KENT RIDGE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2024

ADDITIONAL MATHEMATICS **4049/01**
PAPER 1

SECONDARY 4 EXPRESS/ 5 NORMAL ACADEMIC

Thursday 22 August 2024 **2 hour 15 minutes**

Name: ______________________________ ( ) **Class: Sec** __________

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.

Do not open this question paper until you are told to do so.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue, correction fluid or correction tape.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Total	/ 90

This Question Paper consists of 20 printed pages, including this page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **Do not use a calculator in this question.**

(a) Use the identity for $\tan 2A$ to find the value of $\tan A$ in the form $a + b\sqrt{2}$, where a and b are rational when $\tan 2A = 1$. It is given that A is acute. [4]

(b) Hence find the exact value of $\sec^2 A$. [2]

2 Two perpendicular lines L_1 and L_2 meet at $A\ (2, -3)$.
L_1 has equation $2x + y = 1$ and L_2 has equation $y = mx + c$.
L_2 meets the y – axis at point B.

(a) Find the coordinates of B. [3]

The distance from A to a point $C(4, k)$ is 5.

(b) Find the exact value(s) of k. [3]

3 It is given that $f(x) = 2\cos\left(\frac{x}{2}\right)$ and $g(x) = 3\sin x + 1$.

(a) State the minimum and maximum values of $f(x)$. [1]

(b) State the minimum and maximum values of $g(x)$. [1]

(c) Sketch, on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0° \leq x \leq 360°$. [4]

(d) State the number of solutions to $3\sin x + 1 = 2\cos\left(\frac{x}{2}\right)$ for $0° \leq x \leq 360°$. [1]

4 (a) Find, in simplest form, the first 3 terms in the expansion, in ascending powers of x of $(a - x^2)^6$, where a is a non-zero constant. [3]

(b) The first 2 non-zero terms in the expansion of $\left(\frac{1}{x} + x\right)^2 (a - x^2)^6$ in ascending powers of x are $\frac{a^6}{x^2} + bx^2$. There is no term independent of x. Find the value of constant a and constant b. [5]

5 (a) Explain why the function $f(x)=\frac{x+1}{x-3}, x \neq 3$ is a decreasing function. [4]

(b) Express $\mathrm{f}(x)$ in the form $a + \frac{b}{x-3}$. Hence find $\int \frac{x+1}{x-3}\, dx$. [4]

6 Roast chicken and fish baked in foil are taken out from different ovens.
The temperature, T_c °C, of the chicken t minutes after it is removed from the oven is modelled by the formula $T_c = 75e^{-0.02t}$.

(a) State the initial temperature of the chicken. [1]

(b) Find the time taken for the temperature of the chicken to drop to 65 °C. [3]

The temperature, T_f °C, of the fish kept wrapped in foil t minutes after it is removed from the oven is modelled by the formula $T_f = 63e^{kt}$.

(c) After $\frac{1}{4}$ of an hour, the temperature of the fish is 54.2 °C. Find k, correct to 2 decimal places. [3]

(d) Using the value of k corrected to 2 decimal places in part (c), find the time when the fish has the same temperature as the chicken. [3]

7 (a) Differentiate $x \ln x$ with respect to x. [2]

(b) Given that $f'(x) = \frac{1}{x} - 1$ and $f(x)$ has a maximum point at $(1,0)$, find $f(x)$. [3]

8 (a) Show, with algebraic reasoning, that the curves $y = 2x^2 - 8x$ and $y = -x^2 - 4x - 3$ do not intersect. [3]

(b) By expressing each of $2x^2 - 8x$ and $-x^2 - 4x - 3$ in the form $a(x+b)^2 + c$, where a, b and c are constants, use a sketch to show that the two curves $y = 2x^2 - 8x$ and $y = -x^2 - 4x - 3$ do not intersect. [5]

