Calculator Model: ______________

NAME:	CLASS:	INDEX NO:

QUEENSWAY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXPRESS /
SECONDARY 5 NORMAL ACADEMIC

Parent's Signature:

ADDITIONAL MATHEMATICS

Paper 2

4049/02
27 August 2024

2 hour 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Gives answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

This document consists of **18** printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \cdots + \binom{n}{r} a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = cos^2 A - sin^2 A = 2cos^2 A - 1 = 1 - 2sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The Singapore government issued a savings bond in January 2024 with a yield of 2.75% per year. Mr Tan invested \$15 000 in the bond. The total amount he will receive, after t years, is given by $A = 15000(1.0275)^t$.

(a) Calculate the total amount he will receive in January 2030, correct to the nearest dollar. [2]

(b) In which year will the amount first exceed \$22 000? [2]

2 (a) Without using a calculator, evaluate the value of 6^x given that $2^{2x+6} \times 3^{5x-1} = 27^{x+1}$. [5]

(b) Solve the equation $log_x 9 = 5log_3 x$, giving your answers correct to 2 significant figures. [4]

3 (a) A curve has the equation $y = (p-1)x^2 + 2(p-3)$, where p is a constant. A line has the equation $y = 6x + 3$. Find the range of values of p if the curve lies completely above the line.

[5]

(b) By expressing $y = -2x^2 + 10x - 5$ in the form $y = a(x+b)^2 + c$, where a, b and c are constants, find the maximum value of y.

[3]

4 (a) In the binomial expansion of $\left(1 - \frac{2}{7}x\right)^n$, the sum of the coefficients of the second and third terms is zero. Calculate the value of n and hence, find the sixth term.

[4]

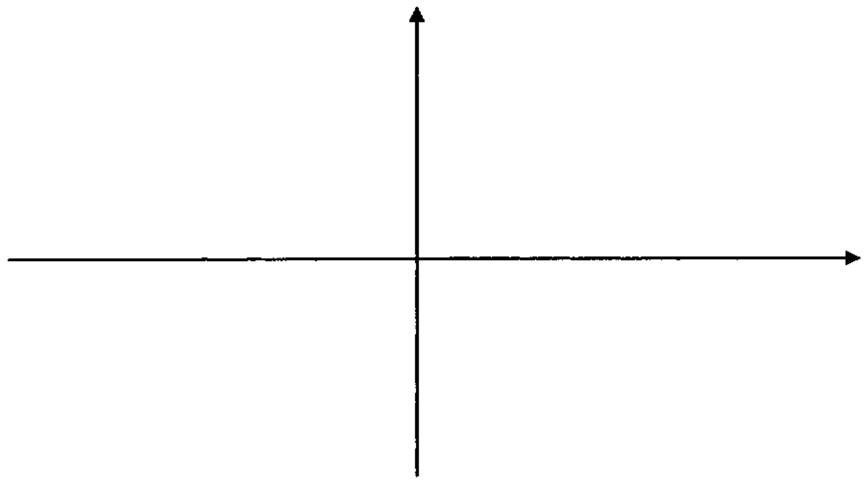
(b) Write down the general term in the binomial expansion of $\left(\frac{1}{x^3} - 2x\right)^8$.
Hence, find the value of the constant term in the expansion of
$\left(3 + \frac{x^2}{2}\right)^2 \left(\frac{1}{x^3} - 2x\right)^8$.

[6]

5 (a) The function f is defined as $f(x) = p - q\sin(rx)$, for $-\pi \leq x \leq \pi$, where p, q and r are positive integers. Given that the amplitude of the function is 6, the period is π and the maximum value is 9.

(i) State the values of p, q and r. [3]

(ii) Hence, sketch the graph of $f(x)$. [2]



(b) The acute angles A and B are such that $\cot(A - B) = \frac{1}{3}$ and $\cot A = \frac{1}{5}$.
Without using a calculator, find the exact value of $\cos B$. [5]

6 A circle passes through the points $(-5, 12)$ and $(9, 14)$. The centre of the circle lies on the line $2y + x = 15$.

(a) Find the equation of the circle. [7]

(b) Explain why the line $y = mx + 6$ intersects the circle at 2 distinct points for all values of m.

[4]

7 (a) A curve has the equation $y = \frac{3x-5}{4x+1}$ for $x > 0$. Explain, with working, why the curve has no stationary points.

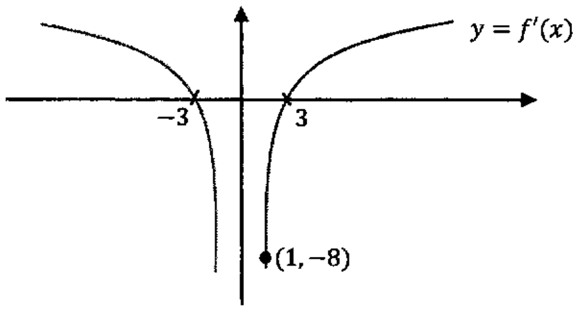
[3]

(b) Given that $f(x)$ is such that $f'(x) = \cos 4x - 3\sin 2x$ and $f(\pi) = 0$, show that $f''(x) + 4f(x) = -3(\sin 4x + 2)$.

