HUA YI SECONDARY SCHOOL
PRELIMINARY EXAM 2024

4-G3 / 5-G2

NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS
PAPER 2

4049/02

08 October 2024

2 hour 15 minutes

Candidates answer on the Question Paper

No Additional Materials is required.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class, and Index Number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue, or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in the brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	90

This document consists of **18** printed pages.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Show that $x = \frac{1}{2}$ is a solution of the equation $2x^3 + x^2 - 3x + 1 = 0$ and hence solve the equation completely. [5]

2 **(a)** By considering the general term in the binomial expansion of $\left(px + \frac{1}{x^3}\right)^9$, where p is a constant, explain why there are no even powers of x in this expansion. [3]

(b) Given that the coefficient of x^8 is equal to the coefficient of x in the expansion of $(2x^3 + 1)\left(px + \frac{1}{x^3}\right)^9$, find the value of p. [4]

(c) Using the value of p in **(b)**, find the term independent of x in the expansion of $(2x^3 + 1)\left(px + \frac{1}{x^3}\right)^9$. [2]

3 (a) Given that $y = \frac{2x}{(3x+1)^{\frac{1}{2}}}$, show that $\frac{dy}{dx} = \frac{3x+2}{(3x+1)^{\frac{3}{2}}}$. [4]

(b) Hence find the value of $\int_0^2 \frac{x}{(3x+1)^{\frac{3}{2}}}\, dx$. [5]

4 Show that the equation $5e^x = \frac{1}{e^x} - 4$ has only one solution and find its value correct to 2 decimal places. [4]

5 The equation of a curve is $y = -2x^2 + 3x + 5$.

(a) Find the set of values for x for which the curve lies below the line $y = 3$ and represent this set of values on a number line. [4]

The line $y = x + k$ is a tangent to the curve at the point Z.

(b) Find the value of the constant k. [3]

(c) Find the coordinates of Z. [2]

6 **(a)** The speed V m/s of a vehicle, t s after passing a fixed point O, is given for $t \geq 0$, as

$$V = 1 + pe^{qt}, \text{ where } p \text{ and } q \text{ are constants.}$$

Explain how a straight line can be drawn to represent the formula, and state how the value of p and q can be obtained from the line. [4]

(b)(i) Data of the speeds of the vehicle was collected. The table shows the corresponding values of V and t.

t	2	4	6	8	10
V	10.35	8.40	6.70	5.42	5.95

Using **(a)**, draw the straight line graph on the next page. [3]

(ii) Estimate the values of p and q. [3]

(iii) Estimate a value of V to replace one incorrect recording of V found in the straight line graph. [2]

7 A motorcyclist, travelling along a straight road, passes a lamp post X, with speed of h km/h. A while later, the motorcyclist passes a second lamp post Y, with a speed of 60 km/h.
Between the two lamp posts, the speed is given by $V = 20e^{50t} + 10\,\text{km/h}$, where t, the time after passing lamp post X is measured in hours.

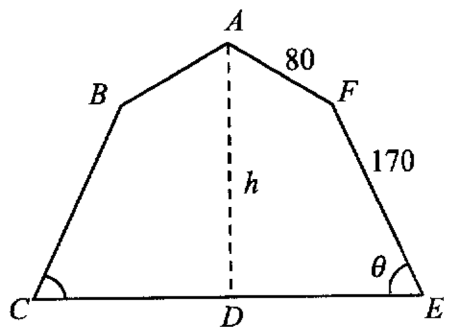
(a) Find the value of h. [2]

(b) Calculate to the nearest second, the time taken to travel from X to Y. [3]

(c) Find the acceleration of the motorcyclist as he passes Y. [3]

(d) Find the distance *XY*. [5]

8 The diagram shows the side view $ABCDEF$ of an ornament. The ornament rests with CE on horizontal ground and is symmetrical about the vertical AD, where D is the midpoint of CE.
Angle DEF = Angle $DAF = \theta$ radians and the lengths of AF and FE are 80 cm and 170 cm respectively. The vertical height of the ornament is h cm.



