SWISS COTTAGE SECONDARY SCHOOL
SECONDARY FOUR AND FIVE
PRELIMINARY EXAMINATION

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Name: ______________________________ (   )   Class: ________

ADDITIONAL MATHEMATICS

4049/01

Paper 1

Monday 9 September 2024
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

Questions	1	2	3	4
Marks				

This document consists of **18** printed pages and **2** blank pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + ... + \binom{n}{r} a^{n-r} b^r + ... + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)........(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

Section A (17 marks)

1 The equation of a curve is $y = 3x^3 + ax^2 + b$, where a and b are constants. If $a > 0$, find, in terms of a and/or b, the range of values of x for which y is increasing. [3]

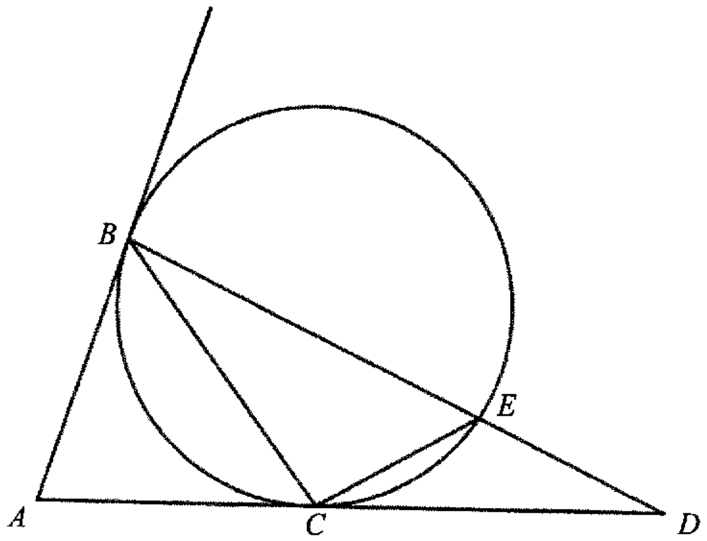
2 The area of a rectangle is $\left(7 + b\sqrt{2}\right)$ cm^2. Given that the length of the rectangle is $\left(a + 4\sqrt{2}\right)$ cm and the breadth of the rectangle is $\left(5 - \sqrt{2}\right)$ cm, find the value of a and of b. [4]

3 The equation of a curve is $y = 2x^2 + 12x + 11$.

(a) Express $2x^2 + 12x + 11$ in the form $a(x+b)^2 + c$ where a, b and c are constants. [2]

(b) Find the range of values of p for which the line $y = px + 11$ intersects the curve at two distinct points. [3]

4



The diagram shows a triangle BCE whose vertices lie on the circumference of a circle. AD is a tangent to the circle at point C and AB is a tangent to the circle at point B. BED is a straight line.

(a) Prove that angle $ABC +$ angle $CED = 180^\circ$. [3]

(b) Show that there **does not** exist a circle that passes through points A, B, E and C. [2]

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Name: ______________________________ (   )

Class: __________

Questions	5	6	7	8	9	10	11	12	13
Marks									

Section B (73 marks)

5 **(a)** Solve the equation $\log_5 x + 2 = 3\log_x 5$. [5]

(b) Sketch the graph $y = \log_{0.5} x$. [2]

6 It is given that $f(x)=3\sin\left(\frac{x}{2}\right)+4$.

(a) State the least and greatest value of $f(x)$. [2]

(b) State the period of $f(x)$. [1]

(c) Sketch the graph of $y=f(x)$ for $0\le x\le 4\pi$. [2]

(d) By drawing a suitable straight line on the same set of axes as the graph of $y=f(x)$, state the number of solutions of the equation $\sin\left(\frac{x}{2}\right)=-\frac{x}{4\pi}$ for $0\le x\le 4\pi$. [2]

7 A curve is such that $\frac{d^2y}{dx^2} = 3e^{-2x} + \cos 2x$. The curve passes through the point A (0, 3) and has a gradient of 5 at A. Find the equation of the curve. [7]

8 **(a)** The expansion of $\left(3x-\frac{2}{x^2}\right)^n$ has a term independent of x. By considering the general term in the expansion, explain why n is a multiple of 3. [3]

(b) It is given that $n = 9$.

