Name:

## MATHEMATICS

Paper 1

## CRESCENT GIRLS' SCHOOL SECONDARY FOUR PRELIMINARY EXAMINATION

Candidates answer on the Question Paper.

## READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80 .

| For Examiner's Use |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Qn No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Marks | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 4 | 3 |
| $\begin{aligned} & \text { Qn } \\ & \text { No. } \end{aligned}$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Marks | 2 | 3 | 3 | 3 |  | 3 | 6 | 7 | 4 | 4 |
| $\begin{gathered} \text { Qn } \\ \text { No. } \end{gathered}$ | 21 | 22 | Total No. of Marks |  |  |  |  |  |  |  |
| Marks | 9 | 8 |  |  |  |  |  |  |  | 0 |

This document consists of 15 printed pages and 1 blank page.

## Compound Interest

$$
\text { Total amount }=P\left(1+\frac{r}{100}\right)^{n}
$$

Mensuration

$$
\begin{aligned}
& \text { Curved surface area of a cone }=\pi r l \\
& \text { Surface area of a sphere }=4 \pi r^{2} \\
& \text { Volume of a cone }=\frac{1}{3} \pi r^{2} h \\
& \text { Volume of a sphere }=\frac{4}{3} \pi r^{3} \\
& \text { Area of triangle } A B C=\frac{1}{2} a b \sin C
\end{aligned}
$$

Arc length $=r \theta$, where $\theta$ is in radians
Sector area $=\frac{1}{2} r^{2} \theta$, where $\theta$ is in radians

Trigonometry

$$
\begin{aligned}
& \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
& a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{aligned}
$$

## Statistics

$$
\begin{aligned}
\text { Mean } & =\frac{\sum f x}{\sum f} \\
\text { Standard Deviation } & =\sqrt{\frac{\sum f x^{2}}{\sum f}-\left(\frac{\sum f x}{\sum f}\right)^{2}}
\end{aligned}
$$

Answer all the questions.
1 By approximating each number to 2 significant figures, estimate the value of $\frac{12.1 \times \sqrt{48.8}}{\sqrt[3]{27.3}}$. Show your working and give your answer to a reasonable degree of accuracy.
Answer

2 (a) Express the ratio of 1 minute and 30 seconds to 1 hour in its simplest form.
(b) If $5 a=4 b$ and $8 b=3 c$, find $a: b: c$.

Answer (a)
(b)

3 Twelve workers are hired to build a wall in 9 days, assuming that they all work at the same rate. After 3 days, two workers left. How many days would the remaining workers take to finish building the wall ?

4 The value of a new house depreciated $10 \%$ each year for 3 years in a row. Then, for the next 3 years, the value of the house increased $10 \%$ each year. Did the value of the house increase or decrease after 6 years? Explain your answer.

Answer: $\qquad$
$\qquad$
$\qquad$
$\qquad$

5 The exterior angles of a hexagon are in the ratio $2: 3: 3: 4: 4: 8$. Find the smallest interior angle of the hexagon.

Answer - [2]

6 Express 0.00952 nanoseconds in megaseconds, giving your answer in standard form.
( 1 nano unit $=1 \times 10^{-9}$ unit; 1 mega unit $=1 \times 10^{6}$ unit)

7 On a particular day at noon, the temperature 15 m above the sea level is $4^{\circ} \mathrm{C}$. The temperature 30 m below the sea level is $-23^{\circ} \mathrm{C}$. Calculate
(a) the difference between these temperatures,

Answer $\qquad$ ${ }^{\circ} C[1]$
(b) the temperature at sea level at noon, assuming that the temperature changes uniformly with height.

8 A tablet is sold at \$1450 after a discount of $20 \%$.
(a) Find the marked price of the tablet.

Answer \$
(b) A customer bought the tablet at the discounted price and he paid for it using a hire purchase scheme according to the following terms: a down-payment of $60 \%$ and the remaining to be paid in monthly instalments over 16 months at a simple interest rate of $x \%$ per annum. Given that the total interest he paid is $\$ 34.80$, find $x$.

$$
\begin{equation*}
\text { Answer } x= \tag{2}
\end{equation*}
$$

9 (a) Factorise completely $4+6 a b-9 a^{2}-b^{2}$.

Answer
(b) Without using a calculator, show that $2^{17}-2^{14}$ is divisible by 7 .

Answer
In triangle $A B C, \angle A B C=90^{\circ}, A B=24 \mathrm{~cm}$ and $\sin \angle A C B=\frac{3}{5}$. Without the use of calculator, find the value of
(i) $A C$
(ii) $5 \cos \left(180^{\circ}-\angle A C B\right)$


Answer (i)
(ii)

11 Solve the following inequality $6<2 x+\frac{3 x-5}{4} \leq 4 x+2$.

> Answer

12 (a) A designer has to design a box in the shape of a cube of length $l \mathrm{~cm}$ so as to store rectangular bricks of dimensions 45 cm by 21 cm by 15 cm . To save cost, he must ensure that the bricks fit exactly into the box, leaving no gaps in between. What is the smallest possible value of $l$ ?
(b) How many bricks can fit into the box?

13 A map is drawn to a scale of $1: 20000$
(a) The distance from one end of a grassland to the other end is represented by a line of length 8 cm on the map. Calculate the actual distance, giving your answer in kilometres.

Answer
km [1]
(b) The actual area of the grassland is 2.5 hectares. Calculate the area on the map, giving your answer in square centimetres. [ 1 hectare $=10000 \mathrm{~m}^{2}$ ]

Answer
$\mathrm{cm}^{2}$ [2]
14 In the diagram, $A B C D E$ is a regular pentagon where $A$ is due north of $E$. Find the bearing of
(i) $B$ from $A$,
(ii) $D$ from $A$,
(iii) $C$ from $E$.


Answer (i)

${ }^{o}$ [1]
(ii)
(iii) ${ }^{o}$ [1]

15 In the axes provided below, sketch the graphs of the following. State intercept(s) if any.
(a) $y=(x-1)^{3}$,
(b) $y=\frac{x+1}{x}$.

Answer
(a)

(b)


16
(a) Consider the sequence $\frac{2}{3}, \frac{5}{8}, \frac{13}{21}, \frac{34}{55}, \ldots \ldots$

Write down the next two terms of the sequence.

> Answer
(b) Write down an expression, in terms of $n$, for the $n$th term of the sequence $3,6,10,15,21, \ldots$.

17 The volume of cylinder $A$ of radius $r \mathrm{~cm}$ and height $h \mathrm{~cm}$ is $240 \mathrm{~cm}^{3}$.
(a) Find the volume of cylinder $B$ of radius $2 r \mathrm{~cm}$ and height $\frac{1}{3} h \mathrm{~cm}$.

Answer $\qquad$ $\mathrm{cm}^{3}$ [2]
(b) Cylinder $C$ is similar to cylinder $A$. If the radius of cylinder $C$ is $\frac{1}{2} r \mathrm{~cm}$, find its volume.

Answer $\qquad$ $\mathrm{cm}^{3}$ [2]
(c) A cone $D$ has the same volume as cylinder $A$. If the height of cone $D$ is $h \mathrm{~cm}$, find the ratio of the radius of cone $D$ to that of cylinder $A$.

18 Given that $\overrightarrow{A B}=\binom{8}{-6}, \overrightarrow{O B}=\binom{-6}{12}$ and $C$ is the point on $O B$ such that $O C: C B=1: 2$.
(a) Find
(i) $|\overrightarrow{A B}|$,
(ii) the position vector of $C$,
(iii) $\overrightarrow{A C}$
(b) Given that $\binom{2}{m}$ and $\overrightarrow{A B}$ are parallel vectors. Find the value of $m$.
(c) Find the coordinates of $D$, such that $A B C D$ is a parallelogram.
$\qquad$
(ii)
(iii) $\overrightarrow{A C}=$
(b) $m=$
(c) $D$ is

19 The diagrams below show four containers (not drawn to scale), $A, B, C$ and $D$ each with a height of $h \mathrm{~cm}$. The containers are initially empty. It takes $t$ seconds to fill each container with water at a constant rate.

(a) On the axes in the answer space below, sketch the graph of the depth of the water against time for each of the four containers.

(b) It takes 12 seconds to fill container $D$ to the brim. Find the time it takes to fill container $D$ to half its height.

20 The diagram shows the speed-time graphs of a car and a lorry travelling on the road for a period of 100 seconds. After accelerating, the car travels at a constant speed of $15 \mathrm{~m} / \mathrm{s}$ for the next 40 seconds before decelerating to a stop, while the lorry travels at a constant speed of $10 \mathrm{~m} / \mathrm{s}$ throughout.

(a) Find the time when the speed of the car is $10 \mathrm{~m} / \mathrm{s}$.

Answer
$\sec [1]$
(b) Find the speed of the car 10 seconds before it comes to rest.

Answer $\mathrm{m} / \mathrm{s}$ [1]
(c) Find the time when the car overtakes the lorry.

Answer
$\sec$ [2]

21 (a) A box contains five slips of paper. Each slip has one of the numbers $4,6,7,8$ or 9 written on it. There are two players for the game. The first player reaches into the box and draws two slips and adds the two numbers. If the sum is even, the player wins. If the sum is odd, the player loses. What is the probability that the first player wins.

## Answer:

(b) A game is such that a fair die is rolled respectively until a ' 6 ' is obtained. Find the probability that the game ends by the fourth roll.

Answer .
Suppose now that the game is such that the same die is rolled repeatedly until two ' 6 's are obtained. Find the probability that
(i) the game ends on the third roll,
(ii) the game ends on the third roll and the sum of the scores is odd.

> Answer (i)
(ii)
(c) Of the 33 students in a class, 25 own tablet PC and 9 own desktop computers.

It is given that $\xi=\{$ students in the class $\}$,
$A=\{$ students who own a tablet PC $\}$,
$B=\{$ students who own a desktop computer $\}$ and $n(A \cap B)=x$
(i) Express $n\left(A^{\prime} \cap B^{\prime}\right)$ in terms of $x$.
(ii) Express in set notation \{ students who own tablet PC but not desktop computer\}.