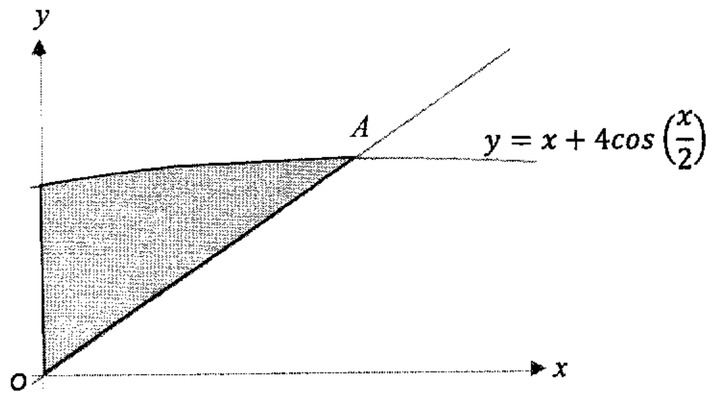
(a) Explain clearly why $h = 80cos\theta + 170sin\theta$. [2]

(b) Express h in the form $Rsin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [3]

(c) Find the greatest possible value of h and the value of θ at which this occurs. [3]

(d) Find the values of θ when $h = 180$ cm. [2]

9



The diagram shows the curve $y = x + 4cos\left(\frac{x}{2}\right)$ for $0 \leq x \leq \frac{\pi}{2}$ radians.
The point A is the stationary point of the curve and OA is a straight line.

(a) Find the exact coordinates of A. [5]

(b) Show that the area of the shaded region is $4-\frac{\sqrt{3}}{3}\pi$ units^2. [5]

10 A tangent to a circle at the point (3,2) cuts the y-axis at 5. The line with the equation $3y = 2x + 5$ is normal to the circle.

(a) Find the equation of the circle, showing your working clearly. [7]

(b) Find the equations of tangents to the circle that are parallel to the x-axis. [2]

End of Paper

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ANSWER SCHEME

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where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

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$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}ab\sin C$$

1 Show that $x = \frac{1}{2}$ is a solution of the equation $2x^3 + x^2 - 3x + 1 = 0$ and hence solve the equation completely. [5]

$f(x) = 2x^3 + x^2 - 3x + 1$

$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 1$ ---------------------M1

$= 0$

$f(x) = (2x - 1)(x^2 + bx - 1)$ -------------------------M1

Compare coeff. of x^2, $1 = 2b - 1$

b= 1 ----------------------------M1

$f(x) = (2x - 1)(x^2 + x - 1)$-------------------------M1

$x = \frac{1}{2}, \frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}$ ---------------------------------------A1

Alternative Method : Using long division to find $(x^2 + x - 1)$.

2 **(a)** By considering the general term in the binomial expansion of $\left(px + \frac{1}{x^3}\right)^9$, where p is a constant, explain why there are no even powers of x in this expansion. [3]

General Term $= \binom{9}{r} (px)^{9-r} \left(\frac{1}{x^3}\right)^r$ -----------------M1

$= \binom{9}{r} p^{9-r} x^{9-4r}$ -------------------M1

Since $9 - 4r = 1 + 4(2 - r)$, one added to any even number will give an odd value, hence there are no even powers of x in this expansion. --------A1

(b) Given that the coefficient of x^8 is equal to the coefficient of x in the expansion of $(2x^3+1)\left(px+\frac{1}{x^3}\right)^9$, find the value of p. [4]

$(2x^3+1)\left(\ldots\ldots\binom{9}{1}(px)^8\left(\frac{1}{x^3}\right)^1+\binom{9}{2}(px)^7\left(\frac{1}{x^3}\right)^2\right)^9$ --------------------M1

$=(2x^3+1)\,(\ldots+9p^8x^5+36p^7x+\cdots)$ ---------------------------------M1

$2(9p^8)=36p^7$ --M1

$p=2$ ---A1

(c) Using the value of p in **(b)**, find the term independent of x in the expansion of $(2x^3+1)\left(px+\frac{1}{x^3}\right)^9$. [2]

Term independent of $x=(2)\binom{9}{3}(2^6)$ ---------------------M1

$=10752$ ---------------------------A1

3 (a) Given that $y = \frac{2x}{(3x+1)^{\frac{1}{2}}}$, show that $\frac{dy}{dx} = \frac{3x+2}{(3x+1)^{\frac{3}{2}}}$. [4]

$\frac{dy}{dx} = \frac{2(3x+1)^{\frac{1}{2}} - 2x\left(\frac{1}{2}\right)(3)(3x+1)^{-\frac{1}{2}}}{(3x+1)^1}$ ------------------M2 (numerator and denominator)

