times \pi \times 8^2}$ $= 4.26308$ $\therefore\ h = 4.26$ (correct to 2 decimal places)	M1 A1

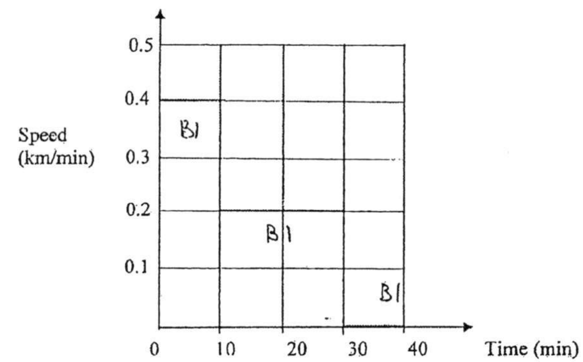
19(b)	Arc length $= \frac{42°}{360°} \times 2\pi \times 8 = 5.86431$ cm Curve surface area $= 5.86431 \times 4.26308 = 25.00002267$ cm^2 Total surface area $= \left(2 \times \frac{42°}{360°} \times \pi \times 8^2\right) + (2 \times 8 \times 4.26308) + 25.00002267$ cm^2 $= 140.1238$ cm^2 ~ 140 cm^2 (3sf)	M1 M1 A1
20(a)	gradient $= \frac{5}{8}$ $\therefore$ equation of line is $y = \frac{5}{8}x + 5$	B1
20(b)	Equation of parallel line is $y = \frac{5}{8}x + c$ At (4,5), $5 = \frac{5}{8}(4) + c$ $c = 5 - \frac{5}{2}$ $= \frac{5}{2}$ $\therefore$ co-ordinates of y-intercept $= \left(0, 2\frac{1}{2}\right)$	M1 A1
20(c)	$x : y = 8 : 5$ $= 16 : 10$ $\therefore$ R = (16, 5+10) = (16,15)	M1 A1

21

Answer



22(a)	Average speed $= \frac{8+8}{60}$ km/min $= \frac{4}{15}$ km/min	B1
22(b)	Method 1 Let distance from $t = 10$ min to $t = 24$ min be h km Using similar triangle, $\frac{h}{4} = \frac{14}{20} = scale \quad factor$ $h = \frac{4 \times 14}{20}$	M1

	$= 2.8$ ∴ distance travelled in the first 24 min = 4 + 2.8km = 6.8km	A1
	Method 2 Equation of line from $t = 10$ min to $t = 24$ min : gradient of line $= \frac{8-4}{30-10}$ $= 0.2$ Equation of line is $y = 0.2x + c$ At (10,4), $4 = 0.2(10) + c$ $= 2 + c$ $c = 4 - 2$ $= 2$ subst $x = 24$ into y, $y = 0.2(24) + 2$ $= 6.8$ ∴ distance travelled in the first 24 min = 6.8 km	M1 A1
22(c)	0 km/min^2	B1
22(d)		

1\. (a) It is given that $S = \frac{a}{2}\sqrt{n^2 - b}$.

(i) Find S when $a = 8$, $b = -7$ and $n = 3$. [1]

(ii) Express n in terms of a, b and S. [3]

(b) Solve the equation $\frac{5}{\sqrt[3]{5}} = 5^{x-1}$. [2]

(c) Factorise $9x^2 - 25 + 12xy - 20y$. [2]

(d) Simplify $\frac{x^2 + 3x - 7}{x - 4} + \frac{9 + 3x}{4 - x}$. [3]

2\. Nancy is planning a holiday to United States. On 1 March 2017, she exchanged S$3000 into US dollars (US$) at Kumar's Money Exchange at a rate of US$1 = S$$x$.

(a) Find an expression, in terms of x, for the amount of US$ she received from Kumar's Money Exchange. [1]

On 15 March 2017, she decided to exchange another S$2100 into US$ at Lee's Money Exchange at a rate of US$1 = S$($x - 0.1$).

(b) Find an expression, in terms of x, for the amount of US$ she received from Lee's Money Exchange. [1]

(c) Given that Nancy received a total of US$3500 from the two Money Exchanges, form an equation in x and show that it simplifies to $70x^2 - 109x + 6 = 0$. [3]

(d) (i) Solve the equation $70x^2 - 109x + 6 = 0$, giving your answers correct to 4 decimal places. [2]

(ii) Suggest a reason why one of the answers has to be rejected. [1]

(iii) Hence, find the exchange rate between S$ and US$ offered by Lee's Money Exchange. [1]

(e) Is it better for Nancy to change her currency on 1 March or 15 March? Justify your answer with appropriate workings. [2]

3\. A cake shop sells 3 different types of muffins. The table below shows the numbers of muffins sold over 2 days and the price of each type of muffin.

	Chocolate	Blueberry	Cheese
Day 1	12	25	10
Day 2	15	24	9
Price of each muffin, $	2.00	1.50	2.50

It is given that $\mathbf{P} = \begin{pmatrix} 12 & 25 & 10 \\ 15 & 24 & 9 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2.0 \\ 1.5 \\ 2.5 \end{pmatrix}$.

(a) Evaluate the matrix $\mathbf{M} = \mathbf{PQ}$. [1]

(b) State what the elements of **M** represent. [1]

(c) The cake shop is having a promotion by giving a 25% discount on chocolate muffins, a 30% discount on blueberry muffins and a 40% discount on cheese muffins.