Answer (i)
(ii)

22 The diagram shows part of the graph of $5(y+3)=a x^{2}+b x$, where $a$ and $b$ are constants.
The graph cuts the $x$-axis at $A\left(-2 \frac{1}{2}, 0\right)$ and $B\left(\frac{1}{2}, 0\right)$.
The graph meets the $y$-axis at the point $C$.
Find
(i) the value of $a$ and of $b$,
(ii) the coordinates of $C$,
(iii) the coordinates of the minimum point,
(iv) the equation of the line of symmetry,

(v) the area of triangle $A B C$.

$$
\text { Answer (i) } a=\ldots \ldots \ldots . . \quad b=\ldots \ldots \ldots . .
$$

(ii) C..
(iii)
(iv)[1]
(v)

## END OF PAPER

Answer Key

| $\begin{array}{\|l\|} \hline \text { Qn } \\ \text { No. } \\ \hline \end{array}$ |  | $\begin{aligned} & \text { Qn } \\ & \text { No. } \\ & \hline \end{aligned}$ |  |
| :---: | :---: | :---: | :---: |
| 1 | 28 or 30 | 17 | (a) $320 \mathrm{~cm}^{3}$ |
| 2 | (a) $1: 40$ <br> (b) $12: 15: 40$ |  | (b) $30 \mathrm{~cm}^{3}$ <br> (c) $\sqrt{3}: 1$ |
| 3 | 7.2 days | 18 | (a) (i) 10 units |
| 4 | $0.9703 P<P$, decreased |  | (ii) $(-2)$ |
| 5 | $60^{\circ}$ |  | $\text { (ii) } \quad 4$ |
| 6 | $9.52 \times 10^{-18}$ megaseconds |  | (12) |
| 7 | (a) $27^{\circ} \mathrm{C} \quad$ (b) $-5^{\circ} \mathrm{C}$ |  | (iii) $\binom{12}{-14}$ |
| 8 | (a) $\$ 1812.50$ (b) 4.5 |  | (b) $-1 \frac{1}{2}$ |
| 9 | (a) $(2-3 a+b)(2+3 a-b)$ <br> (b) $2^{14} \times 7$ is divisible by 7 |  | (c) $D(-10,10)$ |
| 10 | (i) 40 (ii) -4 | 19(a) | depth of water for $A$ |
| 11 | $x>2 \frac{7}{11}$ |  | $/$ |
| 12 | (a) 315 <br> (b) 2205 |  |  |
| 13 | (a) 1.6 km <br> (b) $0.625 \mathrm{~cm}^{2}$ |  |  |
| 14 | (i) $072^{\circ}$ <br> (ii) $144^{\circ}$ <br> (iii) $072^{\circ}$ |  |  |
| 15 | (a) <br> (b) |  |  |


| 16 | (a) $\frac{89}{144}, \frac{233}{377}$ <br> (b) $T_{n}=\frac{1}{2}(n+1)(n+2)$ | 19(b) | 9 secs |
| :---: | :---: | :---: | :---: |
| 20 | (a) $13.3 \mathrm{sec}, 73.3 \mathrm{sec}$ <br> (b) $3.75 \mathrm{~m} / \mathrm{s}$ <br> (c) 30 sec | 21 | (a) $\frac{2}{5}$ <br> (b) $\frac{671}{1296}$ <br> (i) $\frac{5}{108}$ <br> (ii) $\frac{1}{36}$ <br> (c) (i) $x-1$ <br> (ii) $A \cap B^{\prime}$ |
| 22 | (i) $a=12, b=24$ <br> (ii) $C(0,-3)$ <br> (iii) $\left(-1,-5 \frac{2}{5}\right)$ <br> (iv) $x=-1$ <br> (v) 4.5 units $^{2}$ |  |  |

# CRESCENT GIRLS' SCHOOL SECONDARY FOUR PRELIMINARY EXAMINATION 

## MATHEMATICS

Paper 1

4048/01
14 Aug 2018
2 hours

Candidates answer on the Question Paper.

## READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80 .


This document consists of $\mathbf{1 5}$ printed pages and 1 blank page.

## Compound Interest

$$
\text { Total amount }=P\left(1+\frac{r}{100}\right)^{n}
$$

Mensuration
Curved surface area of a cone $=\pi r l$
Surface area of a sphere $=4 \pi r^{2}$

$$
\begin{aligned}
& \text { Volume of a cone }=\frac{1}{3} \pi r^{2} h \\
& \text { Volume of a sphere }=\frac{4}{3} \pi r^{3}
\end{aligned}
$$

$$
\text { Area of triangle } A B C=\frac{1}{2} a b \sin C
$$

Arc length $=r \theta$, where $\theta$ is in radians

$$
\text { Sector area }=\frac{1}{2} r^{2} \theta \text {, where } \theta \text { is in radians }
$$

Trigonometry

$$
\begin{aligned}
& \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
& a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{aligned}
$$

Statistics

$$
\begin{aligned}
\text { Mean } & =\frac{\sum f x}{\sum f} \\
\text { Standard Deviation } & =\sqrt{\frac{\sum f x^{2}}{\sum f}-\left(\frac{\sum f x}{\sum f}\right)^{2}}
\end{aligned}
$$

## Answer all the questions.

1 By approximating each number to 2 significant figures, estimate the value of $\frac{12.1 \times \sqrt{48.8}}{\sqrt[3]{27.3}}$. Show your working and give your answer to a reasonable degree of accuracy.

$$
\begin{aligned}
\frac{12.1 \times \sqrt{48.8}}{\sqrt[3]{27.3}} & \approx \frac{12 \times \sqrt{49}}{\sqrt[3]{27}} \\
& =\frac{12 \times 7}{3} \\
& =28 \\
& \approx 30
\end{aligned} \quad \text { B1 for estimating all values to } 2 \text { s.f. }
$$

Answer $\qquad$ 28 or 30

2 (a) Express the ratio of 1 minute and 30 seconds to 1 hour in its simplest form.
(b) If $5 a=4 b$ and $8 b=3 c$, find $a: b: c$.
(a) $1 \mathrm{~min} 30 \mathrm{~s}: 1 \mathrm{~h}=90 \mathrm{~s}: 3600 \mathrm{~s}$

$$
=1: 40
$$

(b) $a: b: c$
$4 \times 3: 5 \times 3$
$3 \times 5: 8 \times 5$

Answer (a) $\qquad$ $1: 40$
(b) ......... $12: 15: 40$

3 Twelve workers are hired to build a wall in 9 days, assuming that they all work at the same rate. After 3 days, two workers left. How many days would the remaining workers take to finish building the wall?
After 3 days, number of man-day needed to complete the job $=6 \times 12$
$=72$
Hence number of days needed for remaining workers $=\frac{72}{10}$

$$
=7.2
$$

Answer $\qquad$ 7.2 days [2]

4 The value of a new house depreciated $10 \%$ each year for 3 years in a row. Then, for the next 3 years, the value of the house increased $10 \%$ each year. Did the value of the house increase or decrease after 6 years? Explain your answer.

Answer: Let original value of house be $P$. Price after 3 years is $P(0.9)^{3}$. After another 3 years the value of the house will be $P(0.9)^{3}(1.1)^{3}$, which equals approximately $0.9703 P$. Since $0.9703 P<P$, the value of the house decreased after 6 years, by approximately $3 \%$

B1 for $P(0.9)^{3}$; B1 for $0.9703 P$

5 The exterior angles of a hexagon are in the ratio $2: 3: 3: 4: 4: 8$. Find the smallest interior angle of the hexagon.
Largest exterior angle $=\frac{8}{24} \times 360^{\circ}=120^{\circ}$ B1

Hence smallest interior angle $=180^{\circ}-120^{\circ}=60^{\circ}$ B1

Answer .. $60^{\circ}$.

6 Express 0.00952 nanoseconds in megaseconds, giving your answer in standard form.
( 1 nano unit $=1 \times 10^{-9}$ unit ; 1 mega unit $=1 \times 10^{6}$ unit)
$0.00952 \times 10^{-9}$ seconds $=k \times 10^{6}$ seconds Or $0.00952 \times 10^{-9}$ seconds

$$
\begin{array}{rlrl}
k=0.00952 \times 10^{-15} & & =0.00952 \times 10^{-9-6} \text { megaseconds } \\
& =\frac{9.52}{1000} \times 10^{-15} & & =9.52 \times 10^{-3} \times 10^{-15} \\
& =9.52 \times 10^{-18} & & =9.52 \times 10^{-18}
\end{array}
$$

M1A1

7 On a particular day at noon, the temperature 15 m above the sea level is $4^{\circ} \mathrm{C}$.
The temperature 30 m below the sea level is $-23^{\circ} \mathrm{C}$. Calculate
(a) the difference between these temperatures,

$$
\begin{aligned}
\text { difference } & =4-(-23) \\
& =27^{\circ} \mathrm{C}
\end{aligned}
$$

$\qquad$
(b) the temperature at sea level at noon, assuming that the temperature changes uniformly with height.