Find the value of $\dfrac{\text{coefficient of term independent of } x}{\text{coefficient of } \frac{1}{x^6}}$. [4]

9 (a) Express $\frac{9x^2-4x+8}{(x-2)(x+1)^2}$ in partial fractions. [5]

(b) Hence, find $\int \frac{9x^2-4x+8}{(x-2)(x+1)^2}\,\mathrm{d}x$. [3]

10 A particle moves in a straight line such that its displacement, s cm, from a fixed point O is modelled by $s = -3t + e^{\frac{t}{2}}$, where t is the time in seconds since the start of motion.

(a) Show that the particle reaches instantaneous rest at $t = 2\ln 6$. [3]

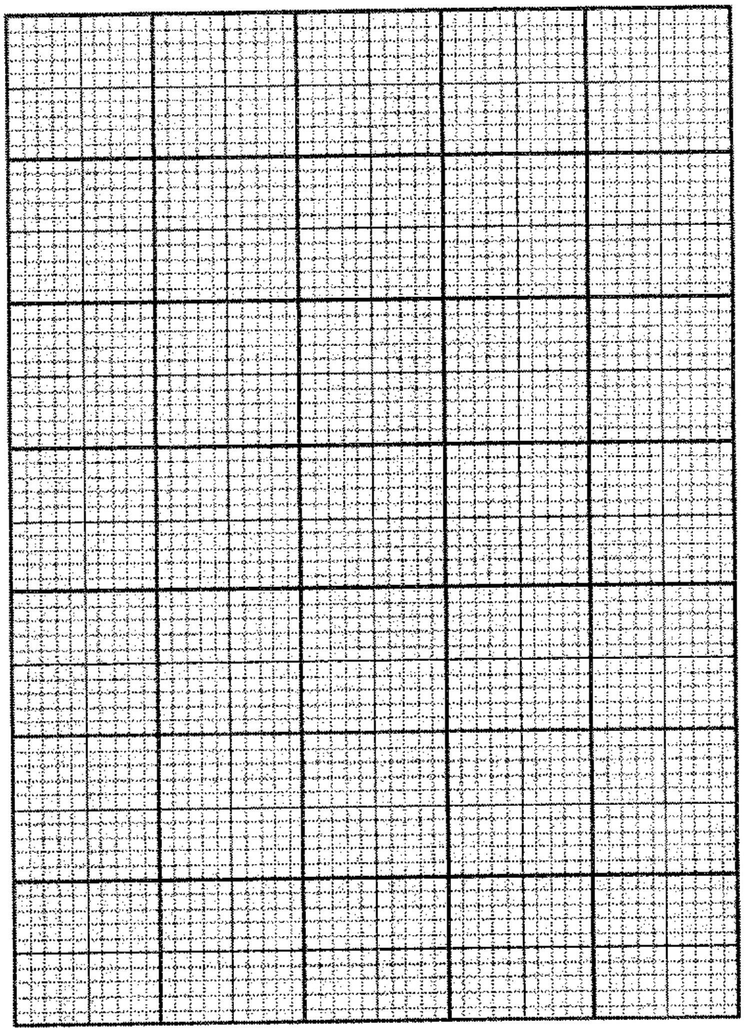
(b) Explain why the particle passes through O during the first second. [2]

(c) Find the total distance travelled by the particle in the interval $t=0$ to $t=4$. [3]

