Geylang Methodist School (Secondary)
Preliminary Examination 2024

Candidate Name			
Class		**Index Number**	

ADDITIONAL MATHEMATICS
Paper 2

4049 / 02

4 Express/5 Normal(A)

Candidates answer on the Question Paper.

No Additional Materials are required.

2 hours 15 minutes

Setters: **Mr Johney Joseph**
Ms Ng Siew Lee

19 August 2024

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets **[ ]** at the end of each question or part question.
The total score for this paper is **90.**

For Examiner's Use
90

This document consists of **19** printed pages and **1** blank page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 Solve the equation $2\sin^4 x + 7\cos^2 x = 4$ for $0^\circ \le x \le 360^\circ$. [6]

2 At a certain time, the mass of a radioactive substance was recorded as 150 g. This mass decreased with time due to decay and after t hours, the recorded mass was M g. It is known that M can be modelled by the formula $M = 150e^{-kt}$, where k is a positive constant. After 50 hours, its mass has decreased to 120g.

(a) Estimate the mass of the substance after 120 hours. [4]

(b) Estimate after how many hours one third of the substance is decayed. [3]

3 A curve has the equation $y = \frac{e^{2x}}{x-2}$, where $x > 2$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the exact value of the coordinates of the stationary point. [3]

(c) Determine the nature of the stationary point. [2]

4 The line $\frac{x}{a}+\frac{y}{b}=1$, where a and b are positive constants, intersects the x-axis at A and the y-axis at B. The perpendicular bisector of the line joining A and B passes through the point $P(-3,-7)$.

(a) Show that $a^2+6a=b^2+14b$. [6]

(b) Given that the gradient of the perpendicular bisector of AB is 2, find the values of a and b. [4]

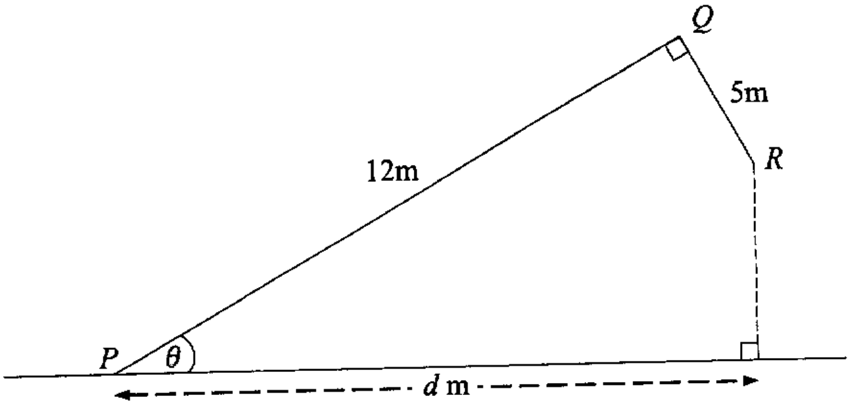
5 A circle, with centre C, has equation $x^2 + y^2 - 10x - 4y + 25 = 0$.

(a) Find the coordinates of C and the radius of the circle. [4]

(b) Explain why the x-axis is a tangent to the circle. [2]

(c) The tangent to the circle at the point where $x = 3$ meets the x-axis at the point P. Find the coordinates of P. [3]

6



The diagram shows two rods PQ and QR, of lengths 12m and 5m respectively. The rods are fixed at Q such that angle $PQR = 90^\circ$ and hinged at P so as to rotate in a vertical plane. The rod PQ makes an angle θ with horizontal ground.

(a) Obtain an expression, in terms of θ, for d, where d is the horizontal distance of R from P. [3]

(b) Express d in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [4]

(c) Given $d = 10$, find the value of θ. [2]

7 The expression $x^3 - 4x^2 + ax + b$, where a and b are constants, has a factor of $x + 1$ and leaves a remainder of -60 when divided by $x + 3$.

(a) Find the value of a and of b. [4]

(b) Using these values of a and b, solve the equation $x^3 - 4x^2 + ax + b = 0$. [4]

(c) Explain how the solution from **part (b)** can be used to solve the equation $9^x + 1 = 4\left(3^x\right) - 6\left(3^{-x}\right)$. [2]

8 A particle P starts from rest at a fixed point O and moves in a straight line. The velocity, v ms^{-2}, of the particle, t s after passing through O is given by $v = 8t - ct^3$ where c is a positive constant.
The velocity of the particle is 12 ms^{-1} after 2 seconds.