(i) Write down a 3×3 matrix **R**, where the product of **RQ** will give the discounted price of each type of muffin. [1]

(ii) Evaluate the matrix $\mathbf{N} = \mathbf{RQ}$. [1]

(d) Evaluate the matrix **PN** and state what the elements of **PN** represent. [2]

(e) Given that $\mathbf{T} = \begin{pmatrix} 1 & 1 \end{pmatrix}$, evaluate the matrix **TPN** and state what the elements of **TPN** represent. [2]

---

4. (a) Bag A contains 1 gold medal, 4 silver medals and 2 bronze medals. Bag B contains 2 gold medals and 5 silver medals. Peter draws a medal at random from bag A, took note of the colour and placed it into bag B. He then draws a medal at random from bag B.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) Find, as a fraction in its simplest form, the probability that

(a) the medal drawn from bag B is a gold medal, [2]

(b) the medal drawn from bag B is a bronze medal. [1]

(b) (i) The nth term of a sequence is given by $T_n = n^2 + 2n - 5$.

(a) Write down the first 5 terms of the sequence. [2]

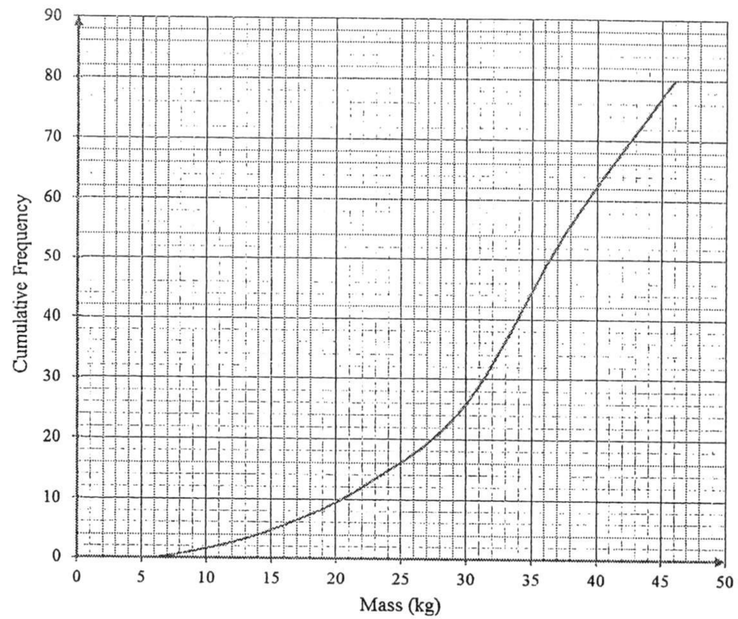
(b) Which term of the sequence has value 163? [2]

(ii) The first five terms of another sequence are 1, 6, 13, 22, 33, ...

(a) By comparing this sequence with the sequence in part (i), write down the nth term of the sequence 1, 6, 13, 22, 33, ... [1]

(b) Hence, find the 99th term. [1]

5. In Plantation A, the total mass of the mangoes produced by each of 80 mango trees were measured. The cumulative frequency curve below shows the distribution of the masses.



(a) Use the graph to find the median mass. [1]

(b) In the following grouped frequency table of the mass of mangoes in the plantation, write down the values of p and of q. [2]

Mass (x kg)	$6 \le x < 14$	$14 \le x < 22$	$22 \le x < 30$	$30 \le x < 38$	$38 \le x < 46$
Frequency	4	p	14	q	24

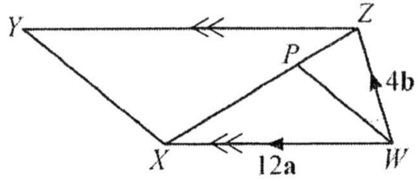
(c) Using your grouped frequency table, calculate an estimate of

(i) the mean mass, [2]

(ii) the standard deviation. [1]

(d) In Plantation B, the total mass of the mangoes produced by each of 80 mango trees were measured. Their mean and standard deviation were found to be 29.1 kg and 10.4 kg respectively. Make two comparisons between the mass of the mangoes produced by the trees in both plantations. [2]

6. In the diagram, $WXYZ$ is a trapezium and WX is parallel to ZY. The point P on XZ is such that $ZP : PX = 1 : 3$ and $WX : ZY = 3 : 4$.



(a) Give that $\overrightarrow{WX} = 12\mathbf{a}$ and $\overrightarrow{WZ} = 4\mathbf{b}$, express in terms of **a** and/or **b**,

(i) $\overrightarrow{ZX}$, [1]

(ii) $\overrightarrow{WP}$, [1]

(b) Determine with justification, if the line XY is parallel to the line WP. [2]

(c) Find

(i) $\dfrac{\text{Area of } \Delta WZP}{\text{Area of } \Delta WXP}$, [1]

(ii) $\dfrac{\text{Area of } \Delta WZP}{\text{Area of } \Delta YXZ}$. [2]

(d) Hence, find the area of $WXYZ$ if the area of ΔWZP is 12 units^2. [1]

7. Diagram I shows a trough where the base $BCGF$ and the top $ADHE$ are horizontal rectangles. Each of the vertical sides $ABCD$ and $EFGH$ is a trapezium. It is given that $AD = 15$ cm, $BC = 11$ cm, $BX = 9$ cm and $CG = 20$ cm. It is fully filled with water.

