NCHS

PAPER-2

1 (a) Given that there is a term that ~~is~~ independent of x in the expansion of $\left(5x^2 - \frac{1}{\sqrt{x}}\right)^n$, where n is a positive integer, find the smallest possible value of n. [3]

(b) Using the value of n found in part (a), explain if there is any term independent of x in the expansion of $\left(1-\frac{1}{50x^5}\right)\left(5x^2-\frac{1}{\sqrt{x}}\right)^n$. [4]

2 The expression $10\,\mathrm{f}(x) + 3\,\mathrm{f}'(x) - \mathrm{f}''(x) + 7\sin 2x + 3\cos 2x$, may be written as $10x + 43$, when $\mathrm{f}'(x) = e^{5x} + 2\sin^2 x$. Find $\mathrm{f}(x)$.

[6]

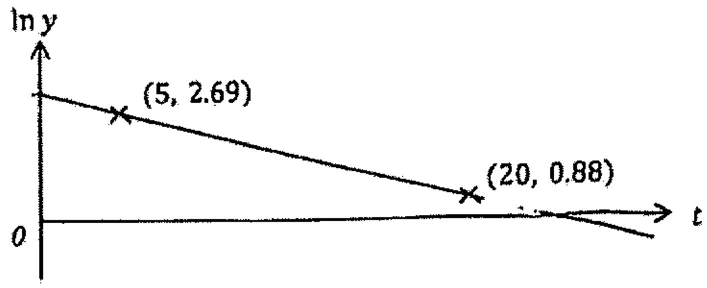
3 **Do not use a calculator in this question.**
It is given that $\tan A = \frac{5}{12}$ and that $\frac{\pi}{2} < A < \frac{3\pi}{2}$.

(a) By expressing $\cos 3A = \cos(2A + A)$, find the exact value of $\cos 3A$. [4]

(b) Find the exact value of $\tan\frac{A}{2}$. [4]

4 Coffee is poured into an empty cup. At time t minutes after the coffee is poured, its temperature exceeds room temperature by y°C. The room temperature is 25°C.

(a)



The variables t and y are related by the equation $y = e^{kt+c}$, where k and c are constants. The diagram above shows the straight line graph obtained by plotting $\ln y$ against t. The line passes through the points (5, 2.69) and (20, 0.88).
Find the value of k and of c. [3]

(b) Calculate the time which the temperature of the coffee would drop to half of its initial temperature. [3]

(c) At the same time, when the coffee was poured into the cup, coffee of the same volume is also poured into an empty tumbler.

Similarly, at time t minutes after the coffee is poured into the tumbler, its temperature exceeds room temperature by y°C and is modelled by another equation.

The solution to the equation $e^{(k+0.2)t} = e^{5-c}$ is the timing where the temperature of the coffee in both the cup and tumbler are equivalent. By using the diagram in part (a), outline the steps to find this timing. [4]