11 The table shows, to 3 significant figures, the value, $\$C$, in thousands, of a car t years from 1st January 2024.

t	1	2	3	4
C	68.2	58.6	50.7	44.3

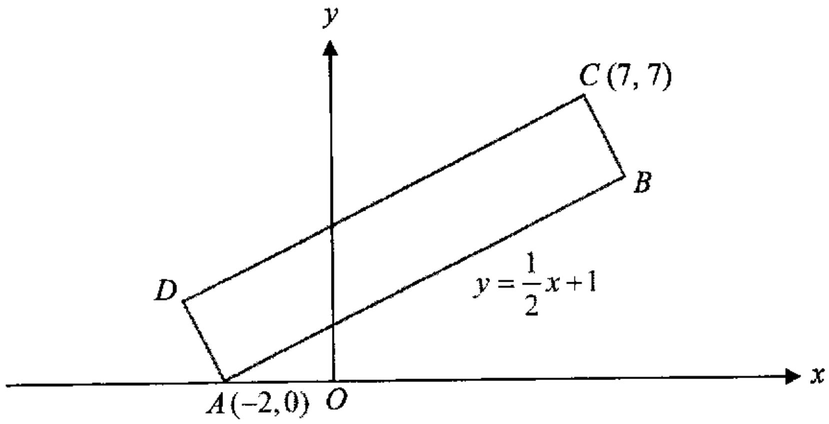
(a) On the grid below plot $\ln(C-15)$ against t and draw a straight line graph. The vertical $\ln(C-15)$-axis should start at 3.0 and have a scale of 2 cm to 0.2 units. The horizontal t-axis should have a scale of 2 cm to 1 unit. [3]



(b) Use your graph to find the gradient of your straight line and hence express C in the form $C = Ae^{-kt} + 15$, where A and k are constants. [4]

(c) Assuming that the model is still appropriate, find the year for which the value of the car is first below \$35 000. [3]

12



The diagram shows a parallelogram with vertices $A(-2,0)$, B, $C(7,7)$ and D. The side AB has equation $y=\frac{1}{2}x+1$ and the length of $AB=5\sqrt{5}$ units.

(a) Find the coordinates of B. [4]

(b) Prove that *ABCD* is a rectangle. [3]

(c) Calculate the area of *ABCD*. [2]

13 The equation of a curve is $y = \dfrac{6}{(2x-5)^3}$.

(a) Show that for $x > 2.5$, the curve has no stationary points. [3]

(b) The normal to the curve at $x = 1$ intersects another curve $36y = x^2 + 90x - 78$ at points A and B. Express the difference of the x-coordinates of A and B in the form $\sqrt{k}$, where k is an integer to be found. [7]

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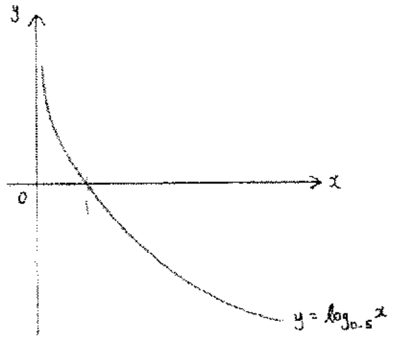
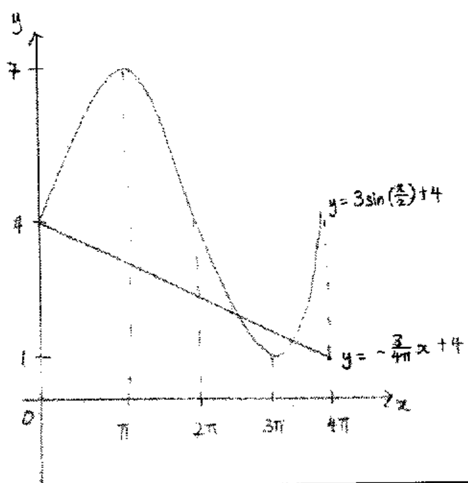
Preliminary Examinations 2024 Paper 1 Marking Scheme Level: Sec 4E/5N

Additional Mathematics (90 marks)