Take $\pi = \dfrac{22}{7}$.

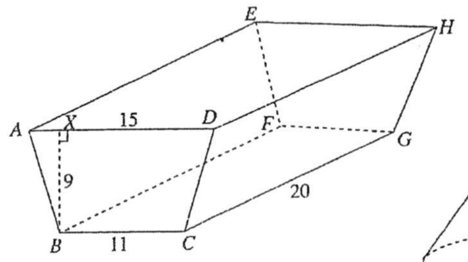


Diagram I

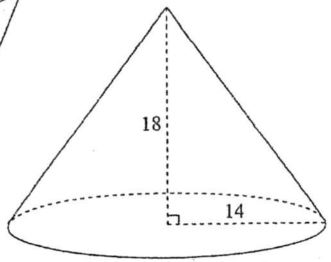


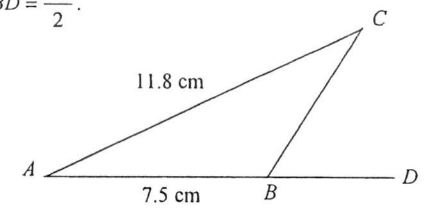
Diagram II

(a) Calculate the volume of the water in the trough in Diagram I. [2]

(b) All the water in Diagram I is made to flow down through a hole at the top of the cone in Diagram II. The water flows at a rate of 0.2 litres per minute. Calculate the time taken for the water in the trough to be transferred, giving your answer in minutes and seconds. [2]

(c) The cone in Diagram II is of base radius 14 cm and height 18 cm. The cone is made to stand on its base. Calculate the height of water in the cone. [4]

8. (a) In the diagram below, ABD is a straight line. $AB = 7.5$ cm, $AC = 11.8$ cm and $\sin\angle CBD = \frac{\sqrt{3}}{2}$.



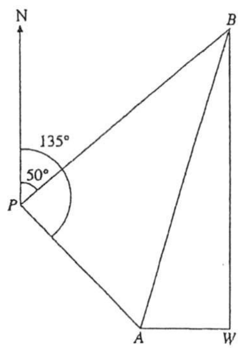
Calculate

(i) $\sin\angle ACB$, giving your answer correct to 3 significant figures, [2]

(ii) $\angle BAC$, [2]

(iii) the area of ΔABC. [1]

(b) Two military ships, Amaze and Brave left port P at 1000. Amaze sailed at 12 km/h on a bearing of $135°$. Brave sailed at 18 km/h on a bearing of $050°$. After sailing for 3 hours, Amaze is at Island A and Brave is at Island B.



Calculate

(i) the length of AB. [3]

(ii) the bearing of A from B. [2]

(iii) Amaze later sailed to Island W which is due east of Island A. It is given that Island B is due north of Island W. Find the distance BW. [2]

(iv) A helicopter is hovering at a height of 10 km vertically above B. Find the greatest angle of elevation Amaze can have of the helicopter as it sailed towards W. [2]

9. **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation $y = \frac{1}{2}x^2 + \frac{18}{x} - 10$.

Some corresponding values of x and y, correct to 1 decimal place, are given in the table below.

x	1	1.5	2	2.5	3	4	4.5	5
y	8.5	3.1	1	0.3	0.5	k	4.1	6.1

(a) Find the value of k. [1]

(b) Using a scale of 2 cm to represent 1 unit on both axes, draw the graph of $y = \frac{1}{2}x^2 + \frac{18}{x} - 10$ for $0 \le x \le 5$. [3]

(c) Use your graph to solve $\frac{1}{2}x^2 + \frac{18}{x} = 15$. [2]

(d) By drawing a tangent, find the gradient of the curve at $x = 3$. [2]

(e) On the same axes, draw the graph of $y = 6 - x$ for $0 \le x \le 5$. [1]

(f) (i) Write down the x-coordinates of the points where the two graphs intersect. [2]

(ii) These values of x are solutions of the equation $x^3 + Ax^2 + Bx + C = 0$. Find the value of A, the value of B and the value of C. [2]

10. The table below shows the admission rates for three different tourist locations in Singapore.

	Per adult ($)	Per child ($)
Zoological Gardens	33	22
Bird Park	29	19
Night Safari	30	20
3-parkhopper package	69	49

Note: The 3-parkhopper package allows admission into all 3 parks within a period of 10 days.

(a) Mr Tan, a Malaysian, brought his wife and three children below 12 years old to visit Singapore in June 2016. First, they visited the Zoo and the Bird Park. Later, they decided to go to the Night Safari.

(i) Calculate the amount Mr Tan spent visiting the 3 parks with his family. [1]

(ii) If he had planned earlier and purchased the 3-parkhopper package, how much would he have saved? [2]

(b) His 3 children enjoyed their excursions at the parks and requested Mr Tan to bring them to the parks during subsequent school holidays. Mr Tan came to know about Wildlife Membership package. The details are as follows:

Membership Package Type	Annual Fee ($)
Individual	
Adult	112
Child	72
Family	
2 adults and 1 child	260

* Add $15 for every additional child up to a maximum of 5 children in a family Membership. Family membership is limited to a maximum of 2 adults and 5 children.