Rate of temperature increase $=\frac{27}{45}=0.6^{\circ} \mathrm{C}$
Hence temperature at sea level $=-23+30 \times 0.6$

$$
=-5{ }^{\circ} \mathrm{C}
$$

Answer $\qquad$ -5. $\qquad$ C [1]

8 A tablet is sold at $\$ 1450$ after a discount of $20 \%$.
(a) Find the marked price of the tablet.

$$
\frac{1450}{80} \times 100=1812.50
$$

Answer \$........1812.50.
(b) A customer bought the tablet at the discounted price and he paid for it using a hire purchase scheme according to the following terms: a down-payment of $60 \%$ and the remaining to be paid in monthly instalments over 16 months at a simple interest rate of $x \%$ per annum. Given that the total interest he paid is $\$ 34.80$, find $x$.

$$
\begin{aligned}
& (40 \% \times 1450) \times \frac{x}{100} \times \frac{16}{12}=34.80 \\
& x=\frac{34.80}{580} \times 100 \times \frac{12}{16} \\
& =4.5
\end{aligned}
$$

9 (a) Factorise completely $4+6 a b-9 a^{2}-b^{2}$.

$$
\begin{aligned}
& 4+6 a b-9 a^{2}-b^{2} \\
& =4-\left(9 a^{2}-6 a b+b^{2}\right) \\
& =4-(3 a-b)^{2} \\
& =(2-3 a+b)(2+3 a-b)
\end{aligned}
$$

$$
\text { M1 for }(3 a-b)^{2}
$$

$$
\text { Answer }(2-3 a+b)(2+3 a-b)
$$

(b) Without using a calculator, show that $2^{17}-2^{14}$ is divisible by 7 .

$$
\begin{aligned}
& 2^{17}-2^{14} \\
& =2^{14} \cdot 2^{3}-2^{14} \\
& =2^{14}(8-1) \\
& =2^{14} \times 7
\end{aligned}
$$

Answer ..As 7 is a factor of $2^{17}-2^{14}, 2^{17}-2^{14}$ is divisible by 7

10 In triangle $A B C, \angle A B C=90^{\circ}, A B=24 \mathrm{~cm}$ and $\sin \angle A C B=\frac{3}{5}$. Without the use of calculator, find the value of
(i) $A C$
(ii) $5 \cos \left(180^{\circ}-\angle A C B\right)$
(i) $\sin \angle A C B=\frac{24}{A C}$


$$
\begin{aligned}
A C & =24 \times \frac{5}{3} \\
& =40
\end{aligned}
$$

(ii) $B C^{2}=40^{2}-24^{2}$

$$
\begin{aligned}
& B C=32 \\
& 5 \cos \left(180^{\circ}-\angle A C B\right)=-5 \cos \angle A C B \quad \mathrm{M} 1 \\
&=-5 \times \frac{32}{40} \\
&=-5 \times \frac{32}{40} \quad \mathrm{~A} 1
\end{aligned}
$$

$\qquad$ .40
$\qquad$
11
Solve the following inequality $6<2 x+\frac{3 x-5}{4} \leq 4 x+2$.

$$
\begin{array}{rlrl}
6 & <2 x+\frac{3 x-5}{4} \text { and } & 2 x+\frac{3 x-5}{4} & \leq 4 x+2 \\
24 & <8 x+3 x-5 & 8 x+3 x-5 & \leq 16 x+8 \\
29 & <11 x & -5 x & \leq 13 \\
x & >\frac{29}{11} & x & \geq-\frac{13}{5} \\
x & >2 \frac{7}{11} & x & \geq-2 \frac{3}{5}
\end{array}
$$

deduct 1 mark from full mark of 2, if 'and' is missing
B1 for any one inequality correct

$$
\begin{equation*}
\text { Answer } \ldots \ldots x>2 \frac{7}{11} . \tag{2}
\end{equation*}
$$

12 (a) A designer has to design a box in the shape of a cube of length $l \mathrm{~cm}$ so as to store rectangular bricks of dimensions 45 cm by 21 cm by 15 cm . To save cost, he must ensure that the bricks fit exactly into the box, leaving no gaps in between. What is the smallest possible value of $l$ ?

$$
\begin{aligned}
45 & =3^{2} \times 5 & & \\
21 & =3 \times 7 & & \text { M1 for prime factorisation } \\
15 & =3 \times 5 & & \\
\text { LCM } & =3^{2} \times 5 \times 7 & & \\
& =315 & & \text { A1 }
\end{aligned}
$$

$$
\text { Answer ...... } 315 .
$$

(b) How many bricks can fit into the box?

$$
\begin{aligned}
\text { Number of bricks } & =\frac{315^{3}}{45 \times 21 \times 15} \\
& =2205
\end{aligned}
$$

$\qquad$ 2205
13 A map is drawn to a scale of 1:20000
(a) The distance from one end of a grassland to the other end is represented by a line of length 8 cm on the map. Calculate the actual distance, giving your answer in kilometres.
$1 \mathrm{~cm}: 0.2 \mathrm{~km}$
$8 \mathrm{~cm}: 1.6 \mathrm{~km}$

Answer $\qquad$ 1.6 $\qquad$ km [1]
(b) The actual area of the grassland is 2.5 hectares. Calculate the area on the map, giving your answer in square centimetres. [ 1 hectare $=10000 \mathrm{~m}^{2}$ ]

$$
\begin{aligned}
2.5 \text { hectares } & =25000 \mathrm{~m}^{2} \\
& =\frac{25000}{10^{6}} \\
& =0.025 \mathrm{~km}^{2}
\end{aligned}
$$

B1

Area scale is $1 \mathrm{~cm}^{2}: 0.04 \mathrm{~km}^{2}$

$$
\text { Area on the map }=\frac{0.025}{0.04}
$$

$$
=0.625 \mathrm{~cm}^{2}
$$

$\qquad$ .0 .625 $\mathrm{cm}^{2}$ [2]
14 In the diagram, $A B C D E$ is a regular pentagon where $A$ is due north of $E$. Find the bearing of
(i) $B$ from $A$,
(ii) $D$ from $A$,
(iii) $C$ from $E$.
(i) Exterior angle $=\frac{360}{5}=72^{\circ}$

$$
\text { Bearing }=072^{\circ}
$$


(ii) $\angle B A D=72^{\circ}$

Bearing $=144^{\circ}$
(iii) Bearing $=072^{\circ}$

Answer (i)............ $072^{\circ}$....... [1]
(ii)......... $144^{\circ} \ldots \ldots .$. [1]
(iii) ............ $072^{\circ}$....... [1]

15 In the axes provided below, sketch the graphs of the following. State intercept(s) if any.
(a) $y=(x-1)^{3}$,
(b) $y=\frac{x+1}{x}, y=1+\frac{1}{x}$

Answer
(a)

(b)

[2]
(a) Shape with point of inflexion on $x$-axis i.e. check that graph curve towards origin B1; $y$-intercept and $x$-intercept B1
(b) Shape and dash asymptote B1; $x$-intercept and $y=1 \mathrm{~B} 1$

16 (a) Consider the sequence $\frac{2}{3}, \frac{5}{8}, \frac{13}{21}, \frac{34}{55}, \ldots \ldots$
Write down the next two terms of the sequence.

$$
\frac{34+55}{55+89}=\frac{89}{144}, \frac{233}{377}
$$

$$
\begin{equation*}
\text { Answer ........... } \frac{89}{144}, \frac{233}{377} \ldots . . . \tag{1}
\end{equation*}
$$

(b) Write down an expression, in terms of $n$, for the $n$th term of the sequence $3,6,10,15,21, \ldots$.
$1+2,1+2+3,1+2+3+4,1+2+3+4+5, \ldots \ldots . . T_{n}=(n+2) \frac{n+1}{2}$ M1A1

$$
\begin{equation*}
\text { Answer } \ldots T_{n}=(n+2) \frac{n+1}{2} . \tag{2}
\end{equation*}
$$

17
The volume of cylinder $A$ of radius $r \mathrm{~cm}$ and height $h \mathrm{~cm}$ is $240 \mathrm{~cm}^{3}$.
(a) Find the volume of cylinder $B$ of radius $2 r \mathrm{~cm}$ and height $\frac{1}{3} h \mathrm{~cm}$.

$$
\text { Volume of cylinder } \begin{aligned}
B & =\pi(2 r)^{2} \frac{1}{3} h \\
& =\frac{4}{3} \pi r^{2} h \\
& =\frac{4}{3} \times 240 \\
& =320
\end{aligned}
$$

M1A1

Answer: $\qquad$ 320 $\qquad$ $\mathrm{cm}^{3}$ [2]
(b) Cylinder $C$ is similar to cylinder $A$. If the radius of cylinder $C$ is $\frac{1}{2} r \mathrm{~cm}$, find its volume.

Answer: $\qquad$ 30. $\qquad$ $\mathrm{cm}^{3}$ [2]
(c) A cone $D$ has the same volume as cylinder $A$. If the height of cone $D$ is $h \mathrm{~cm}$, find the ratio of the radius of cone $D$ to that of cylinder $A$.

$$
\begin{aligned}
& \frac{1}{3} \pi\left(r_{1}\right)^{2} h=\pi r^{2} h \quad \text { M1A1 } \\
& \left(\frac{r_{1}}{r}\right)^{2}=3 \\
& \frac{r_{1}}{r}=\sqrt{3}
\end{aligned}
$$

$\qquad$ $\sqrt{3}: 1$

$$
\begin{aligned}
& \frac{\text { volume of } C}{240}=\left(\frac{1}{2}\right)^{3} \\
& \text { Volume of } C=\frac{1}{8} \times 240 \quad \text { M1A1 } \\
& =30
\end{aligned}
$$

18
Given that $\overrightarrow{A B}=\binom{8}{-6}, \overrightarrow{O B}=\binom{-6}{12}$ and $C$ is the point on $O B$ such that $O C: C B=1: 2$.
(a) Find
(i) $|\overrightarrow{A B}|$,
(ii) the position vector of $C$,
(iii) $\overrightarrow{A C}$
(b) Given that $\binom{2}{m}$ and $\overrightarrow{A B}$ are parallel vectors. Find the value of $m$.
(c) Find the coordinates of $D$, such that $A B C D$ is a parallelogram.
(a) (i) $|\overrightarrow{A B}|=10$ units
(ii) $\overrightarrow{O C}=\frac{1}{3} \overrightarrow{O B}$
(b) $\frac{m}{2}=\frac{-6}{8}$
$=\binom{\frac{-6}{3}}{\frac{12}{3}}=\binom{-2}{4}$
(iii) $\overrightarrow{A C}=\overrightarrow{A B}+\overrightarrow{B O}+\overrightarrow{O C}$
(c) $\overrightarrow{C D}=\overrightarrow{B A}=\binom{-8}{6}$

$$
\begin{aligned}
& =\binom{8}{-6}-\binom{-6}{12}+\binom{-2}{4} \\
& =\binom{12}{-14}
\end{aligned}
$$

$$
\text { Hence } D \text { is }(-2-8,4+6)
$$

$$
=(-10,10)
$$

M1A1
M1A1


19 The diagrams below show four containers (not drawn to scale), $A, B, C$ and $D$ each with a height of $h \mathrm{~cm}$. The containers are initially empty. It takes $t$ seconds to fill each container with water at a constant rate.