Qn. #	Solution	Mark Allocation
1	$\frac{dy}{dx}=9x^2+2ax$ $9x^2+2ax>0$ $x(9x+2a)>0$ $x<-\frac{2}{9}a$ or $x>0$	**M1** (Find $\frac{dy}{dx}$) **M1** ($\frac{dy}{dx}>0$) **A1**
2	$(5-\sqrt{2})(a+4\sqrt{2})=7+b\sqrt{2}$ $5a+20\sqrt{2}-a\sqrt{2}-8=7+b\sqrt{2}$ $(5a-8)+(20-a)\sqrt{2}=7+b\sqrt{2}$ $5a-8=7$ and $20-a=b$ $a=3, b=17$	**M1** (expansion) **M1** (compare coefficient) **A2**
3(a)	$2x^2+12x+11=2(x^2+6x)+11$ $=2\left[(x+3)^2-3^2\right]+11$ $=2(x+3)^2-7$	**B1** (either $(x+3)^2$ or -7 correct) **B2** (all correct)
3(b)	$2x^2+12x+11=px+11$ $2x^2+(12-p)x=0$ $(12-p)^2-4(2)(0)>0$ $(12-p)^2>0$ $p\neq 12$	**M1** (sim eqn) **M1** (Find discriminant) **A1**
4(a)	Let $\angle ABC=x$. $AB=AC$ (tangents from external point) $\angle ACB=\angle ABC=x$ (base angles of isosceles triangle) $\angle CEB=\angle ACB=x$ (alternate segment theorem) $\angle CED=180^\circ-\angle ACB$ (adj. angles on straight line) $=180^\circ-x$ $\angle ABC+\angle CED=x+(180^\circ-x)$ $=180^\circ$	**M1** ($\angle ACB=\angle ABC$) **M1** ($\angle CEB=\angle ACB$) Note: If first M1 not awarded, maximum 2 out of 3 marks **A1**
4(b)	Suppose there exists a circle that passes through *A*, *B*, *E* and *C*. $\angle BAC=180^\circ-\angle ABC-\angle ACB$ (sum of angles of triangle) $=180^\circ-2x$ $\angle BAC=180^\circ-\angle CEB$ (opp angles of cyclic quad) $=180^\circ-x$	**M1** (opp angles of cyclic quad)

Qn. #	Solution	Mark Allocation
	For $x \neq 0$, $180^\circ - 2x \neq 180^\circ - x$ Hence, there is no circle that passes through A, B, E and C.	**A1** (contradiction)
5(a)	$\log_5 x + 2 = 3\log_x 5$ $\log_5 x + 2 = \dfrac{3}{\log_5 x}$ Let $u = \log_5 x$ $u + 2 = \dfrac{3}{u}$ $u^2 + 2u - 3 = 0$ $(u-1)(u+3) = 0$ $u = 1$ or $u = -3$ $\log_5 x = 1$ or $\log_5 x = -3$ $x = 5$ or $x = 5^{-3}$ $x = \dfrac{1}{125}$	**M1** (change of base) **M1** (form quad eqn) **M1** (solve quad eqn) **A2**
5(b)		**B1** (shape) **B1** (x-int and y-axis asymptote)
6(a)	Least value $= 1$ Greatest value $= 7$	**B1** **B1**
6(b)	Period $= 4\pi$ or 720°	**B1**
6(c)		**B1** (shape + correct number of cycles) **B1** (coordinates of start/end point + max/min points)

Qn. #	Solution	Mark Allocation
6(d)	$\sin\left(\frac{x}{2}\right)=-\frac{x}{4\pi}$ $3\sin\left(\frac{x}{2}\right)=-\frac{3}{4\pi}x$ $3\sin\left(\frac{x}{2}\right)+4=-\frac{3}{4\pi}x+4$ $y=-\frac{3}{4\pi}x+4$ After drawing line: Number of solutions $=3$	**M1** (find eqn of line) **A1** (draw line + number of solutions)

7	$\frac{dy}{dx}=\int 3e^{-2x}+\cos 2x\ dx$
	$=-\frac{3}{2}e^{-2x}+\frac{1}{2}\sin 2x+c$
	Sub $x=0$, $\frac{dy}{dx}=5$
	$5=-\frac{3}{2}+c$
	$c=\frac{13}{2}$
	$\frac{dy}{dx}=-\frac{3}{2}e^{-2x}+\frac{1}{2}\sin 2x+\frac{13}{2}$
	$y=\int -\frac{3}{2}e^{-2x}+\frac{1}{2}\sin 2x+\frac{13}{2}\ dx$
	$=\frac{3}{4}e^{-2x}-\frac{1}{4}\cos 2x+\frac{13}{2}x+c_1$
	Sub (0, 3)
	$3=\frac{3}{4}-\frac{1}{4}+c_1$
	$c_1=\frac{5}{2}$
	$y=\frac{3}{4}e^{-2x}-\frac{1}{4}\cos 2x+\frac{13}{2}x+\frac{5}{2}$
		M1 (integrate $3e^{-2x}$)
		M1 (integrate $\cos 2x$)
		M1 (find c)
		M1 (integrate $-\frac{3}{2}e^{-2x}$)
		M1 (integrate $\frac{1}{2}\sin 2x$)
		M1 (integrate $\frac{13}{2}$)
		A1