Membership benefits:

- Unlimited admission to Bird Park, Night Safari and Singapore Zoo;
- Complimentary tram rides at Bird Park and Singapore Zoo;
- Complimentary English language commentary tram rides at Night Safari on all nights;
- 10% discount at all F&B outlets at all parks;
- 10% discount off regular-priced products at all retail outlets at all parks.

Should Mr Tan take up a membership package? If so, which package should he buy? Justify the decision you make and show your calculations clearly. [5]

[End of Paper]

1. (a) It is given that $S = \frac{a}{2}\sqrt{n^2 - b}$.

(i) Find S when $a = 8$, $b = -7$ and $n = 3$. [1]

(ii) Express n in terms of a, b and S. [3]

(b) Solve the equation $\frac{5}{\sqrt[3]{5}} = 5^{x-1}$. [2]

(c) Factorise $9x^2 - 25 + 12xy - 20y$. [2]

(d) Simplify $\frac{x^2 + 3x - 7}{x - 4} + \frac{9 + 3x}{4 - x}$. [3]

(a)(i) $S = \frac{8}{2}\sqrt{(3)^2 - (-7)}$

$= 16$ B1

(ii) $S = \frac{a}{2}\sqrt{n^2 - b}$

$\frac{2S}{a} = \sqrt{n^2 - b}$ M1

$n^2 - b = \frac{4S^2}{a^2}$ M1

$n = \pm\sqrt{\frac{4S^2}{a^2} + b}$ A1

(b) $\frac{5}{\sqrt[3]{5}} = 5^{x-1}$

$x - 1 = 1 - \frac{1}{3}$ M1

$x = 1\frac{2}{3}$ A1

(c)

$9x^2 - 25 + 12xy - 20y$

$= (3x+5)(3x-5) + 4y(3x-5)$ M1

$= (3x-5)(3x+5+4y)$ A1

(d) $\frac{x^2 + 3x - 7}{x - 4} + \frac{9 + 3x}{4 - x}$

$= \frac{x^2 + 3x - 7}{x - 4} - \frac{9 + 3x}{(x - 4)}$ M1

$= \frac{x^2 + 3x - 7 - 9 - 3x}{x - 4}$ B1

$= \frac{x^2 - 16}{x - 4}$

$= x + 4$ A1

2. Nancy is planning a holiday to United States. On 1 March 2017, she exchanged S$3000 into US dollars (US$) at Kumar's Money Exchange at a rate of US$1 = S$$x$.

(a) Find an expression, in terms of x, for the amount of US$ she received from Kumar's Money Exchange. [1]

On 15 March 2017, she decided to exchange another S$2100 into US$ at Lee's Money Exchange at a rate of US$1 = S$($x - 0.1$).

(b) Find an expression, in terms of x, for the amount of US$ she received from Lee's Money Exchange. [1]

(c) Given that Nancy received a total of US$3500 from the two Money Exchanges, form an equation in x and show that it simplifies to $70x^2 - 109x + 6 = 0$. [3]

(d) (i) Solve the equation $70x^2 - 109x + 6 = 0$, giving your answers correct to 4 decimal places. [2]

(ii) Suggest a reason why one of the answers has to be rejected. [1]

(iii) Hence, find the exchange rate between S$ and US$ offered by Lee's Money Exchange. [1]

(e) Is it better for Nancy to change her currency on 1 March or 15 March? Justify your answer with appropriate workings. [2]

(a) $\frac{3000}{x}$ B1

(b) $\frac{2100}{x - 0.1}$ B1

(c) $\frac{3000}{x} + \frac{2100}{x - 0.1} = 3500$ M1

$\frac{3000(x - 0.1) + 2100x}{x(x - 0.1)} = 3500$ M1

$5100x - 300 = 3500(x^2 - 0.1x)$

$3500x^2 - 5450x + 300 = 0$ B1

$70x^2 - 109x + 6 = 0$ (*shown*)

(d)(i)

$x = \frac{-(-109) \pm \sqrt{(-109)^2 - 4(70)(6)}}{2(70)}$ M1

$= \frac{109 \pm \sqrt{10201}}{140}$

$= 1.5$ (exact value) or 0.0571 A1 (for both answers)

(ii) The answer 0.0571 has to be rejected as substituting it into (x-0.1) will make the value negative, and thus inapplicable. R1

(iii) The exchange rate offered by Lee is US$1 = S$1.40. A1

(e) On 1 Mar:

$S\$3000 = US\$\left(\frac{3000}{1.5}\right) = US\$2000$ M1; evaluation step needed

On 15 Mar:

$S\$3000 = US\$\left(\frac{3000}{1.5-0.1}\right) = US\$2142.86$

Since Nancy can exchange for more US$ on 15 Mar as compared to 1 Mar for the same S$3000, she should changed her money on 15 Mar. R1

3. A cake shop sells 3 different types of muffins. The table below shows the numbers of muffins sold over 2 days and the price of each type of muffin.

	Chocolate	Blueberry	Cheese
Day 1	12	25	10
Day 2	15	24	9
Price of each muffin, $	2.00	1.50	2.50

It is given that $\mathbf{P} = \begin{pmatrix} 12 & 25 & 10 \\ 15 & 24 & 9 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2.0 \\ 1.5 \\ 2.5 \end{pmatrix}$.

(a) Evaluate the matrix $\mathbf{M} = \mathbf{PQ}$. [1]

(b) State what the elements of **M** represent. [1]

(c) The cake shop is having a promotion by giving a 25% discount on chocolate muffins, a 30% discount on blueberry muffins and a 40% discount on cheese muffins.