(a) On the axes in the answer space below, sketch the graph of the depth of the water against time for each of the four containers.




B2 or B1(any two correct) [2]
(b) It takes 12 seconds to fill container $D$ to the brim. Find the time it takes to fill container $D$ to half its height.
Ratio of volume of prisms $=\frac{1}{4}$. Required time $=\frac{3}{4} \times 12=9$ seconds M1A1
$\qquad$ 9.

20 The diagram shows the speed-time graphs of a car and a lorry travelling on the road for a period of 100 seconds. After accelerating, the car travels at a constant speed of $15 \mathrm{~m} / \mathrm{s}$ for the next 40 seconds before decelerating to a stop, while the lorry travels at a constant speed of $10 \mathrm{~m} / \mathrm{s}$ throughout.

(a) Find the time when the speed of the car is $10 \mathrm{~m} / \mathrm{s}$.

Acceleration $=\frac{15}{20}=0.75 \mathrm{~m} / \mathrm{s}^{2} \quad$ Deceleration $=0.375 \mathrm{~m} / \mathrm{s}$ in 1 sec
Hence time $=\frac{10}{0.75}=13.3 \quad$ Hence time $=100-\frac{10}{0.375}$

$$
=73.3
$$

Answer $\qquad$ . $13.3,73.3$ $\qquad$ . $\sec [1]$
(b) Find the speed of the car 10 seconds before it comes to rest.

Deceleration $=\frac{15}{40} \mathrm{~m} / \mathrm{s}^{2}$
Required speed $=\frac{15}{40} \times 10=3.75$
Answer $\qquad$ 3.75 $\mathrm{m} / \mathrm{s}$ [1]
(c) Find the time when the car overtakes the lorry.

$$
\begin{aligned}
& \frac{1}{2}(t+t-20) \times 15=10 t \\
& (t-10)=\frac{2}{3} t \\
& t=30
\end{aligned}
$$

$\qquad$ 30 $\sec [2]$

21 (a) A box contains five slips of paper. Each slip has one of the numbers $4,6,7,8$ or 9 written on it. There are two players for the game. The first player reaches into the box and draws two slips and adds the two numbers. If the sum is even, the player wins. If the sum is odd, the player loses. What is the probability that the first player wins.
(b) A game is such that a fair die is rolled respectively until a ' 6 ' is obtained. Find the probability that the game ends by the fourth roll.

$$
\begin{array}{ll}
1-\left(\frac{5}{6}\right)^{4}=\frac{671}{1296} & \text { M1A1 } \\
& \text { Answer ............ } \frac{671}{1296}
\end{array}
$$

Suppose now that the game is such that the same die is rolled repeatedly until two ' 6 's are obtained. Find the probability that
(i) the game ends on the third roll,
(ii) the game ends on the third roll and the sum of the scores is odd.
(i) $2\left(\frac{1}{6} \times \frac{5}{6} \times \frac{1}{6}\right)=\frac{5}{108}$
(ii) $2\left(\frac{1}{6} \times \frac{1}{2} \times \frac{1}{6}\right)=\frac{1}{36}$

$$
\begin{array}{r}
\text { Answer(i) } \ldots \ldots . . \frac{5}{108} . \\
\text { (ii) .......... } \frac{1}{36} \tag{1}
\end{array}
$$

(c) Of the 33 students in a class, 25 own tablet PC and 9 own desktop computers.

It is given that

$$
\begin{aligned}
& \xi=\{\text { students in the class }\}, \\
& A=\{\text { students who own a tablet PC }\}, \\
& B=\{\text { students who own a desktop computer }\} \text { and } n(A \cap B)=x
\end{aligned}
$$

(i) Express $n\left(A^{\prime} \cap B^{\prime}\right)$ in terms of $x$.
(ii) Express in set notation \{ students who own tablet PC but not desktop computer\}.

$$
\begin{align*}
n\left(A^{\prime} \cap B^{\prime}\right) & =n(A \cup B)^{\prime}  \tag{i}\\
& =33-(25-. \\
& =x-1
\end{align*}
$$

$$
=33-(25-x)-9 \quad \text { M1A1 }
$$

Answer (i)...............x-1.
(ii) $. . . . \ldots \ldots . A \cap B^{\prime}$

$$
\begin{align*}
& \mathrm{P}(\text { even and even })+\mathrm{P}(\text { odd and odd })=\left(\frac{3}{5} \times \frac{2}{4}\right)+\left(\frac{2}{5} \times \frac{1}{4}\right) \\
& =\frac{2}{5} \quad \text { M1A1 } \\
& \text { Answer: ...... } \frac{2}{5} \tag{2}
\end{align*}
$$

22 (a) The diagram shows part of the graph of $5(y+3)=a x^{2}+b x$, where $a$ and $b$ are constants. The graph cuts the $x$-axis at $A\left(-2 \frac{1}{2}, 0\right)$ and $B\left(\frac{1}{2}, 0\right)$. The graph meets the $y$-axis at the point $C$.
Find
(i) the value of $a$ and of $b$,
(ii) the coordinates of $C$,
(iii) the coordinates of the minimum point,
(iv) the equation of the line of symmetry,
(v) the area of triangle $A B C$.
(i) $\quad a=60-2 b$
$12=5 a-2 b$

Sub (1) into (2)

$$
\begin{align*}
& b=24  \tag{2}\\
& a=12
\end{align*}
$$


(ii) $\quad C(0,-3)$
(iii) $x=\frac{-2 \frac{1}{2}+\frac{1}{2}}{2}$

M1A1

$$
=-1
$$

Hence minimum point is $\left(-1,-5 \frac{2}{5}\right)$
(iv) $x=-1$
(v) Area of triangle $A B C=\frac{1}{2} \times 3 \times 3=4.5$ units $^{2} \quad$ M1A1 Answer (i) ... $a=12 \ldots, \ldots b=24 \ldots$ [2]
(ii) $\ldots \ldots \ldots . C(0,-3) \ldots \ldots \ldots$ [1]
(iii) .......... $\left(-1,-5 \frac{2}{5}\right) \ldots \ldots \ldots$ [2]
(iv) ... ..... $x=-1$............. [
(v) ............4.5...... units $^{2}$ [2]

## END OF PAPER

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| Name: | Register No.: | Class: |
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## CRESCENT GIRLS' SCHOOL <br> SECONDARY FOUR <br> PRELIMINARY EXAMINATION 2018

MATHEMATICS
Paper 2

Additional Materials: Answer Paper
Graph Paper (1 sheet)
Mark Sheet

## READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
The use of a scientific calculator is expected, where appropriate.
For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question. The total of the marks for this paper is 100 .


This document consists of $\mathbf{1 3}$ printed pages.

## Mathematical Formulae

Compound Interest

$$
\text { Total amount }=P\left(1+\frac{r}{100}\right)^{n}
$$

## Mensuration

> Curved surface area of a cone $=\pi r l$
> Surface area of a sphere $=4 \pi r^{2}$

$$
\text { Volume of a cone }=\frac{1}{3} \pi r^{2} h
$$

$$
\text { Volume of a sphere }=\frac{4}{3} \pi r^{3}
$$

Area of triangle $A B C=\frac{1}{2} a b \sin C$
Arc length $=r \theta$, where $\theta$ is in radians
Sector area $\frac{1}{2} r^{2} \theta$, where $\theta$ is in radians

Trigonometry

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{gathered}
$$

## Statistics

$$
\begin{gathered}
\text { Mean }=\frac{\Sigma f x}{\Sigma f} \\
\text { Standard deviation }=\sqrt{\frac{\sum f x^{2}}{\Sigma f}-\left(\frac{\sum f x}{\Sigma f}\right)^{2}}
\end{gathered}
$$

## Answer all the questions.

1 (a) Solve the equation $\frac{x}{x^{2}-3 x-4}-\frac{x+5}{4-x}=1$.
(b) Make $t$ the subject in the formula $x=2 t \sqrt{\frac{k^{2}}{2 k^{2}+3 t^{2}}}$.
(c) Simplify the expression $\sqrt[3]{\frac{y}{x^{2}}} \times \frac{y}{x} \div \sqrt{\frac{x^{3}}{9 y^{-2}}}$.

2 A shop sells two flavours of ice-cream, Rum Raisin and Super Chunkies.
Each flavour is sold in cups of three different sizes, small, medium and large, and of different prices. The sales in two successive days are given in the table below.

|  | Saturday |  |  | Sunday |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Size | Small | Medium | Large | Small | Medium | Large |
| Cost of ice-cream per <br> cup | $\$ 2.50$ | $\$ 3.20$ | $\$ 4.50$ | $\$ 2.50$ | $\$ 3.20$ | $\$ 4.50$ |
| Number of cups of <br> Rum Raisin sold | 12 | 17 | 8 | 14 | 12 | 10 |
| Number of cups of <br> Super Chunkies sold | 18 | 15 | 11 | 13 | 21 | 16 |

The information for Saturday's sales can be represented by the matrix,
$\mathbf{M}=\left(\begin{array}{lll}12 & 17 & 8 \\ 18 & 15 & 11\end{array}\right)$ and the cost of each flavour for each size can be represented by the matrix
$\mathbf{C}=\left(\begin{array}{l}2.5 \\ 3.2 \\ 4.5\end{array}\right)$. The information for the Sunday's sales can be represented by a $2 \times 3$ matrix $\mathbf{N}$.
(a) Write down the matrix $\mathbf{N}$.
(b) Calculate $\mathbf{P}=(\mathbf{M}+\mathbf{N})$.
(c) Describe what is represented by the elements in $\mathbf{P}$.
(d) Calculate $\mathbf{Q}=\frac{1}{2} \mathbf{P C}$.
(e) Describe what is represented by the elements of $\mathbf{Q}$.
(f) Calculate and describe what is represented by the elements of $\mathbf{R}=\left(\begin{array}{ll}1 & 1\end{array}\right) \mathbf{P C}$.