5

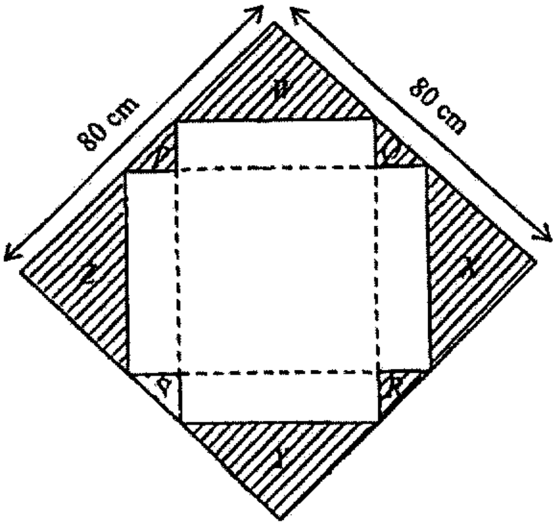


Diagram 1

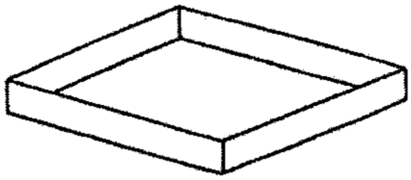


Diagram 2

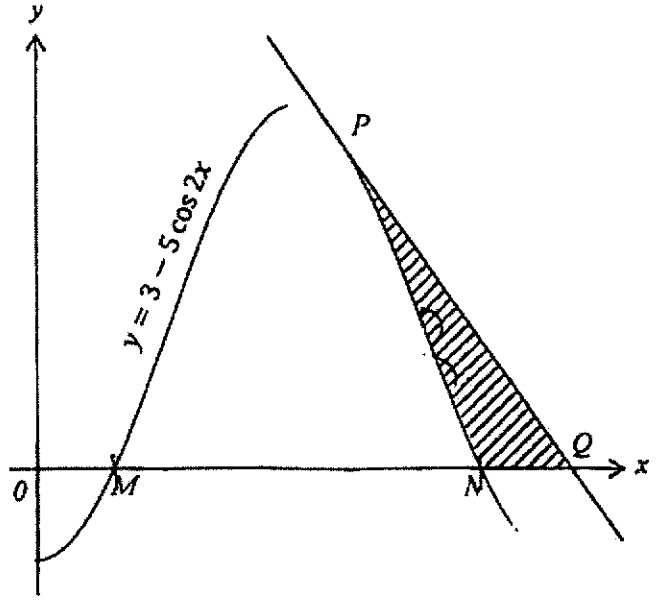
Diagram 1 shows a piece of square cardboard of side 80 cm.
Four small identical isosceles triangles, P, Q, R and S and four big identical isosceles triangles, W, X, Y and Z are removed. The remaining cardboard is folded along the dotted lines to form an open container as shown in Diagram 2.

(a) Let the height of the open container be h cm, show that the total exterior area, A cm^2, of the open container is $3200 - 80\sqrt{2}h + h^2$. [4]

(b) Find the value of h for which the total exterior area of the open container is a maximum. [3]

(c) Hence, find the maximum total exterior area of the open container that can be obtained from the piece of square cardboard. [2]

6



The diagram shows part of the curve $y = 3 - 5\cos 2x$, which cuts the x-axis at M and N. The tangent to the curve at P is -5 and this tangent cuts the x-axis at Q. Find the area of the shaded region PNQ. [9]

Continuation of working space for Question 6.

7 (a) Show that $x + 2y$ is a factor of $4x^3 + x^2y - 11xy^2 + 6y^3$ and hence factorise $4x^3 + x^2y - 11xy^2 + 6y^3$ completely. [3]

(b) Hence, solve the equation $4^{p+1} + 2^{p+1} = 44 - 48(2^{-p})$. [5]

8 (a) The graph of $y = a\sin bx + c$ has one minimum point at $\left(\frac{\pi}{6}, 1\right)$ and the next maximum point after this has coordinates $\left(\frac{\pi}{2}, 9\right)$. Find the values of the constants a, b and c. [3]

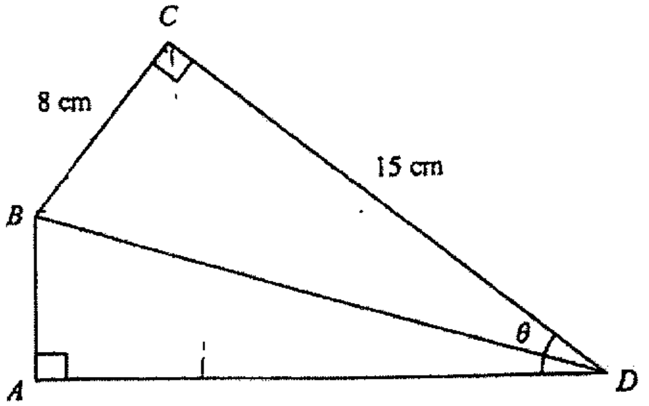
(b) A particle, travelling in a straight line, passes through a fixed point O. The velocity, v m/s, at time t seconds, is given by $v = 3t^2 - 5t + 7$ for $0 \leq t \leq 5$.

(i) Find the acceleration of the particle when $t = 4$. [2]

After $t > 5$ seconds, the particle travels at a velocity, in m/s, where $v = -4t + 77$.