(i) Write down a 3×3 matrix **R**, where the product of **RQ** will give the discounted price of each type of muffin. [1]

(ii) Evaluate the matrix $\mathbf{N} = \mathbf{RQ}$. [1]

(d) Evaluate the matrix **PN** and state what the elements of **PN** represent. [2]

(e) Given that $\mathbf{T} = \begin{pmatrix} 1 & 1 \end{pmatrix}$, evaluate the matrix **TPN** and state what the elements of **TPN** represent. [2]

(a) $\mathbf{M} = \begin{pmatrix} 12 & 25 & 10 \\ 15 & 24 & 9 \end{pmatrix}\begin{pmatrix} 2.0 \\ 1.5 \\ 2.5 \end{pmatrix}$

$= \begin{pmatrix} 86.5 \\ 88.5 \end{pmatrix}$ A1

(b) The elements of **M** represent the total amount of money collected from the sale of 3 types of muffins in Day 1 and Day 2 respectively. A1 (accept alternatives with keywords) (c)(i) $\mathbf{R} = \begin{pmatrix} 0.75 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.6 \end{pmatrix}$ [A1]

(c)(ii) $\mathbf{N} = \begin{pmatrix} 0.75 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.6 \end{pmatrix}\begin{pmatrix} 2.0 \\ 1.5 \\ 2.5 \end{pmatrix} = \begin{pmatrix} 1.5 \\ 1.05 \\ 1.5 \end{pmatrix}$ [A1]

(d) $\mathbf{PN} = \begin{pmatrix} 12 & 25 & 10 \\ 15 & 24 & 9 \end{pmatrix}\begin{pmatrix} 1.5 \\ 1.05 \\ 1.5 \end{pmatrix} = \begin{pmatrix} 59.25 \\ 61.2 \end{pmatrix}$ [A1]

The elements of **PN** represent the total amount of money collected from the sale of muffins during the promotion period on Day 1 and Day 2 respectively. [A1]

(e) $\mathbf{TPN} = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 59.25 \\ 61.2 \end{pmatrix} = \begin{pmatrix} 120.45 \end{pmatrix}$ [A1]

The elements of **TPN** represent the total amount of money collected from the sale of muffins during the promotion period over 2 days. [A1]

4. (a) Bag A contains 1 gold medal, 4 silver medals and 2 bronze medals. Bag B contains 2 gold medals and 5 silver medals. Peter draws a medal at random from bag A, took note of the colour and placed it into bag B. He then draws a medal at random from bag B.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

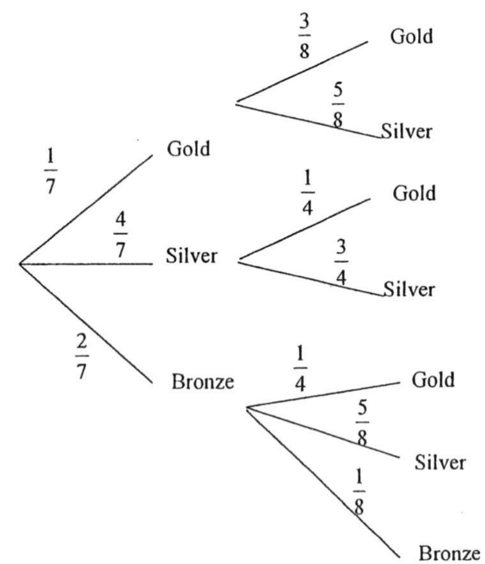
(ii) Find, as a fraction in its simplest form, the probability that

(a) the medal drawn from bag B is a gold medal, [2]

(b) the medal drawn from bag B is a bronze medal. [1]

(a)(i) Answer:

[A2; deduct 1 mark for each error]



(b)(i) P(medal from B is a gold medal)

$= \frac{1}{7}\times\frac{3}{8}+\frac{4}{7}\times\frac{2}{8}+\frac{2}{7}\times\frac{2}{8}$ [M1]

$= \frac{15}{56}$ [A1]

(b) P(medal from B is a bronze medal)

$= \frac{2}{7}\times\frac{1}{8}$

$= \frac{1}{28}$ [A1]

4(b) (i) The nth term of a sequence is given by $T_n = n^2 + 2n - 5$.

(a) Write down the first 5 terms of the sequence. [2]

(b) Which term of the sequence has value 163? [2]

(ii) The first five terms of another sequence are 1, 6, 13, 22, 33, …

(a) By comparing this sequence with the sequence in part (i), write down the nth term of the sequence 1, 6, 13, 22, 33, … [1]

(b) Hence, find the 99th term. [1]

(b)(i)(a) First 5 terms: $-2, 3, 10, 19, 30$ [A2; deduct 1 mark for each error]

(b) $n^2 + 2n - 5 = 163$ [M1]

$n^2 + 2n - 168 = 0$

$(n-12)(n+14) = 0$

$n = 12 \quad or \quad -14 \quad (rejected)$

The 12th term has value 163. [A1]

(ii)(a) $T_n = (n^2 + 2n - 5) + 3 = n^2 + 2n - 2$ [A1]

(b)

$T_n = (99)^2 + 2(99) - 2$

$= 9997$ [A1]

5(a) Use the graph to find the median mass. [1]