The points $A, B, C, D$ lie on a circle, centre $O . N$ and $M$ are midpoints of $A B$ and $C D$ respectively. It is given that $O N=O M$.
(a) Show that the triangles $A B E$ and $D C E$ are congruent.
(b) It is given that $A B=6 \mathrm{~cm}$ and $O N=(r-1) \mathrm{cm}$, where $r$ is the radius of the circle. Find the value of $r$.

4 In the diagram, $A B C D$ is a square whose diagonals are each 2 cm long. Given that $O$ is the centre of the bigger circle and $B D$ and $A D$ are the diameters of the bigger and smaller circle respectively.
Find the area of the shaded region.


5 The first four terms in a sequence of numbers, $p_{1}, p_{2}, p_{3}, p_{4}, \ldots$, are given below.
$p_{1}=1^{2}+2^{2}+2^{2}=3^{2}$
$p_{2}=2^{2}+3^{2}+6^{2}=7^{2}$
$p_{3}=3^{2}+4^{2}+12^{2}=13^{2}$
$p_{4}=4^{2}+5^{2}+20^{2}=21^{2}$
(a) Write down an expression for $p_{5}$ and show that $p_{5}=961$.
(b) Given that $p_{10}=10^{2}+11^{2}+s^{2}=k$, express $k$ as a perfect square in terms of $s$.
(c) Given that $p_{w}=w^{2}+(w+1)^{2}+r^{2}=5257^{2}$, find the value of $r$ and of $w$.
(d) Show that $p_{n}=n^{4}+2 n^{3}+3 n^{2}+2 n+1$.

## 6 Answer the whole of this question on a sheet of graph paper.

The variables $x$ and $y$ are connected by the equation $y=x+\frac{7}{x}-6$.
The table below gives some values of $x$ and the corresponding values of $y$.

| $x$ | 0.5 | 1 | 2 | 3.5 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $p$ | 2 | -0.5 | $q$ | 0.4 | 1.17 | 2 | 2.88 |

(a) Find the values of $p$ and $q$.
(b) Using a scale of 2 cm to represent 1 unit on each axis, draw the graph of $y=x+\frac{7}{x}-6$ for the values of $x$ in the range $0<x \leq 8$.
(c) Draw the line $y=-\frac{1}{2} x+3$.
(d) Use your graph to find the $x$-coordinate of a point on the curve $y=x+\frac{7}{x}-6$ at which the gradient of the tangent is equal to -0.5 .
(e) Find the range of values of $x$ for which $x^{2}-6 x+7 \leq 2.5 x$.
(f) By drawing a suitable line on your graph, find the solutions of the equation $2 x^{2}-13 x+14=0$.

7 (a) In the diagram, $O A R B$ is a sector of a circle with centre $O$, radius 12 cm and angle $A O B=1.2$ radians. $C$ is the centre of the circle enclosed inside the sector, $O C R$ is a straight line and the circle touches the sector at $P, Q$ and $R$.

(i) Show that the radius of the enclosed circle is 4.3305 cm , correct to 4 decimal places.
(ii) Calculate the perimeter of the shaded region $P O Q$.
(b) In the diagram below, $A B D, A F G, A C E, B F C$ and $D G E$ are straight lines.
$B F C$ is parallel to $D G E$ and $D B A$ is parallel to $G C$.
$A B=6 \mathrm{~cm}, B F=3 \mathrm{~cm}, F C=5 \mathrm{~cm}$ and $G C=10 \mathrm{~cm}$.

(i) Prove that triangle $B F A$ is similar to triangle $C F G$.
(ii) Calculate $G E$.
(iii) Given the area of triangle $A B F$ is $6.4 \mathrm{~cm}^{2}$, find the area of trapezium $A C G D$.

8 Amanda ran the 21 km of a half-marathon race at an average speed of $x \mathrm{~km} / \mathrm{h}$.
(a) Write down, in terms of $x$, an expression for the number of hours it took her to complete the race.
(b) Deborah ran the same race at an average speed which is $3 \mathrm{~km} / \mathrm{h}$ faster than Amanda's speed. Write down, in terms of $x$, an expression for the number of hours which Deborah took.
(c) Given that the difference between the two times was 20 minutes, write down an equation in $x$ and show that it reduces to $x^{2}+3 x-189=0$.
(d) Solve the equation $x^{2}+3 x-189=0$, giving your answers correct to 2 decimal places.
(e) Find, in hours and minutes, the time it took Amanda to complete the race.

## 9


[The volume of tetrahedron $=\frac{1}{3} \times$ base area $\times$ height ]
The diagram shows a tetrahedron, $V A B C$, which has a horizontal equilateral triangular base $A B C$ of side 20 cm .
The slant edge of the tetrahedron $(V A, V B$ and $V C)$ are each of length 27 cm . $M$ is the mid-point of $A B$ and the vertical line $V N$ meets the plane $A B C$ at $N$ where $M N: N C=1: 2$. Calculate
(a) $C M$,
(b) the angle of elevation of $V$ from $A$,
(c) the volume of the tetrahedron.

10 In the figure below, the $x$-intercept and $y$-intercept of the line $A B$ are -12 and -6 respectively. Both the $x$-intercept and $y$-intercept of the line $C D$ are -10 .


Find
(a) the equation of the line $A B$ and $C D$,
(b) the coordinates of $E$,
(c) the area of $O C E B$,
(d) the coordinates of $F$ given that point $F$ lies on $A B$ produced such that $A F$ : $F B=5: 3$,
(e) find the coordinates of point $G$ where $G$ is the point on the $x$-axis such that $O E$ is parallel to $G D$.

11 The cumulative frequency curve below shows the weights of a sample of 160 boys from a school when they enter Secondary One.

(a) Use the graph to find an estimate for
(i) the number of boys whose weight is more than 60 kg ,
(ii) the median weight,
(iii) the percentage of boys whose weight is less than or equal to 52 kg .
(b) The lightest $25 \%$ of the boys are classified as "slim".

The heaviest $25 \%$ of the boys are classified as "big-sized".
The remaining boys are classified as "medium".
Find an estimate for
(i) the greatest possible difference in weight between 2 boys classified as "bigsized",
(ii) the least possible difference between the weight of a boy classified as "slim" and the weight of a boy classified as "big-sized".
(c) The following box-and-whisker plot shows the distribution of weights of 160

Secondary One girls from the same school. Compare the data provided and record 2 observations about the distribution of weights between boys and girls.


12 Mrs Tan, a mother of two children, saw a newspaper article as shown.

## Price of formula milk in Singapore has soared

The average price of a 900 g tin of formula milk has increased sharply over the last 5 years, outstripping the price increases of other dairy products and household staples.

On Monday, the Government announced it is tightening rules to encourage greater price competition.

The table below shows the price (in SGD) per 100 grams of different brands of formula milk in year 2012 and 2017.

| Date Brand | Similac | Friso | Nan | S26 | Mamil |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Dec 2012 | 5.71 | 5.22 | 5.20 | 5.13 | 4.96 |
| Mar 2017 | 7.05 | 6.56 | 7.45 | 6.36 | 6.41 |
| \% increase |  | 43.3 | 24.0 | 29.2 |  |

Coffee stain was found on the newspaper article covering some of the information.
(a) (i) Calculate the percentage increase in the price of the Similac and Friso formula milk covered by the coffee stain..
(ii) Hence, do you agree with the headline of the newspaper article? Support your answer with a reason.
A few days later, Mrs Tan saw another article regarding the price of similar brands of formula milk sold in Singapore, Malaysia and China.

| Country | Similac | Friso | Nan | S26 | Mamil |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Singapore | 7.05 | 6.56 | 7.45 | 6.36 | 6.41 |
| Malaysia | 3.92 | 3.54 | 4.29 | 4.13 | 3.51 |
| China | 4.79 | 5.58 | 9.06 | 4.25 | 4.00 |

Price (in SGD) per 100 grams of formula milk in Singapore, Malaysia and China.

Upon seeing the article, Mrs Tan intends to purchase some cans of formula milk in China and ship them back during her holidays.

Mrs Tan did an online research and found the following shipping rate from China to Singapore by SHIPPER Company.

| Weight of parcel, $\boldsymbol{x} \mathbf{( k g})$ | Shipping Rate <br> $1^{\text {st }} \mathrm{kg} 150 \mathrm{RMB}$ follow by |
| :---: | :---: |
| $x \leq 10$ | $75 R M B / \mathrm{kg}$ |
| $10<x \leq 20$ | $35 R M B / \mathrm{kg}$ |
| $20<x \leq 50$ | $31 R M B / \mathrm{kg}$ |
| $50<x \leq 75$ | $27 R M B / \mathrm{kg}$ |
| $75<x \leq 100$ | $25 R M B / \mathrm{kg}$ |
| $100<x \leq 200$ | $24 R M B / \mathrm{kg}$ |
| $x>200$ | $22 R M B / \mathrm{kg}$ |

The information below shows a can of the 900 g formula milk that Mrs Tan intends to purchase in China and the online currency conversion.


1 Singapore Dollar equals
4.89 Chinese Yuan Renminbi (RMB)
$\square$ Singapore Dollar


Chinese Yuan Renminbi

Mrs Tan intends to spend at most $\mathrm{S} \$ 650$ for both the formula milk and shipping fee.
(b) Calculate the maximum number of cans of formula milk that Mrs Tan can buy.