8(a)	$T_{r+1}=\binom{n}{r}(3x)^{n-r}\left(-\frac{2}{x^2}\right)^r$ $=\binom{n}{r}3^{n-r}x^{n-r}(-2)^r(x^{-2})^r$ $=\binom{n}{r}3^{n-r}(-2)^r x^{n-3r}$	**M1** (general term) **M1** (simplification)

Qn. #	Solution	Mark Allocation
	$n-3r=0$ $n=3r$ where r is a positive integer Thus n is a multiple of 3.	**A1** (explanation)
8(b)	Term independent of x: $9=3r$ $r=3$ $T_4=\binom{9}{3}3^6(-2)^3$ $=-489888$ For $\frac{1}{x^6}$ term: $9-3r=-6$ $r=5$ $T_6=\binom{9}{5}3^4(-2)^5x^{-6}$ $=-\frac{326592}{x^6}$ $\frac{\text{coefficient of term independent of } x}{\text{coefficient of } \frac{1}{x^6}}=\frac{-489888}{-326592}$ $=\frac{3}{2}$	**B1** (Obtain -489888) **M1** (Find r for $\frac{1}{x^6}$ term) **M1** (Find $\frac{1}{x^6}$ term) **A1**
9(a)	$\frac{9x^2-4x+8}{(x-2)(x+1)^2}=\frac{A}{x-2}+\frac{B}{(x+1)^2}+\frac{C}{x+1}$ $9x^2-4x+8=A(x+1)^2+B(x-2)+C(x-2)(x+1)$ Sub $x=-1$ $9(-1)^2-4(-1)+8=B(-1-2)$ $B=-7$ Sub $x=2$ $9(2)^2-4(2)+8=A(2+1)^2$ $A=4$ Sub $x=0$ $9(0)^2-4(0)+8=4(1)^2-7(-2)+C(-2)(1)$ $C=5$ $\frac{9x^2-4x+8}{(x-2)(x+1)^2}=\frac{4}{x-2}-\frac{7}{(x+1)^2}+\frac{5}{x+1}$	**M1** (form 3 fractions) **M1** (form identity) **M2** (A, B, C correct) **M1** (1 of 3 constants correct) **A1**
9(b)	$\int\frac{9x^2-4x+8}{(x-2)(x+1)^2}\,dx=\int\frac{4}{x-2}-\frac{7}{(x+1)^2}+\frac{5}{x+1}\,dx$ $=4\ln(x-2)+\frac{7}{x+1}+5\ln(x+1)+c$	**B3 (B1** for each term) Note: Subtract 1 mark if there is no "+ c"

Qn. #	Solution	Mark Allocation
10(a)	$v=\frac{ds}{dt}$ $v=-3+\frac{1}{2}e^{\frac{t}{2}}$ $0=-3+\frac{1}{2}e^{\frac{t}{2}}$ $6=e^{\frac{t}{2}}$ $\ln 6=\frac{t}{2}$ $t=2\ln 6$	 **M1** (find v) **M1** ($v=0$) **A1**
10(b)	At $t=0$, $s=1$ At $t=1$, $s=-1.35$ (3sf) Since displacement changes from positive to negative, the particle passes through $s=0$ some time between $t=0$ and $t=1$. Hence particle passes through O in first second.	**M1** (both values of s) **A1** (explanation)
10(c)	At $t=2\ln 6$, $s=-4.7506$ At $t=4$, $s=-4.6109$ Total distance $=(1+4.7506)+(4.7506-4.6109)$ $=5.89$ cm (3sf)	**M1** (both values of s) **M1** (sum of distances) **A1**
11(a)	Refer to attached graph	**B1** (table of values) **B1** (plot points) **B1** (draw line)
11(b)	Using points (0, 4.17) and (2, 3.78), Gradient $=\frac{4.17-3.78}{0-2}$ $=-0.195$ (accept -0.225 to -0.165) $C=Ae^{-kt}+15$ $\ln(C-15)=\ln A-kt$ $k=0.195$ (3 s.f.) (accept 0.165 to 0.225) $\ln A=4.17$ (accept 4.14 to 4.2) $A=64.7$ (3 s.f.) (accept 62.8 to 66.7) $C=64.7e^{-0.195t}+15$ OR $\ln(C-15)=-0.195t+4.17$ $C-15=e^{-0.195t+4.17}$ $C-15=e^{-0.195t}\times e^{4.17}$ $C=64.7e^{-0.195t}+15$	 **B1** (Gradient) **M1** (Form linear eqn) **A1** (Find A) **A1** **M1** (remove ln) **A2** (**A1** to find A, **A1** for eqn)
11(c)	$64.7155e^{-0.195t}+15<35$	**M1** (accept $=35$)