(b) In the following grouped frequency table of the mass of mangoes in the plantation, write down the values of p and of q. [2]

Mass (x kg)	$6 \le x < 14$	$14 \le x < 22$	$22 \le x < 30$	$30 \le x < 38$	$38 \le x < 46$
Frequency	4	p	14	q	24

(c) Using your grouped frequency table, calculate an estimate of

(i) the mean mass, [2]

(ii) the standard deviation. [1]

(d) In Plantation B, the total mass of the mangoes produced by each of 80 mango trees were measured. Their mean and standard deviation were found to be 29.1 kg and 10.4 kg respectively. Make two comparisons between the mass of the mangoes produced by the trees in both plantations. [2]

(a) Median mass = 34 kg [A1]

(b) $p = 12 - 4 = 8$ [A1] $q = 56 - 26 = 30$ [A1]

(c)(i) Estimated mean mass

$= \frac{4\times10 + 8\times18 + 14\times26 + 30\times34 + 24\times42}{80}$ kg [M1; for mid-values into formula]

$= 32.2$ kg [A1]

(ii) Standard deviation

$= \sqrt{\frac{89472}{80} - \left(\frac{2576}{80}\right)^2}$ kg

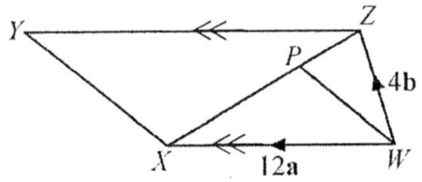
$= 9.031$ kg

$= 9.03$ kg [A1] (to 3 s.f.)

(d) - Since mean mass of A is higher than mean mass of B, the average mass of mangoes produced in plantation A is heavier than that in plantation B. [A1]

- Since standard deviation of A is less than standard deviation of B, the mass of mangoes produced in plantation A is more consistent than that in plantation B. [A1]

6. In the diagram, $WXYZ$ is a trapezium and WX is parallel to ZY. The point P on XZ is such that $ZP : PX = 1 : 3$ and $WX : ZY = 3 : 4$.



(a) Give that $\overrightarrow{WX} = 12\mathbf{a}$ and $\overrightarrow{WZ} = 4\mathbf{b}$, express in terms of **a** and/or **b**,

(i) $\overrightarrow{ZX}$, [1]

(ii) $\overrightarrow{WP}$, [1]

(b) Determine with justification, if the line XY is parallel to the line WP. [2]

(c) Find

(i) $\dfrac{\text{Area of } \Delta WZP}{\text{Area of } \Delta WXP}$, [1]

(ii) $\dfrac{\text{Area of } \Delta WZP}{\text{Area of } \Delta YXZ}$. [2]

(d) Hence, find the area of $WXYZ$ if the area of ΔWZP is 12 units^2. [1]

(a)(i) $\overrightarrow{ZX} = \overrightarrow{ZW} + \overrightarrow{WX}$
$= -4\mathbf{b} + 12\mathbf{a}$ A1

(ii) $\overrightarrow{ZP} = \frac{1}{4}\overrightarrow{ZX}$
$= -\mathbf{b} + 3\mathbf{a}$

$\overrightarrow{WP} = \overrightarrow{WZ} + \overrightarrow{ZP}$
$= 4\mathbf{b} + (-\mathbf{b} + 3\mathbf{a})$
$= 3\mathbf{b} + 3\mathbf{a}$ A1

(b) $\overrightarrow{ZY} = \frac{4}{3}\overrightarrow{WX} = 16\mathbf{a}$

$\overrightarrow{XY} = \overrightarrow{XZ} + \overrightarrow{ZY}$
$= (4\mathbf{b} - 12\mathbf{a}) + 16\mathbf{a}$ M1
$= 4\mathbf{b} + 4\mathbf{a}$
$= \frac{4}{3}\overrightarrow{WP}$

Since $\overrightarrow{XY} = \frac{4}{3}\overrightarrow{WP}$, XY is parallel to WP. R1

Note: 1 mark overall is penalized if students missed out the vector symbol

(c)(i) $\dfrac{\text{Area of } \Delta WZP}{\text{Area of } \Delta WXP} = \dfrac{\frac{1}{2} \times ZP \times h}{\frac{1}{2} \times PX \times h} = \dfrac{1}{3}$ A1

(ii) $\dfrac{\text{Area of } \Delta WZP}{\text{Area of } \Delta YXZ}$

$= \dfrac{\text{Area of } \Delta WZP}{\text{Area of } \Delta WZX} \times \dfrac{\text{Area of } \Delta WZX}{\text{Area of } \Delta YXZ}$

$= \dfrac{1}{4} \times \dfrac{\frac{1}{2} \times WX \times h}{\frac{1}{2} \times ZY \times h}$ M1

$= \dfrac{1}{4} \text{ x } \dfrac{WX}{ZY}$

$= \dfrac{1}{4} \text{ x } \dfrac{3}{4}$

$= \dfrac{3}{16}$ A1

(d) Area of $\Delta WZX = 4 \times 12 = 48$ units^2

Area of $\Delta YXZ = \dfrac{16}{3} \times 12$ units^2
$= 64$ units^2

Area of $WXYZ = 48 + 64$ units^2
$= 112$ units^2 A1

7. Diagram I shows a trough where the base $BCGF$ and the top $ADHE$ are horizontal rectangles. Each of the vertical sides $ABCD$ and $EFGH$ is a trapezium. It is given that $AD = 15$ cm, $BC = 11$ cm, $BX = 9$ cm and $CG = 20$ cm. It is fully filled with water.