## END OF PAPER

## ANSWER KEY



| $\mathbf{7}$ | aii | 21.1 cm |
| :--- | :--- | :--- |
|  | $\mathbf{b i i}$ | $13 \frac{1}{3} \mathrm{~cm}$ |
|  | $\mathbf{b i i i}$ | $74.0 \mathrm{~cm}^{2}$ |
| $\mathbf{8}$ | $\mathbf{a}$ | $\frac{21}{x} \mathrm{hr}$ |
|  | $\mathbf{b}$ | $\frac{21}{x+3} \mathrm{hr}$ |
|  | $\mathbf{d}$ | $x=12.33(2 \mathrm{dp}) \quad$ or $\quad x=-15.33(2 \mathrm{dp})$ |
|  | $\mathbf{e}$ | 1 hr 42 mins |
| $\mathbf{9}$ | $\mathbf{a}$ | 17.3 cm |
|  | $\mathbf{b}$ | $64.7^{\circ}$ |
|  | $\mathbf{c}$ | 1410 cm |
| $\mathbf{1 0}$ | $\mathbf{a}$ | $y=-x-10$ |
|  | $\mathbf{b}$ | $(-8,-2)$ |
|  | $\mathbf{c}$ | $34 \mathrm{units}{ }^{2}$ |
|  | $\mathbf{d}$ | $(18,-15)$ |
|  | $\mathbf{e}$ | $(40,0)$ |
| $\mathbf{1 1}$ | $\mathbf{a i}$ | 40 |
|  | aii | 56 kg |
|  | aiii | $27.5 \%$ |
|  | $\mathbf{b i}$ | 15 kg |
|  | $\mathbf{b i i}$ | 8.5 kg |
| $\mathbf{1 2}$ | $\mathbf{a i}$ | $\%$ increase of Similac $=23.5 \%$ <br> $\%$ increase of the Friso $=25.7 \%$ |
|  | $\mathbf{b}$ | 13 tins |
|  |  |  |


| Name: MARK SCHEME | Register No.: | Class: |
| :--- | :--- | :--- |



## CRESCENT GIRLS' SCHOOL <br> SECONDARY FOUR <br> PRELIMINARY EXAMINATION 2018

MATHEMATICS
Paper 2

Additional Materials: Answer Paper
Graph Paper (1 sheet)
Mark Sheet

## READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
The use of a scientific calculator is expected, where appropriate.
For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question. The total of the marks for this paper is 100 .


This document consists of 21 printed pages.

## Mathematical Formulae

Compound Interest

$$
\text { Total amount }=P\left(1+\frac{r}{100}\right)^{n}
$$

## Mensuration

> Curved surface area of a cone $=\pi r l$
> Surface area of a sphere $=4 \pi r^{2}$

$$
\text { Volume of a cone }=\frac{1}{3} \pi r^{2} h
$$

$$
\text { Volume of a sphere }=\frac{4}{3} \pi r^{3}
$$

Area of triangle $A B C=\frac{1}{2} a b \sin C$
Arc length $=r \theta$, where $\theta$ is in radians
Sector area $\frac{1}{2} r^{2} \theta$, where $\theta$ is in radians

Trigonometry

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{gathered}
$$

## Statistics

$$
\begin{gathered}
\text { Mean }=\frac{\Sigma f x}{\Sigma f} \\
\text { Standard deviation }=\sqrt{\frac{\sum f x^{2}}{\Sigma f}-\left(\frac{\sum f x}{\Sigma f}\right)^{2}}
\end{gathered}
$$

## Answer all the questions.

1 (a) Solve the equation $\frac{x}{x^{2}-3 x-4}-\frac{x+5}{4-x}=1$.
(b) Make $t$ the subject in the formula $x=2 t \sqrt{\frac{k^{2}}{2 k^{2}+3 t^{2}}}$.
(c) Simplify the expression $\sqrt[3]{\frac{y}{x^{2}}} \times \frac{y}{x} \div \sqrt{\frac{x^{3}}{9 y^{-2}}}$.

1 (a)

$$
\begin{aligned}
& \frac{x}{x^{2}-3 x-4}-\frac{x+5}{4-x}=1 \\
& \frac{x}{(x-4)(x+1)}-\frac{x+5}{4-x}=1 \\
& \frac{x}{(x-4)(x+1)}+\frac{x+5}{x-4}=1
\end{aligned}
$$

Multiply throughout by $(x-4)(x+1)$
$x+(x+5)(x+1)=(x-4)(x+1)$
$x+x^{2}+6 x+5=x^{2}-3 x-4$
$10 x+9=0$
$x=-\frac{9}{10}$
(b)

$$
\begin{array}{ll}
x=2 t \sqrt{\frac{k^{2}}{2 k^{2}+3 t^{2}}} \\
\frac{x}{2 t}=\sqrt{\frac{k^{2}}{2 k^{2}+3 t^{2}}} & \\
\frac{x^{2}}{4 t^{2}}=\frac{k^{2}}{2 k^{2}+3 t^{2}} & \text { M1 } \\
x^{2}\left(2 k^{2}+3 t^{2}\right)=4 k^{2} t^{2} & \\
2 k^{2} x^{2}+3 t^{2} x^{2}=4 k^{2} t^{2} & \text { M1 } \\
3 t^{2} x^{2}-4 k^{2} t^{2}=-2 k^{2} x^{2} & \\
t^{2}\left(3 x^{2}-4 k^{2}\right)=-2 k^{2} x^{2} & \begin{array}{l}
\text { A1 (No marks if } \pm \text { is not } \\
\text { shown }
\end{array} \\
t= \pm \sqrt{\frac{-2 k^{2} x^{2}}{3 x^{2}-4 k^{2}}} \text { or } t= \pm \sqrt{\frac{2 k^{2} x^{2}}{4 k^{2}-3 x^{2}}} &
\end{array}
$$

(c) $\sqrt[3]{\frac{y}{x^{2}}} \times \frac{y}{x} \div \sqrt{\frac{x^{3}}{9 y^{-2}}}$

$$
\begin{aligned}
& =\frac{y^{\frac{1}{3}}}{x^{\frac{2}{3}}} \times \frac{y}{x} \div \frac{x^{\frac{3}{2}}}{3 y^{-1}} \\
& =\frac{y^{\frac{1}{3}}}{x^{\frac{2}{3}}} \times \frac{y}{x} \times \frac{3 y^{-1}}{x^{\frac{3}{2}}}
\end{aligned}
$$

M1 - indices of variables correct

$$
=\frac{3 y^{\frac{1}{3}}}{x^{\frac{19}{6}}}
$$

A1

2 A shop sells two flavours of ice-cream, Rum Raisin and Super Chunkies.
Each flavour is sold in cups of three different sizes, small, medium and large, and of different prices. The sales in two successive days are given in the table below.

|  | Saturday |  |  | Sunday |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Size | Small | Medium | Large | Small | Medium | Large |
| Cost of ice-cream per <br> cup | $\$ 2.50$ | $\$ 3.20$ | $\$ 4.50$ | $\$ 2.50$ | $\$ 3.20$ | $\$ 4.50$ |
| Number of cups of <br> Rum Raisin sold | 12 | 17 | 8 | 14 | 12 | 10 |
| Number of cups of <br> Super Chunkies sold | 18 | 15 | 11 | 13 | 21 | 16 |

The information for Saturday's sales can be represented by the matrix, $\mathbf{M}=\left(\begin{array}{lll}12 & 17 & 8 \\ 18 & 15 & 11\end{array}\right)$ and the cost of each flavour for each size can be represented by the matrix $\mathbf{C}=\left(\begin{array}{l}2.5 \\ 3.2 \\ 4.5\end{array}\right)$. The information for the Sunday's sales can be represented by a $2 \times 3$ matrix $\mathbf{N}$.
(a) Write down the matrix $\mathbf{N}$.
(b) Calculate $\mathbf{P}=(\mathbf{M}+\mathbf{N})$.
(c) Describe what is represented by the elements in $\mathbf{P}$.
(d) Calculate $\mathbf{Q}=\frac{1}{2} \mathbf{P C}$.
(e) Describe what is represented by the elements of $\mathbf{Q}$.
(f) Calculate and describe what is represented by the elements of $\mathbf{R}=\left(\begin{array}{ll}1 & 1\end{array}\right) \mathbf{P C}$.
(a) $\quad \mathbf{N}=\left(\begin{array}{lll}14 & 12 & 10 \\ 13 & 21 & 16\end{array}\right)$

B1
(b)

$$
\mathbf{P}=\left(\begin{array}{lll}
26 & 29 & 18 \\
31 & 36 & 27
\end{array}\right) \quad \leftarrow \text { Flavours }
$$

B1
(c) Total number of cups of Rum Raisin and Super

Chunkies ice-cream of different sizes sold on Saturday and Sunday.
(d) $\quad \mathbf{Q}=\frac{1}{2} \mathbf{P C}$

$$
\begin{aligned}
& =\frac{1}{2}\binom{238.8}{314.2} \\
& =\binom{119.4}{157.1} \leftarrow \text { Flavours }
\end{aligned}
$$

(e) The average sum received from the sales of each
(e) The average sum received from the sales of ea
flavour of ice-cream on Saturday and Sunday.
(f) $\quad \mathbf{R}=\left(\begin{array}{ll}1 & 1\end{array}\right)\binom{238.8}{314.2}=\left(\begin{array}{l}553\end{array}\right)$

It represents the total amt. collected from the sales of all flavours and cup sizes of ice-cream on Saturday and B1 Sunday.

B1

3


The points $A, B, C, D$ lie on a circle, centre $O . N$ and $M$ are midpoints of $A B$ and $C D$ respectively. It is given that $O N=O M$.
(a) Show that the triangles $A B E$ and $D C E$ are congruent.
(b) It is given that $A B=6 \mathrm{~cm}$ and $O N=(r-1) \mathrm{cm}$, where $r$ is the radius of the circle. Find the value of $r$.
(a) $\angle A B E=\angle D C E$ (angles in the same segment)
$\angle B A E=\angle C D E$ (angles in the same segment)
$O N=O M$ (given)
$\therefore A B=C D$ (equal chords, equidistant from centre)
$\therefore \triangle A B E \equiv \triangle D C E$ (ASA)
(b) $O B^{2}=O N^{2}+N B^{2}$
$r^{2}=3^{2}+(r-1)^{2}$
$r^{2}=9+r^{2}-2 r+1$
$2 r=10$
$r=5 \mathrm{~cm}$
M2 - All 3 reasons are correctly given
M1 - Any 2 correct reasons are given

A1 - congruency statement

M1

4 In the diagram, $A B C D$ is a square whose diagonals are each 2 cm long. Given that $O$ is the centre of the bigger circle and $B D$ and $A D$ are the diameters of the bigger and smaller circle respectively.
Find the area of the shaded region.