$= \frac{2(3x+1)-3x}{(3x+1)^{\frac{3}{2}}}$ ----------------------------------M1

$= \frac{3x+2}{(3x+1)^{\frac{3}{2}}}$ --A1

(b) Hence find the value of $\int_0^2 \frac{x}{(3x+1)^{\frac{3}{2}}}\, dx$. [5]

$From\ \frac{dy}{dx} = \frac{3x+2}{(3x+1)^{\frac{3}{2}}}$,

$$\frac{dy}{dx} = \frac{3x}{(3x+1)^{\frac{3}{2}}} + \frac{2}{(3x+1)^{\frac{3}{2}}}$$

$$\frac{3x}{(3x+1)^{\frac{3}{2}}} = \frac{dy}{dx} - \frac{2}{(3x+1)^{\frac{3}{2}}}$$

$\int_0^2 \frac{3x}{(3x+1)^{\frac{3}{2}}} dx = \frac{2x}{(3x+1)^{\frac{1}{2}}} - \int_0^2 \frac{2}{(3x+1)^{\frac{3}{2}}}\, dx$ --------------------------------M1

$$\int_0^2 \frac{x}{(3x+1)^{\frac{3}{2}}} dx = \frac{1}{3}\left[\frac{2x}{(3x+1)^{\frac{1}{2}}}\right]_0^2 - \frac{1}{3}\int_0^2 \frac{2}{(3x+1)^{\frac{3}{2}}}\, dx$$

$= \frac{1}{3}\left[\frac{2x}{(3x+1)^{\frac{1}{2}}}\right]_0^2 - \frac{1}{3}\left[\frac{2(3x+1)^{-\frac{1}{2}}}{\left(-\frac{1}{2}\right)(3)}\right]_0^2$ ---------------------M1, M1

$= \frac{1}{3}\left(\frac{4}{\sqrt{7}} - 0\right) + \left(\frac{4}{9}\right)\left(\frac{1}{\sqrt{7}} - 1\right)$ ----------------------------M1

$= 0.227$ ----------------------------------A1

4 Show that the equation $5e^x = \frac{1}{e^x} - 4$ has only one solution and find its value correct to 2 decimal places. [4]

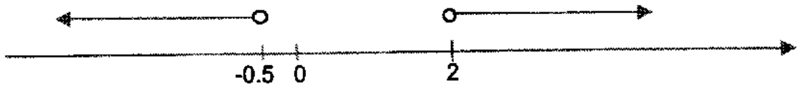
$5e^x = \dfrac{1}{e^x} - 4$

$5e^{2x} = 1 - 4e^x$
$5e^{2x} + 4e^x - 1 = 0$ ------------------------M1
$5(e^x)^2 + 4e^x - 1 = 0$
$(5e^x - 1)(e^x + 1) = 0$ --------------------M1
$e^x = \frac{1}{5}$ or $e^x = -1$
$x = ln\left(\frac{1}{5}\right)$ (no solution, reject)----------A1
$= -1.61$ (2 dp) ------A1

5 The equation of a curve is $y = -2x^2 + 3x + 5$.

(a) Find the set of values for x for which the curve lies below the line $y = 3$ and represent this set of values on a number line. [4]

$-2x^2 + 3x + 5 < 3$ ------------------------M1
$2x^2 - 3x - 2 > 0$
$(2x + 1)(x - 2) > 0$ ----------------------M1
$x < -\frac{1}{2}$ or $x > 2$ ------------------------A1



A1

The line $y = x + k$ is a tangent to the curve at the point Z.

(b) Find the value of the constant k. [3]

$-2x^2 + 3x + 5 = x + k$
$2x^2 - 2x + k - 5 = 0$ --------------------------------M1
Line is tangent to curve => $b^2 - 4ac = 0$

$(-2)^2 - 4(2)(k - 5) = 0$ ----------------------------M1
$4 - 8k + 40 = 0$
$k = \frac{11}{2}$ or $5\frac{1}{2}$ ---A1

(c) Find the coordinates of Z. [2]

$y = x + \frac{11}{2}$--------------eqn 1
Sub eqn 1 into eqn of curve,
$-2x^2 + 3x + 5 = x + \frac{11}{2}$ ----------------M1
$-4x^2 + 6x + 10 = 2x + 11$
$-4x^2 + 4x - 1 = 0$
$4x^2 - 4x + 1 = 0$
$(2x - 1)^2 = 0$
$x = \frac{1}{2}, y = 6$
$Z = \left(\frac{1}{2},\ 6\right)$ ------------------------------A1