Take $\pi = \dfrac{22}{7}$.

Take $\pi = \dfrac{22}{7}$.]

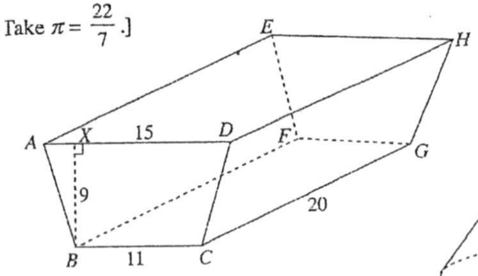


Diagram I

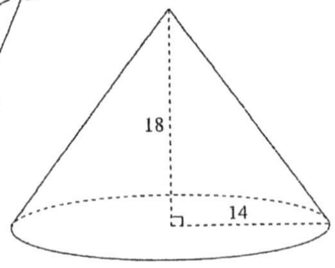


Diagram II

(a) Calculate the volume of the water in the trough in Diagram I. [2]

(b) All the water in Diagram I is made to flow down through a hole at the top of the cone in Diagram II. The water flows at a rate of 0.2 litres per minute. Calculate the time taken for the water in the trough to be transferred, giving your answer in minutes and seconds. [2]

(c) The cone in Diagram II is of base radius 14 cm and height 18 cm. The cone is made to stand on its base. Calculate the height of water in the cone. [4]

(a) Volume of water

$= \left[\frac{1}{2}\times(11+15)\times 9\right]\times 20\ \text{cm}^3$ M1

$= 2340\ \text{cm}^3$ A1

(b) 0.2 litres = 200 cm^3

Time taken $= \frac{2340}{200}$ min M1

$= 11.7$ min $= 11$ min 42 sec A1

(c) Volume of cone

$= \frac{1}{3}\times\frac{22}{7}\times 14^2\times 18\ \text{cm}^3 = 3696\ \text{cm}^3$

Volume of cone not occupied by water

$= (3696 - 2340)\ \text{cm}^3 = 1356\ \text{cm}^3$ B1

By similar triangles, $\frac{h}{18} = \frac{r}{14}$

$r = \frac{7}{9}h$ B1

$\frac{1}{3}\times\frac{22}{7}\times(\frac{7}{9}h)^2\times h = 1356$

$h^3 = 2139\frac{51}{77}$

$h = 12.8859$ cm

≈ 12.9 cm A1

Height of water in cone $= 18 - 12.8859$ cm $= 5.1141$ cm ≈ 5.11 cm A1

Alternative method for (c):

Volume of cone not occupied by water

$= (3696 - 2340)\ \text{cm}^3$

$= 1356\ \text{cm}^3$ B1

$\left(\frac{h}{18}\right)^3 = \frac{1356}{3696}$ M1

$h = 12.8859$

$h \approx 12.9$ A1

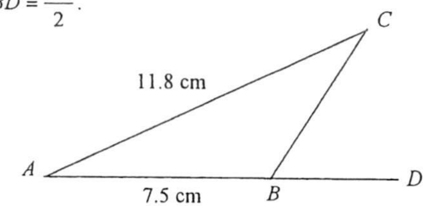
Height of water in cone

$= 18 - 12.8859$ cm

$= 5.1141$ cm A1

≈ 5.11 cm

8. (a) In the diagram below, ABD is a straight line. $AB = 7.5$ cm, $AC = 11.8$ cm and $\sin\angle CBD = \frac{\sqrt{3}}{2}$.



Calculate

(i) $\sin\angle ACB$, giving your answer correct to 3 significant figures, [2]

(ii) $\angle BAC$, [2]

(iii) the area of ΔABC. [1]

(a)(i) $\frac{\sin\angle ACB}{7.5} = \frac{\frac{\sqrt{3}}{2}}{11.8}$ M1

$\sin\angle ACB = 0.550439$

≈ 0.550 (to 3 s.f.) A1

(ii) $\sin\angle CBD = \frac{\sqrt{3}}{2}$

$\angle CBD = 60°$

$\angle CBA = 120°$ M1

$\sin\angle ACB = 0.550439$

$\angle ACB = 33.3971°$

$\angle BAC = 180° - 120° - 33.3971°$ (angle sum of triangle)

$= 26.6029°$

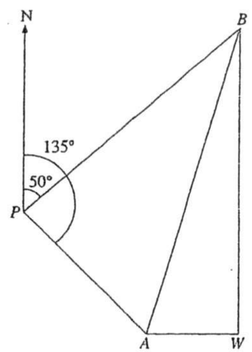
$\sim 26.6°$ (correct to 1 dp) A1

(iii) Area of $\Delta ABC = \frac{1}{2}\times 11.8\times 7.5\times\sin 26.6029°\ \text{cm}^2$

$= 19.8153\ \text{cm}^2$

$\approx 19.8\ \text{cm}^2$ (to 3 s.f.) A1

8. (b) Two military ships, Amaze and Brave left port P at 1000. Amaze sailed at 12 km/h on a bearing of $135°$. Brave sailed at 18 km/h on a bearing of $050°$. After sailing for 3 hours, Amaze is at Island A and Brave is at Island B.