(ii) Find the total distance travelled by the particle in the first 30 seconds. [7]

9



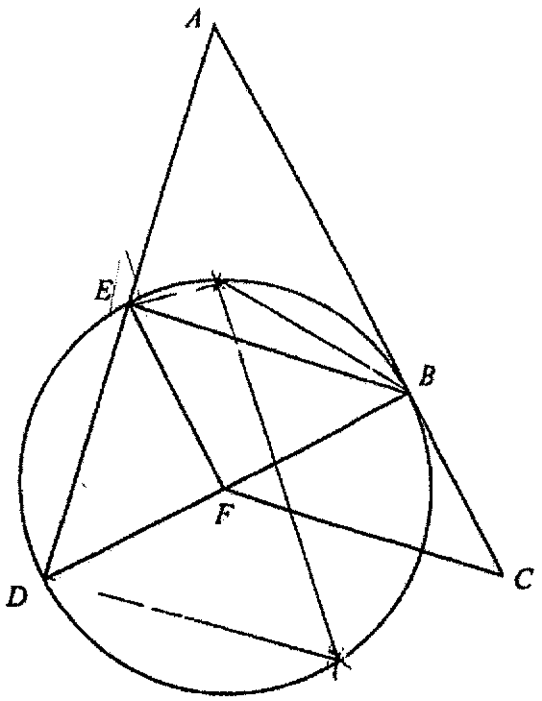
The diagram shows a metal structure $ABCD$ consisting of five metal rods of different lengths. The length of BC and CD are 15 m and 8 m respectively. Angle $ADC = \theta$ for $0° < \theta < 90°$.

(a) Show that the total lengths, P m, of the five metal rods used is $40 + 23\sin\theta + 7\cos\theta$. [3]

(b) Express P in the form $40 + R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [2]

(c) Find the value of θ if the total length of the five metal rods is 60 m. [3]

(d) State the minimum value of $\frac{1}{40+(R\sin(\theta+\alpha))^2}$ and the corresponding value of θ for which it occurs. [3]

10 

The diagram shows a circle, centre *F*, with *BD* as diameter. The point *E* lies on the circle. The tangent at a point *B* on the circle meets *DE* extended at the point *A*. Point *C* lies on *AB* extended and *ED* = *AE*.

(a) State the relationship between the length of *DF* and *AB*. Give reasons to support your answer. [2]

(b) Prove that

(i) triangle ABE is similar to triangle EDF, [2]

(ii) $AD^2 - BD^2 = 2 \times BE \times ED$. [3]

Point G is on minor arc EB such that BG bisects angle EBA.
Point H is on the circle such that $EGHD$ is a cyclic quadrilateral.

(c) Prove that angle $GHD = 90° -$ angle GBE. [3]

-End of Paper-

2024 NCHS Prelim AM Paper 2

Student Solutions

Qn	Solution

1(a)

$$T_{r+1} = \binom{n}{r}(5x^2)^{n-r}\left(-\frac{1}{\sqrt{x}}\right)^r$$
$$= \binom{n}{r}(5)^{n-r}(-1)^r x^{2(n-r)}x^{-\frac{1}{2}r}$$
$$= \binom{n}{r}(5)^{n-r}(-1)^r x^{2n-\frac{5}{2}r}$$

Since there is a term independent of x,

$$x^{2n-\frac{5}{2}r} = x^0$$
$$2n - \frac{5}{2}r = 0$$
$$n = \frac{5}{4}r$$

Since both n and r are positive integers,
Smallest $n = 5$, when $r = 4$.

1(b)

For the term $\left(5x^2 - \frac{1}{\sqrt{x}}\right)^n$,

$$T_{r+1} = \binom{n}{r}(5)^{n-r}(-1)^r x^{2n-\frac{5}{2}r}$$

For constant, $r = 4$,

$$\text{Constant} = \binom{5}{4}(5)^{5-4}(-1)^4$$
$$= 25$$

For x^5,

$$x^{2n-\frac{5}{2}r} = x^5$$
$$2n - \frac{5}{2}r = 5$$
$$r = 2$$

$$\text{Coefficient of } x^5 = \binom{5}{2}(5)^{5-2}(-1)^2$$
$$= 1250$$

Term independent of x

$$= (1)(125) + \left(-\frac{1}{50}\right)(1250)$$
$$= 0$$

Hence, there are no term independent of x.