9

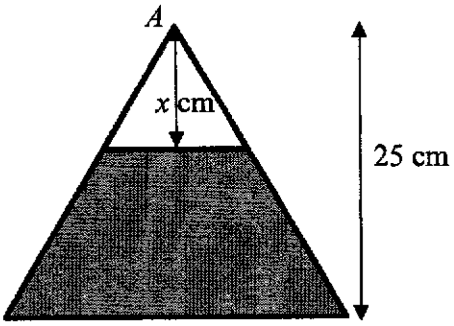


Figure 9A

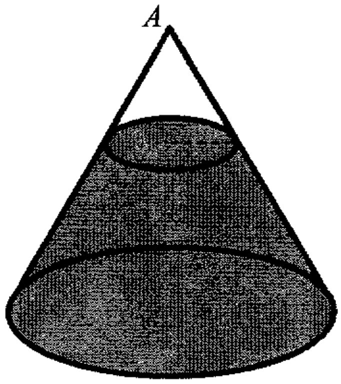


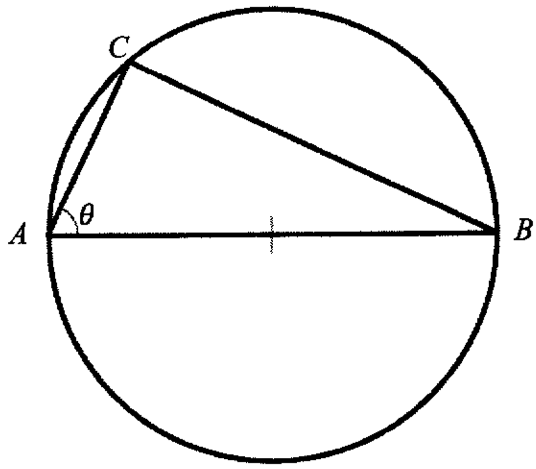
Figure 9B

An empty cone as shown in Figure 9B has base radius 5 cm and height 25 cm. Liquid is poured into it such that in t seconds the top surface of the liquid is x cm below the apex of the cone, as shown labelled in Figure 9A, the vertical section of the cone from its apex A.

(a) Show that the volume of the liquid, $V = \frac{1}{3}\pi(625 - \frac{x^3}{25})$. [2]

(b) Given that x changes with t such that $\frac{dx}{dt} = -\frac{1}{2}t$ cm/s, find the rate of change of the volume of the liquid when $x = 2$ cm. [6]

10



Right-angled triangle ABC is inscribed in a circle with diameter $AB = 8$ cm.

(a) Express the area of triangle ABC in terms of $\sin 2\theta$. [3]

(b) Find the maximum area of triangle ABC. [1]

(c) Show that the perimeter of the triangle $ABC = 8(1 + \sin\theta + \cos\theta)$. [1]

(d) By expressing $\sin\theta + \cos\theta = R\sin(\theta + \alpha)$, where R and α are constants to be found, find the maximum perimeter of triangle ABC. [3]

11 $f(x) = \frac{1}{3}x^3 + ax^2 - 20x - 4 + b$.

It is given that $f(x)$ is divisible by x and leaves a remainder of – 51 when divided by $x - 3$.

(a) Find the value of a and of b. [4]

(b) Find the exact values of the x coordinates of the turning points of the curve $y = f(x)$ and determine the nature of each turning point. [5]

12 (a) Given that $\frac{\sin(A-B)}{\sin(A+B)} = \frac{2}{3}$, prove that $\tan A = 5\tan B$. [3]

(b) Hence or otherwise, solve $3\sin(45° - \theta) = 2\sin(45° + \theta)$ for $0° \leq \theta \leq 360°$. [4]

End of Paper

Kent Ridge Secondary School
Secondary 4 Express/5 Normal Academic Preliminary Examination 2024
Add Math Prelim 2024 P1 Mark scheme