6 **(a)** The speed V m/s of a vehicle, t s after passing a fixed point O, is given for $t \geq 0, V = 1 + pe^{qt}$, where p and q are constants.
Explain how a straight line can be drawn to represent the formula, and state how the value of p and q can be obtained from the line. [4]

$V - 1 = pe^{qt}$
$\ln(V-1) = \ln(pe^{qt})$
$\ln(V-1) = \ln p + qt$ --------------------M1
Draw $\ln(V-1)$ against t ---------------M1

y- intercept = $\ln p$ --------------------A1
gradient = q -------------------------- A1

(b)(i) Data of the speeds of the vehicle was collected. The table below shows the corresponding values of V and t.

t	2	4	6	8	10
V	10.35	8.40	6.70	5.42	5.95

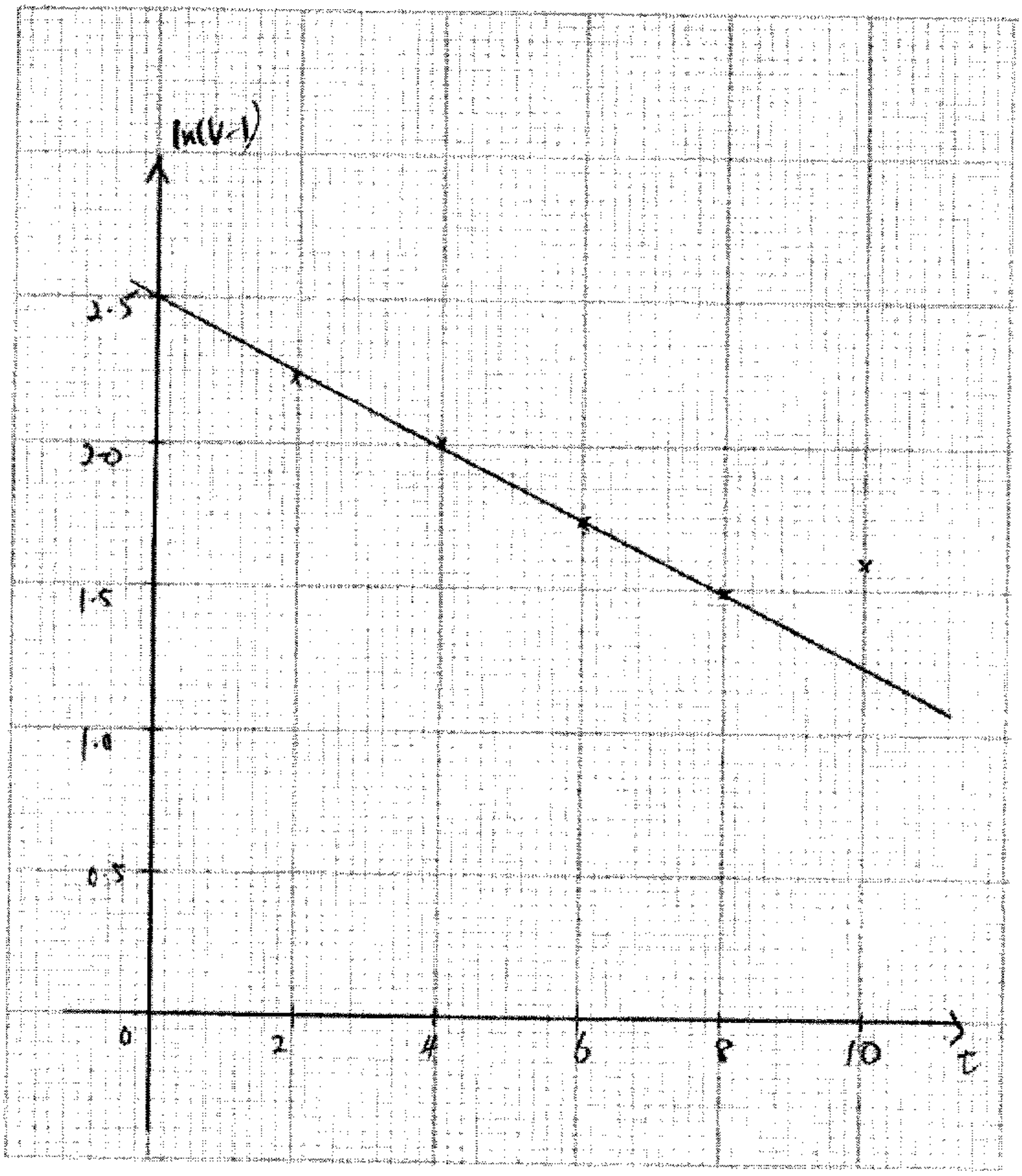
Using **(a),** draw the straight line graph on the next page. [3]