Find

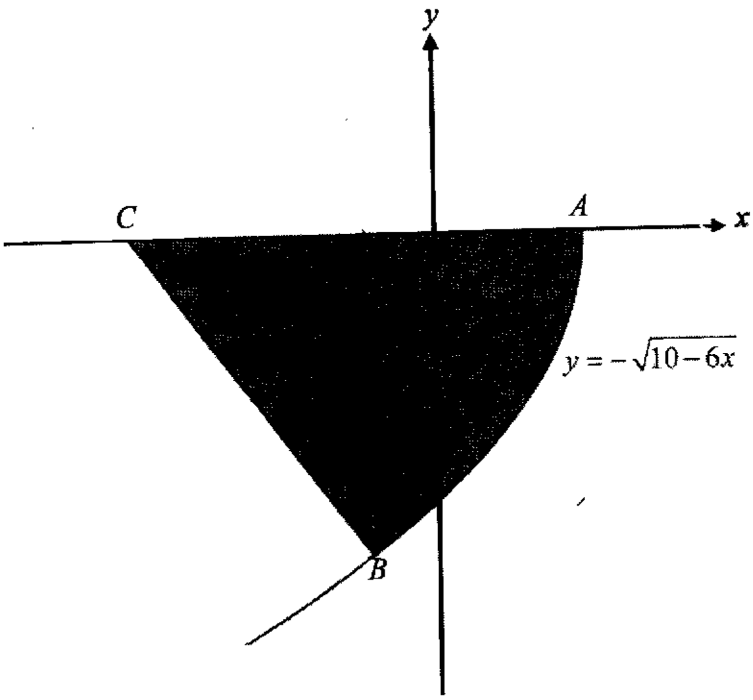
(a) the value of c, [1]

(b) the acceleration after 2 seconds, [2]

(c) the time at which P will change its direction of motion, [2]

(d) the average speed of the particle in the first 5 seconds. [5]

9 The diagram shows part of the curve $y=-\sqrt{10-6x}$ meeting the x-axis at the point A. The normal to the curve at B, where $x=-1$, meets the x-axis at the point C.



(a) Find the equation of the normal at B. [6]

(b) Find the area of the shaded region. [5]

10 **(a)** **(i)** Express $\dfrac{2x^2+3x}{x^2+3x+2}$ in partial fractions. [4]

(ii) Hence evaluate $\displaystyle\int_1^3 \frac{2x^2+3x}{x^2+3x+2}\,\mathrm{d}x$. [3]

(b) Given that $y = x\ln\left(x^2+3x+2\right)$, find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(c) Using the results from **parts (a)(ii)** and **(b)**,
evaluate $\int_1^3 \ln\left(x^2+3x+2\right)\mathrm{d}x$. [2]

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Marking Scheme AM P2 (4049/02)

Qn	Answer	Marks	Partial Marks	Guidance
1	$2\sin^4 x+7\left(1-\sin^2 x\right)=4$			
	$\sin^2 x=\frac{1}{2}$			
	Basic angle $=45^\circ$			
	$x=45^\circ, 135^\circ, 225^\circ, 315^\circ$			
2(a)	$120=150e^{-50k}$			
	$e^{50k}=\frac{150}{120}$ $k=\frac{\ln\left(\frac{150}{120}\right)}{50}$ $=0.00446287$			
	$M=150e^{-120\times 0.00446287}$ $=87.8g$ (3 s.f.)			
2(b)	When M $=\frac{2}{3}\times 150=100\,g$ $100=150e^{-0.00446287t}$ $e^{0.00446287t}=\frac{150}{100}$ $t=\frac{\ln\left(\frac{150}{100}\right)}{0.00446287}$ $=90.9$ hours (3 s.f.)			
3(a)	$\frac{dy}{dx}=\frac{2e^{2x}(x-2)-e^{2x}}{(x-2)^2}$			
3(b)	$\frac{2e^{2x}(x-2)-e^{2x}}{(x-2)^2}=0$ $e^{2x}(2x-4-1)=0$ $x=\frac{5}{2}=2.5$ $y=2e^5$ $\left(2.5,\ 2e^5\right)$			
3(c)	$(2.5, 2e^5)$ is a minimum point.			

[redacted]				
4(a)	A$(a,0)$ and B$(0,b)$ gradient of AB $=-\frac{b}{a}$ gradient of perpendicular bisector $=\frac{a}{b}$ midpoint of AB is $=\left(\frac{a}{2},\frac{b}{2}\right)$ gradient of perpendicular bisector $=\frac{\frac{b}{2}+7}{\frac{a}{2}+3}$ $\frac{a}{b}=\frac{b+14}{a+6}$ $a^2+6a=b^2+14b$ (shown)			
4(b)	$a=2b$ $(2b)^2+6(2b)=b^2+14b$ $3b^2-2b=0$ $b(3b-2)=0$ $b=\frac{2}{3}$ $a=\frac{4}{3}$			
[redacted]				
5(a)	$2g=-10,\quad 2f=-4$ centre$(5,2)$ $r=\sqrt{(-5)^2+(-2)^2-25}$ $=2$ units			
5(b)	The centre of the circle is 2 units above the x-axis and the radius of the circle is 2 units. Hence the x-axis is a tangent to the circle.			
5(c)	When $x=3$ $y=2$ $(3,2)$ and $(5,2)$ gradient $=0$ Equation of tangent is $x=3$ $\therefore$ P$(3,0)$			