Calculate

(i) the length of AB. [3]

(ii) the bearing of A from B. [2]

(iii) Amaze later sailed to Island W which is due east of Island A. It is given that Island B is due north of Island W. Find the distance BW. [2]

(iv) A helicopter is hovering at a height of 10 km vertically above B. Find the greatest angle of elevation Amaze can have of the helicopter as it sailed towards W. [2]

(b)(i) Derive: $PB = 54$ km and $PA = 36$ km **B1**

$AB^2 = 36^2 + 54^2 - 2\,(36)\,(54)\cos 85°$ **M1**

$= 62.2345$

$AB \approx 62.2$ km (to 3 s.f.) **A1**

(ii) $\dfrac{\sin\angle ABP}{36} = \dfrac{\sin 85°}{62.2345}$

$\sin\angle ABP = \dfrac{36\sin 85°}{62.2345}$ **B1**

$\angle ABP = 35.1876°$

Bearing of A from $B = 360° - (180° - 050°) - 035.1876°$

$= 194.8124°$

$\sim 194.8°$ (correct to 1 d.p.) **A1**

(iii) $\angle ABW = 180° - 130° - 35.1876° = 14.8124°$

$BW = (\cos 14.8124°) \times 62.2345$ **M1** * $BW = \cos 14.81° \times 62.234$ has no mark

$= 60.1663$ km

~ 60.2 km (to 3 s.f.)

(iv) Let the greatest angle of elevation be α.

$\tan\alpha = \dfrac{10}{60.166}$

$\alpha = 9.4°$ (to 1 d.p.)

10.

(a)(i) Amount = 2 x \$(33+29+30) + 3 x \$(22+19+20)

= \$367 **A1**

(ii) Amount (under 3-parkhopper package)

= \$(2 x 69) + \$(3 x 49) **M1**

= \$285

He would save (\$367 - \$285) = \$82 **A1**

(b) Membership Individual (MI):
(2 x \$112) + (3 x \$72) = \$440

Membership Family (MF):
\$260 + (\$15 x 2) = \$290 **M1**

Comparing MI and MF:
Savings = \$440 - \$290 = \$150

Mr Tan should take up Membership Family package as he would save \$150. **A1 + R1**

From part (a)(ii): 3-parkhopper package = \$285
Although the <u>3-parkhopper package is cheaper than the MF by \$5</u>, but Mr Tay only needs to pay this extra \$5 to have unlimted admission into all 3 parks in a year, which is more value for money.
[M1] for comparison between 3-parkhopper package and MF
[R1] for logical reasoning to justify the extra \$5 spent

9

(a) When $x=4$, $y=\frac{1}{2}(4)^2+\frac{18}{4}-10=2.5$

$k=2.5$ (A1)

(c) $\frac{1}{2}x^2+\frac{18}{x}=15$

$\frac{1}{2}x^2+\frac{18}{x}-10=5$

Draw $y=5$ (M1)

From the graph, $x=1.28$ or 4.8 (A1) (both answers)

(accept error of ± 0.1)

(d) At $x=3$, gradient of tangent

$=\frac{3-0}{5.4-2.5}=0.98$ (A1)

* (accept 0.8 to 1.0)

* Drawing of tangent (M1)

(e) $y=6-x$

x	0	2	4
y	6	4	2

(f)(i) $x=1.3$ and 3.88 (A2)

(accept error of ± 0.1)

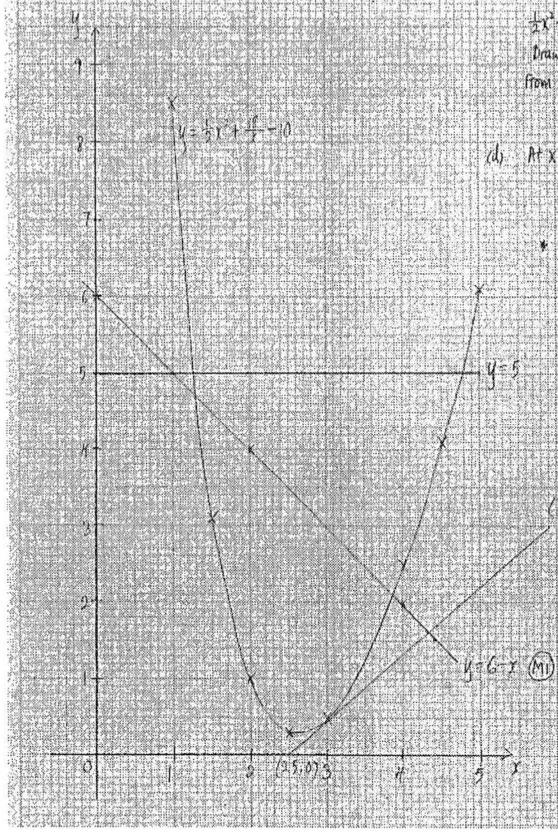
(ii) $\frac{1}{2}x^2+\frac{18}{x}-10=6-x$

$\frac{1}{2}x^2+\frac{18}{x}+x-16=0$

$x^2+\frac{36}{x}+2x-32=0$ (M1)

$x^3+36+2x^2-32x=0$

$\therefore A=2,\ B=-32,\ C=36$ (A1)



Curve sketching:

Plotting of curve (B1)

Smooth curve passing thro' all points (C1)

Axis and scale (B1)