Qn	Solutions	Marks
1a	$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$ $1 = \frac{2\tan A}{1-\tan^2 A}$ $1-\tan^2 A = 2\tan A$ $\tan^2 A + 2\tan A - 1 = 0$ $\tan A = \frac{-2 \pm \sqrt{4+4}}{2}$ $= -1+\sqrt{2}$	M1 A1 M1 A1
1b	$\sec^2 A = 1+\tan^2 A$ $= 1+\left(-1+\sqrt{2}\right)^2$ $= 1+1-2\sqrt{2}+2$ $= 4-2\sqrt{2}$	M1 A1
2a	Gradient $L_1 = -2$ Gradient $L_2 = \frac{1}{2}$ $-3 = \frac{1}{2}(2) + c$ $c = -4$ $B(0,-4)$	M1 M1 A1
2b	$(4-2)^2 + (k+3)^2 = 25$ $(k+3)^2 = 21$ $k = \pm\sqrt{21} - 3$	M1 M1 or apply quad formula to general eqn A1
3a	Min -2 and max 2	B1
3b	Min -3 + 1 = -2, max = 3+1 = 4	B1
3c	f(x) period or shape correct g(x) period or shape correct f(x) fully correct g(x) fully correct	B1 B1 B1 B1
3d	3	B1
4a	$(a-x^2)^6 = a^6 - 6a^5x^2 + \binom{6}{2}a^4x^4 + \ldots$ $= a^6 - 6a^5x^2 + 15a^4x^4 + \ldots$	B1,B1,B1
4b	$\left(\frac{1}{x^2}+2+x^2\right)(a-x^2)^6$ $\left(\frac{1}{x^2}+2+x^2\right)(a^6 - 6a^5x^2 + 15a^4x^4)$ Term independent of x: $-6a^5 + 2a^6 = 0$ $-6+2a = 0$ $a = 3$ Term in x^2: $b = 15a^4 - 12a^5 + a^6 = -972$	M1 M1 A1 M1,A1
5a	$f'(x) = \frac{(x-3)-(x+1)}{(x-3)^2}$	M1

Qn	Solutions	Marks
	$$f'(x) = \frac{-4}{(x-3)^2}$$ Since $(x-3)^2 > 0, x \neq 3$ **f '(x) is always negative or** **Gradient of f is always negative**, so it is a decreasing function	A1 B1 B1
5b	$$f(x) = 1 + \frac{4}{x-3}$$ $$\int 1 + \frac{4}{x-3}\, dx = x + 4\ln(x-3) + c$$	M1 M1, M1, A1
6a	75°C	B1
6b	$75e^{-0.02t} = 65$ $e^{-0.02t} = \frac{65}{75}$ $-0.02t = \ln\left(\frac{65}{75}\right)$ $t = \frac{\ln\left(\frac{65}{75}\right)}{-0.02} = 7.16$ min	 M1 M1 A1
6c	$63e^{15k} = 54.2$ $k = \frac{\ln\left(\frac{54.2}{63}\right)}{15} = -0.01003 = -0.01$	M1 M1, A1
6d	$$75e^{-0.02t} = 63e^{-0.01t}$$ $$\frac{e^{-0.02t}}{e^{-0.01t}} = \frac{63}{75}$$ $$e^{-0.01t} = \frac{63}{75}$$ $t = \frac{\ln\left(\frac{63}{75}\right)}{-0.01} = 17.4$ minutes	M1 M1 A1
7a	$$x\left(\frac{1}{x}\right) + \ln x$$ $$= 1 + \ln x$$	M1 – diff ln x correctly A1
7b	$$f(x) = \ln x - x + c$$ $$0 = \ln 1 - 1 + c$$ $$c = 1$$ $$f(x) = \ln x - x + 1$$	M1 M1 A1
8a	$2x^2 - 8x = -x^2 - 4x - 3$ $3x^2 - 4x + 3 = 0$ Discriminant = $(-4)^2 - 4(3)(3) = -20$ Since **discriminant < 0, there are no real roots** to the simultaneous equations. The 2 curves do not intersect	M1 M1 A1
8b	$$2(x^2 - 4x) = 2[(x-2)^2 - 4]$$ $$= 2(x-2)^2 - 8$$ $$-(x^2 + 4x + 3) = -[(x+2)^2 - 1]$$ $$= -(x+2)^2 + 1$$ Sketch min curve with TP (2,-8) passing through O Sketch max curve with TP (-2,1) passing through (0,-3) Non intersecting	M1 M1 A1 B1 B1