By Pythagoras' theorem
$A D=\sqrt{2} \Rightarrow$ Radius of smallcircle $=\frac{\sqrt{2}}{2}$

Area of semi-circle $=\frac{1}{2} \pi\left(\frac{\sqrt{2}}{2}\right)^{2}=\frac{\pi}{4}$ or $0.78539 \mathrm{~cm}^{2}$
Area of segment $A D=\frac{1}{2}(1)^{2}\left[\frac{\pi}{2}-\sin \frac{\pi}{2}\right]=\frac{\pi}{4}-\frac{1}{2}$ or $0.28539 \mathrm{~cm}^{2}$

Area of shaded region $=\frac{\pi}{4}-0.28539=\frac{1}{2}$ or $0.500 \mathrm{~cm}^{2}(3 \mathrm{sf})$

5 The first four terms in a sequence of numbers, $p_{1}, p_{2}, p_{3}, p_{4}, \ldots$, are given below.
$p_{1}=1^{2}+2^{2}+2^{2}=3^{2}$
$p_{2}=2^{2}+3^{2}+6^{2}=7^{2}$
$p_{3}=3^{2}+4^{2}+12^{2}=13^{2}$
$p_{4}=4^{2}+5^{2}+20^{2}=21^{2}$
(a) Write down an expression for $p_{5}$ and show that $p_{5}=961$.
(b) Given that $p_{10}=10^{2}+11^{2}+s^{2}=k$, express $k$ as a perfect square in terms of $s$.
(c) Given that $p_{w}=w^{2}+(w+1)^{2}+r^{2}=5257^{2}$, find the value of $r$ and of $w$.
(d) Show that $p_{n}=n^{4}+2 n^{3}+3 n^{2}+2 n+1$.
(a) $p_{5}=5^{2}+6^{2}+30^{2}$

$$
\begin{align*}
& =25+36+900 \\
& =961 \tag{B1}
\end{align*}
$$

(b) $k=(s+1)^{2}$
(c) $\quad r=5256$

$$
w(w+1)=5256
$$

$$
=72 \times 73
$$

$$
w=72
$$

(d) $\quad p_{n}=n^{2}+(n+1)^{2}+[n(n+1)]^{2}$
$=n^{2}+n^{2}+2 n+1+\left(n^{2}\right)(n+1)^{2}$
$=2 n^{2}+2 n+1+n^{2}\left(n^{2}+2 n+1\right)$
$=2 n^{2}+2 n+1+n^{4}+2 n^{3}+n^{2}$
$=n^{4}+2 n^{3}+3 n^{2}+2 n+1$

## 6 Answer the whole of this question on a sheet of graph paper.

The variables $x$ and $y$ are connected by the equation $y=x+\frac{7}{x}-6$.
The table below gives some values of $x$ and the corresponding values of $y$.

| $x$ | 0.5 | 1 | 2 | 3.5 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $p$ | 2 | -0.5 | $q$ | 0.4 | 1.17 | 2 | 2.88 |

(a) Find the values of $p$ and $q$.
(b) Using a scale of 2 cm to represent 1 unit on each axis, draw the graph of $y=x+\frac{7}{x}-6$ for the values of $x$ in the range $0<x \leq 8$.
(c) Draw the line $y=-\frac{1}{2} x+3$.
(d) Use your graph to find the $x$-coordinate of a point on the curve $y=x+\frac{7}{x}-6$ at which the gradient of the tangent is equal to -0.5 .
(e) Find the range of values of $x$ for which $x^{2}-6 x+7 \leq 2.5 x$.
(f) By drawing a suitable line on your graph, find the solutions of the equation

$$
2 x^{2}-13 x+14=0
$$

(a) $p=8.5$
$q=-0.5$
A1 - both correct answer
(b)

(d) $x=2.15 \pm 0.2$
(e) $x^{2}-6 x+7 \leq 2.5 x$
$x-6+\frac{7}{x} \leq 2.5$
$y \leq 2.5$
$0.9( \pm 0.1) \leq x \leq 7.6( \pm 0.1)$
A1 - both correct answer
(f) $2 x^{2}-13 x+14=0$

$$
x+\frac{7}{x}-6.5=0
$$

$$
x+\frac{7}{x}-6=0.5
$$

$$
y=0.5
$$

M1

Draw $y=0.5$ on the graph
M1

$$
x=0.6 \pm 0.1
$$

$$
=5.9 \pm 0.1
$$

7 (a) In the diagram, $O A R B$ is a sector of a circle with centre $O$, radius 12 cm and angle $A O B=1.2$ radians. $C$ is the centre of the circle enclosed inside the sector, $O C R$ is a straight line and the circle touches the sector at $P, Q$ and $R$.

(i) Show that the radius of the enclosed circle is 4.3305 cm , correct to 4 decimal places.
(ii) Calculate the perimeter of the shaded region $P O Q$.
(b) In the diagram below, $A B D, A F G, A C E, B F C$ and $D G E$ are straight lines.
$B F C$ is parallel to $D G E$ and $D B A$ is parallel to $G C$. $A B=6 \mathrm{~cm}, B F=3 \mathrm{~cm}, F C=5 \mathrm{~cm}$ and $G C=10 \mathrm{~cm}$.

(i) Prove that triangle $B F A$ is similar to triangle $C F G$.
(ii) Calculate $G E$.
(iii) Given the area of triangle $A B F$ is $6.4 \mathrm{~cm}^{2}$, find the area of trapezium $A C G D$.
(i) Let $r$ be the radius of the enclosed circle.

In $\triangle C O P$,

$$
\begin{aligned}
& \begin{aligned}
\sin 0.6 & =\frac{C P}{O C} \\
& =\frac{r}{12-r} \\
\sin 0.6(12-r) & =r \\
r \sin 0.6+r & =12 \sin 0.6 \\
r & =\frac{12 \sin 0.6}{\sin 0.6+1}
\end{aligned} \\
& \begin{aligned}
\therefore r=4.3305 \mathrm{~cm}(4 \mathrm{dp})(\text { shown })
\end{aligned} \\
& \begin{aligned}
\text { (ii) } \angle P C Q & =2 \pi-\frac{\pi}{2}-\frac{\pi}{2}-1.2=\pi-1.2=1.9415 \mathrm{rad}
\end{aligned} \\
& \qquad \begin{aligned}
O P & =6.3298 \mathrm{~cm}=O Q \\
\text { Perimeter of shaded region } & =2(6.3298)+(4.3305)(1.9415) \\
& =21.0672 \\
& \approx 21.1 \mathrm{~cm}(3 \mathrm{sf})
\end{aligned}
\end{aligned}
$$

(b)(i)
$\angle B F A=\angle C F G$ (vertically opposite angles)
$\angle A B F=\angle G C F$ (alternate angles)
$\therefore \triangle B F A$ is similar to $\triangle C F G$
M1 - both reasons are correct
A1
(b)(ii)

$$
\begin{aligned}
\frac{A B}{A D} & =\frac{B C}{D E} \quad(\triangle A B C \text { is similar to } \triangle A D E) \\
\frac{6}{6+10} & =\frac{8}{8+G E} \\
\frac{6}{16} & =\frac{8}{8+G E} \\
8+G E & =\frac{8 \times 16}{6} \\
G E & =13 \frac{1}{3} \mathrm{~cm}
\end{aligned}
$$

$$
\begin{aligned}
\frac{G E}{B C} & =\frac{B C}{A B} \quad(\triangle A B C \text { is similar to } \triangle C G E) \\
\frac{G E}{10} & =\frac{8}{6} \\
G E & =13 \frac{1}{3} \mathrm{~cm}
\end{aligned}
$$

(b)(iii)

$$
\text { Area of } \begin{aligned}
\triangle A B C & =\frac{8}{3} \times \text { Area of } \triangle A B F \\
& =\frac{8}{3} \times 6.4 \\
& =17 \frac{1}{15} \mathrm{~cm}^{2}
\end{aligned}
$$

Area of trapezium $A C G D=\frac{1}{2}(C G+A D) \times$ perpendicular distance from $C G$ to $A D$
Area of triangle $A B C=\frac{1}{2}(A B) \times$ perpendicular distance from $C G$ to $A D$
$\frac{\text { Area of trapezium } A C G D}{\text { Area of triangle } A B C}=\frac{C G+A D}{A B}$
Area of trapezium $A C G D=\frac{16+10}{6} \times 17 \frac{1}{15}$

$$
=74.0 \mathrm{~cm}^{2}(3 \mathrm{sf})
$$

8 Amanda ran the 21 km of a half-marathon race at an average speed of $x \mathrm{~km} / \mathrm{h}$.
(a) Write down, in terms of $x$, an expression for the number of hours it took her to complete the race.
(b) Deborah ran the same race at an average speed which is $3 \mathrm{~km} / \mathrm{h}$ faster than Amanda's speed. Write down, in terms of $x$, an expression for the number of hours which Deborah took.
(c) Given that the difference between the two times was 20 minutes, write down an equation in $x$ and show that it reduces to $x^{2}+3 x-189=0$.
(d) Solve the equation $x^{2}+3 x-189=0$, giving your answers correct to 2 decimal places.
(e) Find, in hours and minutes, the time it took Amanda to complete the race.
(a) $T_{1}=\frac{21}{x} \mathrm{hr}$

A1
A1
(d) $x^{2}+3 x-189=0$

$$
\begin{array}{ll}
x=\frac{-3 \pm \sqrt{3^{2}-4(1)(-189)}}{2} & \text { M1 }  \tag{M1}\\
x=12.33(2 \mathrm{dp}) \quad \text { or } \quad x=-15.33(2 \mathrm{dp}) & \text { A1 }
\end{array}
$$

(e) $\quad x=-15.33$ (rejected)

$$
\begin{aligned}
\text { Time taken } & =\frac{21}{12.33} \\
& =1 \mathrm{hr} 42 \mathrm{mins}
\end{aligned}
$$A1