2

Method 1

$$f'(x) = e^{5x} + 2\sin^2 x$$
$$= e^{5x} + 2(\sin x)^2$$
$$= e^{5x} + 1 - \cos 2x$$

$$f''(x) = \frac{d}{dx}(f'(x))$$
$$= \frac{d}{dx}(e^{5x} + 2(\sin x)^2)$$
$$= 5e^{5x} + 4\sin x\cos x$$
$$= 5e^{5x} + 2\sin 2x$$

$$f(x) = \int f'(x)\,dx$$
$$= \int e^{5x} + 1 - \cos 2x\,dx$$
$$= \frac{e^{5x}}{5} + x - \frac{\sin 2x}{2} + c$$

Given

$$10f(x) + 3f'(x) - f''(x) + 7\sin 2x + 3\cos 2x = 10x + 43$$

$$10\left(\frac{e^{5x}}{5} + x - \frac{\sin 2x}{2} + c\right) + 3(e^{5x} + 2\sin^2 x) - (5e^{5x} + 2\sin 2x) + 7\sin 2x + 3\cos 2x = 10x + 43$$

$$2e^{5x} + 10x - 5\sin 2x + 10c + 3e^{5x} + 6\sin^2 x - 5e^{5x} - 2\sin 2x + 7\sin 2x + 3\cos 2x = 10x + 43$$

$$10c + 6\sin^2 x + 3\cos 2x = 43$$
$$10c + 3(1 - \cos 2x) + 3\cos 2x = 43$$
$$c = 4$$

$$\therefore f(x) = \frac{e^{5x}}{5} + x - \frac{\sin 2x}{2} + 4$$

Method 2

$$f'(x) = e^{5x} + 2\sin^2 x$$

$$f''(x) = \frac{d}{dx}(f'(x))$$
$$= \frac{d}{dx}(e^{5x} + 2(\sin x)^2)$$
$$= 5e^{5x} + 4\sin x\cos x$$
$$= 5e^{5x} + 2\sin 2x$$

Given

$$10f(x) + 3f'(x) - f''(x) + 7\sin 2x + 3\cos 2x = 10x + 43$$

$$10f(x) + 3(e^{5x} + 2\sin^2 x) - (5e^{5x} + 2\sin 2x) + 7\sin 2x + 3\cos 2x = 10x + 43$$

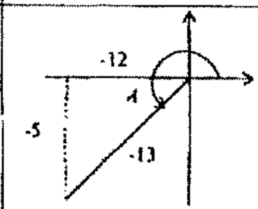
$$10f(x) + 3e^{5x} + 6\sin^2 x - 5e^{5x} - 2\sin 2x + 7\sin 2x + 3\cos 2x = 10x + 43$$

$$10f(x) - 2e^{5x} + 5\sin 2x + 3(1 - \cos 2x) + \cos 2x = 10x + 43$$

$$10f(x) - 2e^{5x} + 5\sin 2x + 3 - 3\cos 2x + \cos 2x = 10x + 43$$

$$10f(x) = 2e^{5x} - 5\sin 2x + 10x + 40$$
$$f(x) = \frac{e^{5x}}{5} + x - \frac{\sin 2x}{2} + 4$$

3(a)



$$\sin A = -\frac{5}{13}$$
$$\cos A = -\frac{12}{13}$$

Method 1

$$\cos 3A = \cos(2A + A)$$
$$= \cos 2A\cos A - \sin 2A\sin A$$
$$= (1 - 2\sin^2 A)\cos A - (2\sin A\cos A)\sin A$$
$$= \left(1 - 2\left(-\frac{5}{13}\right)^2\right)\left(-\frac{12}{13}\right) - \left(2\left(-\frac{5}{13}\right)\left(-\frac{12}{13}\right)\right)\left(-\frac{5}{13}\right)$$
$$= -\frac{828}{2197}$$