t	2	4	6	8	10
ln(V-1)	2.24	2.00	1.74	1.49	1.60

M1 - Calculate ln $(v-1)$, M1 – correct points plotted, M1 – Best fit line

(ii) Estimate the values of p and q. [3]

$q = \frac{2.5-1.5}{0-8}$ --------------------M1
$= -0.125$ ---------------------A1
$lnp = 2.5$
$p = 12.18$ ----------------A1



(iii) Estimate a value of V to replace one incorrect recording of V found in the straight line graph. [2]

Incorrect value of V occurs at $x = 10$.
$\ln(V - 1) = 1.25$ ------------------------------M1
$V = 4.49$ ------------------------------------A1

7 A motorcyclist, travelling along a straight road, passes a lamp post X, with speed of h km/h. A while later, the motorcyclist passes a second lamp post Y, with a speed of 60 km/h.
Between the two lamp posts, the speed is given by $V = 20e^{50t} + 10$ km/h where t, the time after passing lamp post X is measured in hours.

(a) Find the value of h. [2]

$t = 0,\ h = 20e^{0} + 10$ ----------------------------M1
$h = 30\ km/h$ ---------------------------------------A1

(b) Calculate to the nearest second, the time taken to travel from X to Y. [3]

$60 = 20e^{50t} + 10$ --------------M1
$ln(2.5) = 50t$
$t = 0.0183258\ h$ -----------------M1
$= 66$ seconds ----------------A1

(c) Find the acceleration of the motorcyclist as he passes Y. [3]

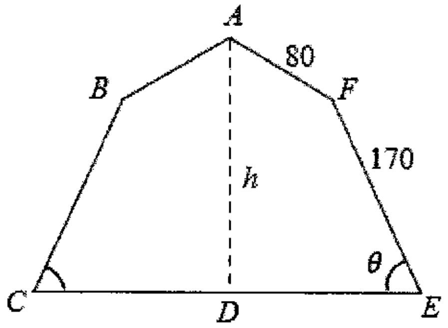
Acceleration $= \frac{dv}{dt}$
$= 20(50)e^{50t}$ -----------------M1
$= 20\ (50)e^{50(0.0183258)}$------------M1
$= 2500\ km/h^2$ ------------------A1

(d) Find the distance XY. [5]

Distance XY
$= \int_0^{\frac{ln2.5}{50}} v\,dt$
$= \int_0^{\frac{ln2.5}{50}} 20e^{50t} + 10dt$ ----------------------M1
$= \left[\frac{20}{50}e^{50t} + 10t\right]_0^{\frac{ln2.5}{50}}$ -----------------------M1, M1

$= \left(\frac{2}{5}e^{ln2.5} + 10\left(\frac{ln2.5}{50}\right)\right) - \frac{2}{5}$ ------------------M1
$= 0.783km$ ---------------------------------A1

8 The diagram shows the side view $ABCDEF$ of an ornament. The ornament rests with CE on horizontal ground and is symmetrical about the vertical AD, where D is the midpoint of CE.
Angle DEF = Angle DAF = θ radians and the lengths of AF and FE are 80 cm and 170 cm respectively. The vertical height of the ornament is h cm.



(a) Explain clearly why $h = 80cos\theta + 170sin\theta$ [2]

$cos\theta = \frac{AM}{80}$ $sin\theta = \frac{FN}{170}$ -----------------M1

$h = AM + MD$
$= 80cos\theta + 170sin\theta$------------------------A1

(b) Express h in the form $Rsin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [3]
$h = 170sin\theta + 80cos\theta$
$R = \sqrt{80^2 + 170^2} = \sqrt{35300}$ or 187.88 ------------------------M1
$\alpha = tan^{-1}\left(\frac{80}{170}\right) = 0.44\, rad$-------------------------------------M1
$\sqrt{35300}\, sin(\theta + 0.44)$ ---A1

(c) Find the greatest possible value of h and the value of θ at which this occurs. [3]