[The volume of tetrahedron $=\frac{1}{3} \times$ base area $\times$ height ]
The diagram shows a tetrahedron, $V A B C$, which has a horizontal equilateral triangular base $A B C$ of side 20 cm .
The slant edge of the tetrahedron $(V A, V B$ and $V C)$ are each of length 27 cm . $M$ is the mid-point of $A B$ and the vertical line $V N$ meets the plane $A B C$ at $N$ where $M N: N C=1: 2$. Calculate
(a) $C M$,
(b) the angle of elevation of $V$ from $A$,
(c) the volume of the tetrahedron.
(a) $C M^{2}=C B^{2}-M B^{2} \quad$ (Pythagoras Theorem)

$$
\begin{aligned}
& =20^{2}-10^{2} \\
& =300 \\
C M & =17.3 \mathrm{~cm}(3 \mathrm{sf})
\end{aligned}
$$

(b)

$$
\begin{aligned}
A N^{2} & =A M^{2}+M N^{2} \quad(\text { Pythagoras } \\
& =10^{2}+\left[\frac{1}{3}(17.320)\right]^{2} \\
& =11.546 \mathrm{~cm} \\
\cos \angle V A N & =\frac{11.546}{27} \\
\angle V A N & =64.7^{\circ}(1 \mathrm{dp}) \\
V N & =V A \sin \angle V A N \\
& =27 \sin 64.68 \\
& =24.4 \mathrm{~cm}
\end{aligned}
$$M1

(c)

Volume of tetrahedron $=\frac{1}{3} \times$ Area of base $\times V N$

$$
\begin{aligned}
& =\frac{1}{3} \times\left(\frac{1}{2} \times 20 \times 20 \sin 60^{\circ}\right) \times 24.406 \\
& =1410 \mathrm{~cm}^{3}
\end{aligned}
$$

10 In the figure below, the $x$-intercept and $y$-intercept of the line $A B$ are -12 and -6 respectively. Both the $x$-intercept and $y$-intercept of the line $C D$ are -10 .


Find
(a) the equation of the line $A B$ and $C D$,
(b) the coordinates of $E$,
(c) the area of $O C E B$,
(d) the coordinates of $F$ given that point $F$ lies on $A B$ produced such that $A F$ : $F B=5: 3$,
(e) find the coordinates of point $G$ where $G$ is the point on the $x$-axis such that $O E$ is parallel to $G D$.
(a) Gradient of the line $A B=\frac{-6-0}{0-(-12)}=-\frac{1}{2}$

Equation of the line $A B: y=-\frac{1}{2} x-6$
Gradient of the line $C D=\frac{-10-0}{0-(-10)}=-1$
Equation of the line CD: $y=-x-10$
(b) $y=-\frac{1}{2} x-6$
$y=-x-10$
(1) $-(2)$
$x=-8$
$y=-2$

Coordinates of $E=(-8,-2)$
(c) Area of $O C E B=$ Area of $\triangle C D O-$ Area of $\triangle E B D$

$$
\begin{aligned}
& =\frac{1}{2} \times 10 \times 10-\frac{1}{2} \times 4 \times 8 \\
& =34 \text { units }^{2}
\end{aligned}
$$

(d) $\frac{A F}{F B}=\frac{5}{3}$

Using similar triangles,

$$
\begin{aligned}
\frac{-12-x}{0-x} & =\frac{5}{3} \\
x & =18 \\
\frac{0-y}{-6-y} & =\frac{5}{3} \\
y & =-15
\end{aligned}
$$

Coordinates of $F=(18,-15)$
(e) Let the coordinates of $G$ be $(k, 0)$
$\frac{0-(-2)}{0-(-8)}=\frac{0-(-10)}{k-0}$
$\frac{1}{4}=\frac{10}{k}$
$k=40$
Coordinates of $G=(40,0)$

## OR

$$
\overrightarrow{O E}=m \overrightarrow{G D}
$$

$$
\binom{-8}{-2}=m\binom{-k}{-10}
$$

$$
-2=-10 m
$$

$$
m=\frac{1}{5}
$$

$$
-8=-m k
$$

$$
k=40
$$

Coordinates of $G=(40,0)$

11 The cumulative frequency curve below shows the weights of a sample of 160 boys from a school when they enter Secondary One.

(a) Use the graph to find an estimate for
(i) the number of boys whose weight is more than 60 kg ,
(ii) the median weight,
(iii) the percentage of boys whose weight is less than or equal to 52 kg .
(b) The lightest $25 \%$ of the boys are classified as "slim".

The heaviest $25 \%$ of the boys are classified as "big-sized".
The remaining boys are classified as "medium".
Find an estimate for
(i) the greatest possible difference in weight between 2 boys classified as "bigsized",
(ii) the least possible difference between the weight of a boy classified as "slim" and the weight of a boy classified as "big-sized".
(c) The following box-and-whisker plot shows the distribution of weights of 160

Secondary One girls from the same school. Compare the data provided and record 2 observations about the distribution of weights between boys and girls.


11 (a) (i) Number of boys whose weight is more than $60 \mathrm{~kg}=40$.
(ii) median weight $=56 \mathrm{~kg}$.
(iii) Percentage of boys whose weight $\leq 52 \mathrm{~kg}=\frac{44}{160} \times 100 \%$

$$
\begin{equation*}
=27.5 \% \tag{B1}
\end{equation*}
$$

(b) (i) Based on distribution of number of boys

Greatest possible difference $=75-60$

$$
\begin{equation*}
=15 \mathrm{~kg} \tag{A1}
\end{equation*}
$$

(ii) Based on distribution of weight

Least possible difference $=60-51.5$

$$
=8.5 \mathrm{~kg}
$$

A1
(c) Median weight of girls lower than boys; girls are lighter ;

For boys, $\mathrm{Q}_{3}-\mathrm{Q}_{1}=8.5 \mathrm{~kg}$. For girls, $\mathrm{Q}_{3}-\mathrm{Q}_{1}=12 \mathrm{~kg}$; Girls' weight has more variation.

12 Mrs Tan, a mother of two children, saw a newspaper article as shown.

## Price of formula milk in Singapore has soared

The average price of a 900 g tin of formula milk has increased sharply over the last 5 years, outstripping the price increases of other dairy products and household staples.

On Monday, the Government announced it is tightening rules to encourage greater price competition.

The table below shows the price (in SGD) per 100 grams of different brands of formula milk in year 2012 and 2017.

| Date Brand | Similac | Friso | Nan | S26 | Mamil |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Dec 2012 | 5.71 | 5.22 | 5.20 | 5.13 | 4.96 |
| Mar 2017 | 7.05 | 6.56 | 7.45 | 6.36 | 6.41 |
| \% increase |  |  | 43.3 | 24.0 | 29.2 |

Coffee stain was found on the newspaper article covering some of the information.
(a) (i) Calculate the percentage increase in the price of the Similac and Friso formula milk covered by the coffee stain..
(ii) Hence, do you agree with the headline of the newspaper article? Support your answer with a reason.

A few days later, Mrs Tan saw another article regarding the price of similar brands of formula milk sold in Singapore, Malaysia and China.

| Country Brand | Similac | Friso | Nan | S26 | Mamil |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Singapore | 7.05 | 6.56 | 7.45 | 6.36 | 6.41 |
| Malaysia | 3.92 | 3.54 | 4.29 | 4.13 | 3.51 |
| China | 4.79 | 5.58 | 9.06 | 4.25 | 4.00 |

Price (in SGD) per 100 grams of formula milk in Singapore, Malaysia and China.
Upon seeing the article, Mrs Tan intends to purchase some cans of formula milk in China and ship them back during her holidays.

Mrs Tan did an online research and found the following shipping rate from China to Singapore by SHIPPER Company.

| Weight of parcel, $\boldsymbol{x}(\mathbf{k g})$ | Shipping Rate <br> $1^{\text {st }} \mathrm{kg} 150 \mathrm{RMB}$ follow by |
| :---: | :---: |
| $x \leq 10$ | $75 \mathrm{RMB} / \mathrm{kg}$ |
| $10<x \leq 20$ | $35 \mathrm{RMB} / \mathrm{kg}$ |
| $20<x \leq 50$ | $31 \mathrm{RMB} / \mathrm{kg}$ |
| $50<x \leq 75$ | $27 \mathrm{RMB} / \mathrm{kg}$ |
| $75<x \leq 100$ | $25 \mathrm{RMB} / \mathrm{kg}$ |
| $100<x \leq 200$ | $24 \mathrm{RMB} / \mathrm{kg}$ |
| $x>200$ | $22 \mathrm{RMB} / \mathrm{kg}$ |

The information below shows a can of the 900 g formula milk that Mrs Tan intends to purchase in China and the online currency conversion.


Mrs Tan intends to spend at most $\mathrm{S} \$ 650$ for both the formula milk and shipping fee.
(b) Calculate the maximum number of cans of formula milk that Mrs Tan can buy.

12 (a) (i) Percentage increase of Similac $=\frac{7.05-5.71}{5.71} \times 100 \%$

$$
\begin{equation*}
=23.5 \% \tag{B1}
\end{equation*}
$$

Percentage increase of the Friso $=\frac{6.56-5.22}{5.22} \times 100 \%$

$$
\begin{equation*}
=25.7 \% \tag{B1}
\end{equation*}
$$

(ii) Mean of percentage increase $=\frac{23.5+25.7+43.3+24.0+29.2}{5}$

$$
=29.14 \%
$$

Agree.
B1
as the mean of percentage increase is greater than $29 \%$ which is much higher than price increase of general food consumption items. (accept any logical answer)
(b) $\quad \mathrm{S} \$ 650=4.89 \times 650$

Let $x$ be the number of can of milk powder.


## Or working in term of Singapore Dollar

Shipping rate of
$150 \mathrm{RMB}=\mathrm{S} \$ 30.675$
$35 \mathrm{RMB}=\mathrm{S} \$ 7.157$

Let $x$ be the number of can of milk powder.


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44.6913x}\leq619.32
x=13 tins of formula milk

\section*{END OF PAPER}```