Qn	Solutions	Marks
9a	$\frac{r}{x}=\frac{5}{25}$ $r=\frac{x}{5}$ Volume of liquid $=\frac{1}{3}\pi(5)^2(25)-\frac{1}{3}\pi\left(\frac{x}{5}\right)^2(x)$ $=\frac{1}{3}\pi(625-\frac{x^3}{25})$	M1 B1
9b	$\frac{dV}{dx}=\frac{1}{3}\pi(-\frac{3x^2}{25})$ $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$ $=\frac{1}{3}\pi(-\frac{3x^2}{25})\left(-\frac{1}{2}t\right)$ $x=\int-\frac{1}{2}t\,dt$ $x=-\frac{1}{2}\left(\frac{t^2}{2}\right)+c$ Sub t = 0, x = 25 $c=25$ $x=-\frac{t^2}{4}+25$ Sub x = 2 $-\frac{t^2}{4}+25=2$ $t^2=92$ $t=\sqrt{92}$ $\frac{dV}{dt}=\frac{1}{3}\pi\left(-\frac{3(2)^2}{25}\right)\left(-\frac{1}{2}\sqrt{92}\right)=2.41\ cm^3/s$	M1 M1 M1 A1 A1 A1
10a	$BC=8\sin\theta$ $AC=8\cos\theta$ Area $=\frac{1}{2}(8\sin\theta)(8\cos\theta)$ $=32\sin\theta\cos\theta$ $=16\sin 2\theta$	M1 – either BC or AC found M1 A1
10b	Max = 16	B1
10c	Perimeter = AB + BC + CA = $8+8\sin\theta+8\cos\theta$ $=8(1+\sin\theta+\cos\theta)$	B1
10d	$\sin\theta+\cos\theta=\sqrt{2}\sin(\theta+45°)$ Max perimeter = $8(1+\sqrt{2})$	B1, B1 B1
11(a)	$f(0)=-4+b=0$ $b=4$ $f(3)=\frac{1}{3}(3)^3+a(3)^2-20(3)-4+4=-51$ $a=0$	M1 A1 M1 A1
11(b)	$f(x)=\frac{1}{3}x^3-20x$ $f'(x)=x^2-20=0$ $x=\pm\sqrt{20}=\pm2\sqrt{5}$	 M1 A1

Qn	Solutions	Marks
	$f''(x) = 2x$ When $x = 2\sqrt{5}$, $f''(x) > 0$, it is a minimum point When $x = -2\sqrt{5}$, $f''(x) < 0$, it is a maximum point	M1 A1 A1
12(a)	$\frac{\sin A \cos B - \cos A \sin B}{\sin A \cos B + \cos A \sin B} = \frac{2}{3}$ $3 \sin A \cos B - 3 \cos A \sin B$ $= 2 \sin A \cos B + 2 \cos A \sin B$ $\sin A \cos B = 5 \cos A \sin B$ $\frac{\sin A \cos B}{\cos A \cos B} = \frac{5 \cos A \sin B}{\cos A \cos B}$ $\tan A = 5 \tan B$	M1 M1 B1
12(b)	$\frac{\sin(45° - \theta)}{\sin(45° + \theta)} = \frac{2}{3}$ $A = 45°, B = \theta$ $\tan 45° = 5 \tan \theta$ $\tan \theta = \frac{1}{5}$ $\alpha = \tan^{-1}\left(\frac{1}{5}\right) = 11.30°$ $\theta = 11.30, 180 + 11.30 = 11.3°, 191.3°$	 M1 M1 A1 A1