Method 2

$$\cos 3A = \cos(2A + A)$$
$$= \cos 2A\cos A - \sin 2A\sin A$$
$$= (2\cos^2 A - 1)\cos A - (2\sin A\cos A)\sin A$$
$$= \left(2\left(-\frac{12}{13}\right)^2 - 1\right)\left(-\frac{12}{13}\right) - \left(2\left(-\frac{5}{13}\right)\left(-\frac{12}{13}\right)\right)\left(-\frac{5}{13}\right)$$
$$= -\frac{828}{2197}$$

3(b)

$$\tan A = \frac{2\tan\frac{A}{2}}{1 - \tan^2\frac{A}{2}}$$
$$\frac{5}{12} = \frac{2\tan\frac{A}{2}}{1 - \tan^2\frac{A}{2}}$$
$$5 - 5\tan^2\frac{A}{2} = 24\tan\frac{A}{2}$$
$$0 = 5\tan^2\frac{A}{2} + 24\tan\frac{A}{2} - 5$$
$$\tan\frac{A}{2} = \frac{-(24) \pm \sqrt{(24)^2 - 4(5)(-5)}}{2(5)}$$
$$= -5 \quad \text{or} \quad \frac{1}{5}$$

Since $\pi < A < \frac{3\pi}{2}$,

$\frac{\pi}{2} < \frac{A}{2} < \frac{3\pi}{4}$ (2nd Quadrant),

$\therefore \tan\frac{A}{2} = -5$

4(a)

$$\text{Gradient} = \frac{0.88 - 2.69}{20 - 5} = -\frac{181}{1500}$$

$$Y - 2.69 = -\frac{181}{1500}(X - 5)$$

$$Y = -\frac{181}{1500}X + \frac{247}{75}$$

$$\ln y = -\frac{181}{1500}t + \frac{247}{75}$$

$$y = e^{-\frac{181}{1500}t + \frac{247}{75}}$$

$$\therefore k = -\frac{181}{1500},\ c = \frac{247}{75}$$

4(b)

Initial Temperature

When $t = 0$,

$$y = e^{\frac{247}{75}}$$

$$\text{Initial Temperature} = 25 + e^{\frac{247}{75}} = 51.9325^\circ C$$

Temperature to drop to half

$$y = \frac{51.9325}{2} - 25 = 0.966245$$

When $y = 0.966245$,

$$\ln(0.966245) = -\frac{181}{1500}t + \frac{247}{75}$$

$$\therefore t = 27.577$$

$$\approx 27.6 \text{ min (to 3sf)}$$

4(c)

$$e^{(k+0.2)t} = e^{5-c}$$

$$e^{kt}e^{0.2t} = \frac{e^5}{e^c}$$

$$e^{kt}e^{c} = \frac{e^5}{e^{0.2t}}$$

$$e^{kt+c} = e^{5-0.2t}$$

Tumbler Model

$$y = e^{5-0.2t}$$

$$\ln y = 5 - 0.2t$$

Step 1:

Draw a straight-line graph in (a), where the gradient of the straight line is -0.2 and the ln y-intercept is 5.

Step 2:

The intersection between the 2 straight line graphs will be the timing where the temperatures are equivalent.

5(a)

Consider ΔP,

$$\text{hyp} = \sqrt{h^2 + h^2} = \sqrt{2}h$$

Consider ΔZ,

$$\text{Side} = \frac{80 - \sqrt{2}h}{2}$$

$$= 2\left(\frac{80 - \sqrt{2}h}{2}\right)^2$$

$$= \frac{(80 - \sqrt{2}h)^2}{2}$$

$$\text{base} = \text{hyp} = \frac{80 - \sqrt{2}h}{\sqrt{2}}$$

$\therefore$ Total exterior area, A

$= 4 \times \text{base} \times \text{height} + \text{base} \times \text{base}$

$$= 4h\left(\frac{80 - \sqrt{2}h}{\sqrt{2}}\right) + \left(\frac{80 - \sqrt{2}h}{\sqrt{2}}\right)^2$$