Qn. #	Solution	Mark Allocation
	$e^{-0.195t} < \frac{20}{64.7155}$ $-0.195t < \ln\left(\frac{20}{64.7155}\right)$ $t > 6.02$ Year 2030	**M1** (apply ln) **A1** (Year)
12(a)	Let $B\left(x, \frac{1}{2}x+1\right)$ $(x+2)^2 + \left(\frac{1}{2}x+1\right)^2 = \left(5\sqrt{5}\right)^2$ $x^2+4x+4+\frac{1}{4}x^2+x+1=125$ $\frac{5}{4}x^2+5x-120=0$ $x^2+4x-96=0$ $(x-8)(x+12)=0$ $x=8$ or $x=-12$ (rej) $y=5$ $B(8,5)$	**M1** (form eqn using length) **M1** (simplification) **M1** (solve quad eqn) **A1**
12(b)	Gradient of $BC = \frac{7-5}{7-8}$ $= -2$ Gradient of $AB \times$ Gradient of $BC = \frac{1}{2} \times -2$ $= -1$ Therefore $\angle ABC = 90^\circ$ Since $ABCD$ is a parallelogram with int angle $= 90^\circ$, $ABCD$ is a rectangle.	**M1** (Gradient of BC) **M1** (Show right angle) **A1** (explanation)
12(c)	Length $BC = \sqrt{(8-7)^2+(5-7)^2}$ $= \sqrt{5}$ units Area of $ABCD = 5\sqrt{5} \times \sqrt{5}$ $= 25$ units^2	**M1** (Find BC) **A1**
13(a)	$\frac{dy}{dx} = 6(-3)(2x-5)^{-4}(2)$ $= -\frac{36}{(2x-5)^4}$ For $x > 2.5$, since numerator of $\frac{dy}{dx} \neq 0$, $\frac{dy}{dx} \neq 0$ Therefore there are no stationary points.	**M1** ($\frac{dy}{dx}$ without $\times 2$) **M2** (correct $\frac{dy}{dx}$) **A1** (with explanation)

Qn. #	Solution	Mark Allocation
13(b)	At $x=1$, $y=-\frac{2}{9}$	**B1** (y – coordinate)
	At $x=1$, $\frac{dy}{dx}=-\frac{4}{9}$	
	Gradient of normal $=\frac{9}{4}$	**M1** (gradient of normal)
	Eqn of normal: $y+\frac{2}{9}=\frac{9}{4}(x-1)$ $y=\frac{9}{4}x-\frac{89}{36}$ $36y=81x-89$	**M1** (form eqn of normal)
	Points of intersection: $x^2+90x-78=81x-89$ $x^2+9x+11=0$	**M1** (sim eqn)
	$x=\frac{-9\pm\sqrt{9^2-4(1)(11)}}{2(1)}$ $=-\frac{9}{2}\pm\frac{\sqrt{37}}{2}$	**M1** (quad formula)
	Difference between x-coordinates $=-\frac{9}{2}+\frac{\sqrt{37}}{2}-\left(-\frac{9}{2}-\frac{\sqrt{37}}{2}\right)$	**M1** (difference)
	$=\sqrt{37}$	**A1**