Greatest value of h =187.88 ----------------A1
$sin(\theta + 0.44) = 1$ --------------------------M1
$\theta + 0.44 = \frac{\pi}{2}$
$\theta = 1.13\, rad$ ------------------------------A1

(d) Find the value of θ when h = 180 cm. [2]

$\sqrt{35300} sin(\theta + 0.44) = 180$
$\theta + 0.44 = 1.280$, 1.862 --------------------------------M1
$\theta = 0.84$ rad , 1.42 rad-------------------------------------A1

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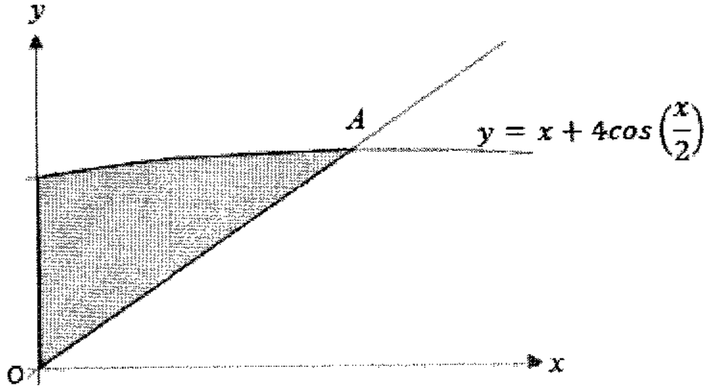

The diagram shows the curve $y = x + 4cos\left(\frac{x}{2}\right)$ for $0 \le x \le \pi$ radians. The point A is the stationary point of the curve and OA is a straight line.

(a) Find the coordinates of A. [5]

$\frac{dy}{dx} = 1 - 2sin\left(\frac{x}{2}\right)$ ----------------------M1, M1

At the stationary point A, $\frac{dy}{dx} = 0$

$1 - 2sin\left(\frac{x}{2}\right) = 0$ ------------M1

$sin\left(\frac{x}{2}\right) = 0.5$

$x = \frac{\pi}{3}$ ------------------------M1

$y = \frac{\pi}{3} + 2\sqrt{3}$

$A = \left(\frac{\pi}{3}, \frac{\pi}{3} + 2\sqrt{3}\right)$ ----------------------------A1

(b) Show that the area of the shaded region is $4 - \frac{\sqrt{3}}{3}\pi$ units^2. [5]

Shaded area

$= \int_0^{\frac{\pi}{3}} x + 4cos\left(\frac{x}{2}\right) dx - \frac{1}{2}\left(\frac{\pi}{3}\right)\left(\frac{\pi}{3} + 2\sqrt{3}\right)$ ------------------- M1 (Area of Δ)

$= \left[\frac{x^2}{2} + \frac{4sin\left(\frac{x}{2}\right)}{\frac{1}{2}}\right]_0^{\frac{\pi}{3}} - \frac{\pi^2}{18} - \frac{\pi\sqrt{3}}{3}$ -------------M2 (Integration of the two terms)

$= \frac{\pi^2}{18} + 8sin\left(\frac{\pi}{2}\right) - 0 - \frac{\pi^2}{18} - \frac{\pi\sqrt{3}}{3}$ --------------------M1

$= 4 - \frac{\sqrt{3}}{3}\pi$ --------------------------A1

10 A tangent to a circle at the point (3,2) cuts the y-axis at 5. The line with the equation $3y = 2x + 5$ is normal to the circle.

(a) Show all your workings, find the equation of the circle. [7]

Gradient of tangent = $\frac{5-2}{0-3} = -1$ ---------------------M1
Gradient of normal =1 -------------------------------M1
Eqn of normal : $y = x - 1$ --------------------------M1

Solve simultaneous eqns of the two normals to get the centre of circle.
----M1
$y = x - 1$ -------------eqn 1
$3y = 2x + 5$ -----------eqn 2
Sub eqn (1) into eqn 2, $3(x - 1) = 2x + 5$
Centre : $x = 8, y = 7$ ------------------M1
Radius of circle = $\sqrt{(8-3)^2 + (7-2)^2} = \sqrt{50}$ -----------M1
Equation of Circle : $(x - 8)^2 + (y - 7)^2 = 50$ ----------A1

(b) Find the tangents to the circle that are parallel to the x-axis. [2]

$y = 7 + \sqrt{50}$ and $y = 7 - \sqrt{50}$-------------B2