$$= \frac{320}{\sqrt{2}}h - 4h^2 + \frac{6400 - 160\sqrt{2}h + 2h^2}{2}$$

$$= \frac{320\sqrt{2}}{2}h - 4h^2 + 3200 - 80\sqrt{2}h + h^2$$

$$= 3200 + 80\sqrt{2}h - 3h^2$$

Method 2

Consider ΔZ,

$$\text{Side} = \frac{80 - \sqrt{2}h}{2}$$

$\therefore$ Total exterior area, A

$= 80 \times 80 - 4 \times \text{small triangle} - 4 \times \text{big triangle}$

$$= 6400 - 4 \times \left(\frac{1}{2}\right)(\sqrt{2}h)^2 - 4 \times \left(\frac{1}{2}\right)\left(\frac{80 - \sqrt{2}h}{2}\right)^2$$

$$= 6400 - 4h^2 - \frac{6400 - 160\sqrt{2}h + 2h^2}{2}$$

$$= 6400 - 4h^2 - 3200 + 80\sqrt{2}h - h^2$$

$$= 3200 + 80\sqrt{2}h - 3h^2$$

5(b)

When the total exterior area is a maximum, $\frac{dA}{dh} = 0$.

$$A = 3200 + 80\sqrt{2}h - 3h^2$$

$$\frac{dA}{dh} = 80\sqrt{2} - 6h$$

$$0 = 80\sqrt{2} - 6h$$

$$h = \frac{80\sqrt{2}}{6} = \frac{40\sqrt{2}}{3} \text{ cm}$$

$$\frac{d^2A}{dh^2} = -6 \ (< 0)$$

$\therefore A$ is maximum.

5(c)

When $h = \frac{40\sqrt{2}}{3}$,

$$A = 3200 + 80\sqrt{2}\left(\frac{40\sqrt{2}}{3}\right) - 3\left(\frac{40\sqrt{2}}{3}\right)^2$$

$$= 4266\frac{2}{3} \text{ cm}^2$$

6

$$y = 3 - 5\cos 2x$$

Point N

When $y = 0$,

$$3 - 5\cos 2x = 0$$

$$\cos 2x = \frac{3}{5}$$

$$\alpha = \cos^{-1}\frac{3}{5} = 0.927295$$

$2x$ lies in 1[st] (rej)/ 4[th] quad.

$$2x = 5.35589$$

$$x = 2.67795$$

Point P

$$y = 3 - 5\cos 2x$$

$$\frac{dy}{dx} = -5(-\sin 2x)(2) = 10\sin 2x$$

When $\frac{dy}{dx} = -5$,

$$10\sin 2x = -5$$

$$\sin 2x = -\frac{1}{2}$$

$$\alpha = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

$2x$ lies in 3[rd]/ 4[th] (rej) quad.

$$2x = \frac{7\pi}{6}$$

$$x = \frac{7\pi}{12}$$

When $x = \frac{7\pi}{12}$,

$$y = 3 - 5\cos 2\left(\frac{7\pi}{12}\right) = 7.330127$$

$$\therefore P\left(\frac{7\pi}{12}, 7.330127\right)$$

Equation PQ, Point Q

$$y - 7.330127 = -5\left(x - \frac{7\pi}{12}\right)$$

$$y = -5x + 16.4931$$

When $y = 0$,

$$-5x + 16.4931 = 0$$

$$x = 3.29862$$

$$\therefore Q(3.29862, 0)$$

Area below the curve from x_P to x_N

$$\text{area} = \int_{\frac{7\pi}{12}}^{2.67795} 3 - 5\cos 2x \, dx$$

$$= \left[3x - \frac{5\sin 2x}{2}\right]_{\frac{7\pi}{12}}^{2.67795}$$

$$= 10.0338 - 6.74779$$

$$= 3.28601 \text{ units}^2$$

$\therefore$ shaded area

$=$ area of triangle $-$ area below curve

$$= \frac{1}{2}(7.330127)\left(3.29862 - \frac{7\pi}{12}\right) - 3.28601$$

$$= 5.37307 - 3.28601$$

$$= 2.0871$$

$$= 2.09 \text{ units}^2$$

7(a) To prove $(x+2y)$ is a factor:

Method 1

Let $f(x) = 4x^3 + x^2y - 11xy^2 + 6y^3$,

$f(-2y) = 4(-2y)^3 + (-2y)^2y - 11(-2y)y^2 + 6y^3$

$= -32y^3 + 4y^3 + 22y^3 + 6y^3$

$= 0$

Since f(−2y), by factor theorem, $(x+2y)$ is a factor.