Qn	Answer	Marks	Partial Marks	Guidance
6(a)	$PT=12\cos\theta$ $SR=5\sin\theta$ $d=12\cos\theta+5\sin\theta$			
6(b)	$R=\sqrt{12^2+5^2}$ $=13$ $\alpha=\tan^{-1}\left(\frac{5}{12}\right)$ $=22.6^\circ$ $d=13\cos\left(\theta-22.6^\circ\right)$			
6(c)	$13\cos\left(\theta-22.6^\circ\right)=10$ $\theta=62.3^\circ$			
7(a)	$f(-1)=0\rightarrow -a+b=5$ $f(-3)=-60\rightarrow -3a+b=3$ $a=1,\ b=6$			
7(b)	$x^3-4x^2+x+6=0$ $(x+1)\left(x^2-5x+6\right)=0$ $x=-1,2,3$			
7(c)	$\left(3^x\right)^2+1=4\left(3^x\right)-\frac{6}{3^x}$ $\left(3^x\right)^3-4\left(3^x\right)^2+3^x+6=0$ The equation can be solved by taking each solution in part b 3^x.			

Qn	Answer	Marks	Partial Marks	Guidance
8 (a)	$12=8(2)-c(2)^3$ $c=\frac{1}{2}$			
8(b)	$\frac{dv}{dt}=a=8-\frac{3}{2}t^2$ $=8-\frac{3}{2}(2)^2$ $=2\ m/s^2$			
8(c)	$8t-\frac{1}{2}t^3=0$ $t\left(8-\frac{1}{2}t^2\right)=0$ $t^2=16$ $t=4$			
	$s=\int\left(8t-\frac{1}{2}t^3\right)dt$ $=\frac{8t^2}{2}-\frac{t^4}{8}+c$ $s=0, t=0 \Rightarrow c=0$ $s=4t^2-\frac{1}{8}t^4$ $t=4,$ $s=4(4)^2-\frac{1}{8}(4)^4$ $=32$ m $t=5,$ $s=4(5)^2-\frac{1}{8}(5)^4$ $=21.875$m Distance travelled = $=32+32-21.875=42.125$ Average speed = $42.125 \div 5 = 8.425$ m/s			

Qn	Answer	Marks	Partial Marks	Guidance
9(a)	$\frac{dy}{dx}=-\frac{1}{2}(10-6x)^{-\frac{1}{2}}(-6)$ $=3(10-6x)^{-\frac{1}{2}}$ $x=-1,\quad \frac{dy}{dx}=\frac{3}{4},$ $y=-4$ Gradient of normal = $-\frac{4}{3}$ Equation of normal : $y+4=-\frac{4}{3}(x+1)$ $3y=-4x-16$			
9(b)	At $C, y=0, x=-4$ At $A, y=0, x=\frac{5}{3}$ Area of triangle = $\frac{1}{2}\times 3\times 4=6$ unit^2 Area under curve = $-\int_{-1}^{\frac{5}{3}}-\sqrt{10-6x}\,dx$ $=\left[\frac{(10-6x)^{\frac{3}{2}}}{\frac{3}{2}\times(-6)}\right]_{-1}^{\frac{5}{3}}=\left[0-\frac{4^3}{9}\right]$ Area of shaded region = $6+\frac{4^3}{9}$ $=13\frac{1}{9}$ or 13.1 unit^2			

Qn	Answer	Marks	Partial Marks	Guidance
10 (a) (i)	$\frac{2x^2+3x}{x^2+3x+2}=2-\frac{3x+4}{x^2+3x+2}$ $\frac{3x+4}{x^2+3x+2}=\frac{A}{x+2}+\frac{B}{x+1}$ $3x+4=A(1+x)+B(x+2)$ $Let\ x=-1, B=1$ $Let\ x=-2, A=2$ $\frac{2x^2+3x}{x^2+3x+2}=2-\frac{2}{x+2}-\frac{1}{x+1}$			
10 (a) (ii)	$\int_1^3 \left(2-\frac{2}{x+2}-\frac{1}{x+1}\right)dx$ $=\left[2x-2\ln(x+2)-\ln(x+1)\right]_1^3$ $=[6-2\ln 5-\ln 4]-[2-2\ln 3-\ln 2]$ $=2.29$			
10 (b)	$\frac{dy}{dx}=\ln(x^2+3x+2)+\frac{x}{x^2+3x+2}\times(2x+3)$ $=\ln(x^2+3x+2)+\frac{2x^2+3x}{2+x-x^2}$			
10 (c)	$\int_1^3\left[\ln(x^2+3x+2)+\frac{2x^2+3x}{x^2+3x+2}\right]dx$ $=\left[x\ln(x^2+3x+2)\right]_1^3$ $\int_1^3 \ln(x^2+3x+2)dx$ $=\left[x\ln(x^2+3x+2)\right]_1^3-\int_1^3\left[\frac{2x^2+3x}{x^2+3x+2}\right]dx$ $=3\ln 20-\ln 6-2.2852$ $=4.91$			