Method 2

$$\begin{array}{r} 4x^2 - 7yx + 3y^2 \\ x + 2y \,\overline{)\, 4x^3 + x^2y - 11xy^2 + 6y^3} \\ \ldots \\ 3y^2x + 6y^3 \\ -(3y^2x + 6y^3) \\ \hline 0 \end{array}$$

Since the remainder is 0, $(x+2y)$ is a factor.

Hence factorise:

Method 1 – Compare coefficients

$f(x) = (x+2y)(4x^2 + bx \ \ldots)$

Comparing coefficients of,

x ... $-7y$

... Division

As shown earlier

$\therefore f(x) = (x+2y)(4x^2 - 7yx + 3y^2)$

$= (x+2y)(x-y)(4x-3y)$

7(b) $4^{p+1} + 2^{p+1} = 44 - 48(2^{-p})$

$2^{2(p+1)} + 2^{p+1} = 44 - \frac{48}{2^p}$

$2^{2p}2^2 + 2^p 2 = 44 - \frac{48}{2^p}$

Let $u = 2^p$,

$4u^2 + 2u = 44 - \frac{48}{u}$

$(\times u)$

$4u^3 + 2u^2 - 44u + 48 = 0$

From (a),

$48 = 6y^3$

$y^3 = 8$

$y = 2$

From (a), $x = u$

From (a),

$f(x) = (x+2y)(x-y)(4x-3y)$

$f(x) = (x+4)(x-2)(4x-6) = 0$

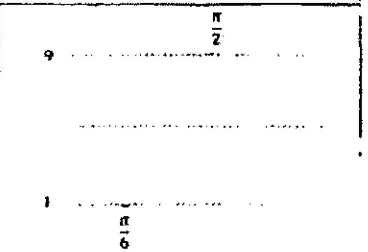
$x = -4$ $x = 2$ $x = \frac{3}{2}$

$2^p = -4$ (rej) $2^p = 2$, $\therefore p = 1$ $2^p = \frac{3}{2}$

$\therefore p = \frac{\lg\frac{3}{2}}{\lg 2}$

$= 0.585$ (to 3sf)

8(a)



$y = a\sin bx + c$		
$c = \frac{9+1}{2} = 5$	$\frac{1}{2}$ period $= \frac{\pi}{2} - \frac{\pi}{6}$ period $= \frac{2\pi}{3}$ $\frac{2\pi}{b} = \frac{2\pi}{3}$ $b = 3$	Amplitude $= 9 - 5 = 4$ $a = -4$

8(b)(i) $a = \frac{dv}{dt}$

$= 6t - 5$

When $t = 4$,

$a = 6(4) - 5$

$= 19\ m/s^2$

8(b)(ii) For $0 \le t \le 5$

$v = 3t^2 - 5t + 7$

Displacement expression:

$s = \int 3t^2 - 5t + 7\ dt$

$= \frac{3t^3}{3} - \frac{5t^2}{2} + 7t + c$

When $t = 0, s = 0$,

$c = 0$

$\therefore s = \frac{3t^3}{3} - \frac{5t^2}{2} + 7t$

Turning point:

When $v = 0$,

$3t^2 - 5t + 7 = 0$

$b^2 - 4ac = (-5)^2 - 4(3)(7)$

$= -59\ (< 0)$

Since $b^2 - 4ac < 0$, there is no turning point.

When $t = 0$, $s = 0$ m

$t = 5$, $s = 97.5$ m

For $t > 5$,

$v = -4t + 77$

Displacement expression:

$s = \int -4t + 77\ dt$

$= -\frac{4t^2}{2} + 77t + d$

When $t = 5, s = 97.5$,

$97.5 = -2(5)^2 + 77(5) + d$

$d = -\frac{475}{2}$

$\therefore s = -2t^2 + 77t - \frac{475}{2}$

Turning point:

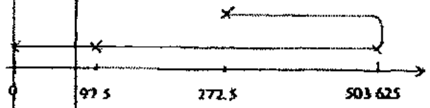
When $v = 0$,

$-4t + 77 = 0$

$t = \frac{77}{4} = 19\frac{1}{4}$

When $t = 19\frac{1}{4}$, $s = 503.625$ m

$t = 30$, $s = 272.5$ m



$\therefore$ total distance

$= 503.625 + (503.625 - 272.5)$

$= 734.75$ m

9(a) Consider $\Delta ABCD$, using Pythagoras theorem,

$BD = \sqrt{8^2 + 15^2} = 17$

Consider ΔBXC,

$CX = 8\cos\theta$

$BX = 8\sin\theta$

Consider ΔCYD,

$CY = 15\sin\theta$

$YD = 15\cos\theta$

Total lengths

$= AB + 8 + 15 + AD + BD$

$= (CY - CX) + 8 + 15 + (BX + YD) + BD$

$= 15\sin\theta - 8\cos\theta + 8 + 15 + 8\sin\theta + 15\cos\theta + 17$

$= 40 + 23\sin\theta + 7\cos\theta$ (shown)

9(b) $P = 40 + 23\sin\theta + 7\cos\theta$

$= 40 + R\sin(\theta + \alpha)$

$R = \sqrt{23^2 + 7^2}$

$= \sqrt{578}$

$= 24.04163 = 24.0$ (to 3sf)

	$\alpha = \tan^{-1}\frac{7}{23} = 16.9275°$ $= 16.9°$ (to 1 dp) $\therefore P = 40 + 24.0\sin(\theta + 16.9°)$
9(c)	When $P = 60$, $40 + 24.04163\sin(\theta + 16.9275°)$ $= 60$ $\sin(\theta + 16.9275°)$ $= 0.83189$ $\alpha = \sin^{-1} 0.83189$ $= 56.2934°$ $(\theta + 16.9275°)$ lies in $1^{st}/2^{nd}$ (rej) quad. $\theta + 16.9275° = 56.2934°$ $\theta = 39.3659°$ $= 39.4°$ (to 1 dp)
9(d)	$\max(R\sin(\theta + \alpha))^2 = (\sqrt{578})^2 = 578$ $\therefore \min \frac{1}{40 + (R\sin(\theta + \alpha))^2}$ $= \frac{1}{40 + 578} = \frac{1}{618}$ $\theta + 16.9275° = 90°$ $\theta = 73.0725°$ $= 73.1°$ (to 1 dp)
10(a)	Since E is the midpoint of AD (given), F is the midpoint of DB (given), By midpoint theorem, $FE = \frac{1}{2}AB$
10(b) (i)	$\angle ABE$ $= \angle EDF$ (alternate segment theorem) From (a), by midpoint theorem, $EF \parallel AB$ $\angle DEF$ $= \angle BAE$ (corresponding angles, $EF \parallel AB$)

	$\therefore \Delta ABE$ is similar to ΔEDF (AA similari
10(b) (ii)	From (i), $\frac{AB}{ED}$ $= \frac{BE}{DF}$ (corresponding sides of similar $AB \times DF = BE \times ED$ $AB \times \frac{1}{2}AB = BE \times ED$ $AB^2 = 2 \times BE \times ED$ Since $\angle DBA = 90°$ (tangent $\perp$ radius), Using Pythagoras' theorem, $AD^2 - BD^2 = 2 \times BE \times ED$
10(c)	<u>Method 1</u> Let $\angle GBE$ be α. $\angle GBA = \alpha$ (BG bisects $\angle EBA$) $\angle GEB$ $= \alpha$ (alternate segment theorem) $\angle DEB = 90°$ (rt $\angle$ in a semicircle) $\angle GHD$ $= 180°$ $- (90° + \alpha)$ ($\angle$ in opp segment) $= 90° - \alpha$ $= 90° - \angle GBE$ <u>Method 2</u> Let $\angle GBE$ be α. $\angle GBA = \alpha$ (BG bisects $\angle EBA$) $\angle DBA = 90°$ (tangent $\perp$ radius) $\angle GBD = 90° - \alpha$ $\angle GHD$ $= \angle GBD$ ($\angle$ in the same segment) $= 90° - \alpha$ $= 90° - \angle GBE$