[4]

(c) Given that $\frac{d}{dx}\left(\frac{2-x}{\sqrt{1-2x}}\right) = \frac{ax+b}{\sqrt{(1-2x)^3}}$, find the value of a and b. [4]

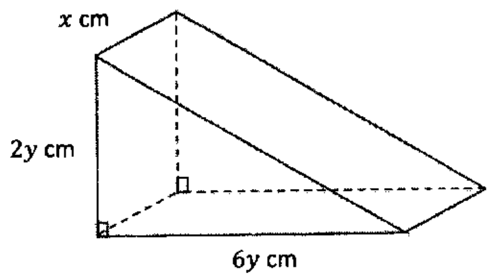
8 A curve $y = f(x)$ passes through the point $(1, 10)$.
The graph of $y = f'(x)$ is shown below.



(a) State the x-coordinates of the stationary points of the curve $y = f(x)$ and hence, determine their nature. [3]

(b) Find the equation of the normal to the curve at the point $(1, 10)$. [2]

9 The diagram below shows a wooden door stopper in the shape of a right prism with a volume of 60 cm^3. The cross-section of the prism is a triangle, with side lengths of $2y$ cm and $6y$ cm respectively, with a width of x cm.



(a) Express x in terms of y and show that the total surface area of the door stopper A, is given as $A = 12y^2 + \frac{20}{y}(\sqrt{10} + 4)$ cm^2.

[3]

(b) Given that y can vary, find the value of y for which A has a minimum value. [3]

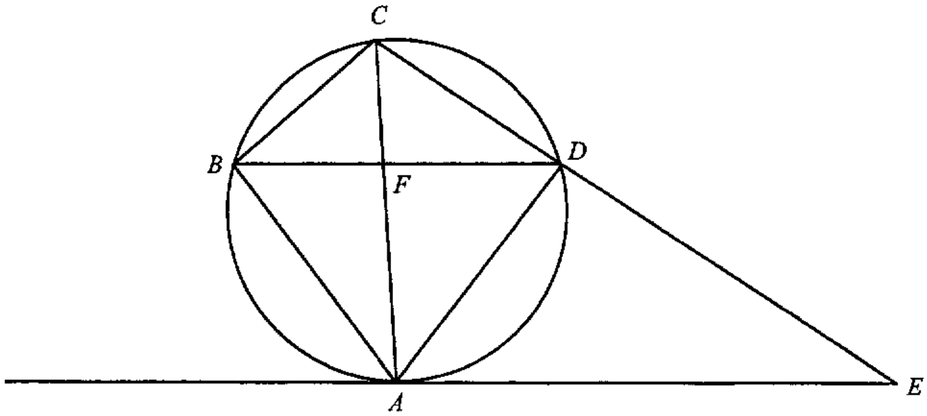
10 (a) Find all angles between 0 and 2π which satisfy $3\cos 2x + 4\sin x = 3$. [4]

(b) Without using a calculator,

(i) show that $\cos\frac{7\pi}{12} = \frac{\sqrt{2}-\sqrt{6}}{4}$. [3]

(ii) and hence, find the exact value of $\sin^2\frac{7\pi}{12}$. [3]

11



Points A, B, C and D are inscribed in a circle such that $AB = AD$. A tangent to the circle at point A meets the line CD produced at E. The lines AC and BD intersects at point F.

(a) Prove that the line BFD is parallel to line AE. [3]

(b) Show that triangle ABC is similar to triangle EDA. [3]

--- End of Paper ---

4E AM Prelim 2024 P2 Solutions

1.		The Singapore government issued a savings bond in January 2024 with a yield of 2.75% per year. Mr Tan invested \$15 000 in the bond. The total amount he will receive, after t years, is given by $A = 15000(1.0275)^t$.
	(a)	Calculate the total amount he will receive in January 2030, correct to the nearest dollar.
		$A = 15000(1.0275)^6$ $= 17651.52$ $Amount = \$\ 17652$
	(b)	In which year will the amount first exceed \$22 000?
		$22000 = 15000(1.0275)^t$ $1.0275^t = \frac{22}{15}$ $t = \ln\left(\frac{22}{15}\right) / \ln 1.0275$ $t = 14.117$ Year 2039.

2.	(a)	Without using a calculator, evaluate the value of 6^x given that $2^{2x+6} \times 3^{5x-1} = 27^{x+1}$
		$$2^{2x+6} \times 3^{5x-1} = 3^{3x+3}$$ $$2^{2x} \times 2^6 \times 3^{5x} \times 3^{-1} = 3^{3x} \times 3^3$$ $$\frac{2^{2x} \times 3^{5x}}{3^{3x}} = \frac{3^3}{2^6 \times 3^{-1}}$$ $$2^{2x} \times 3^{2x} = \frac{3^4}{2^6}$$ $$6^{2x} = \frac{81}{64}$$ $$6^x = \frac{9}{8}$$
	(b)	Solve the equation $log_x 9 = 5log_3 x$, giving your answers correct to 2 significant figures.
		$$log_x 9 = 5log_3 x$$ $$\frac{log_3 9}{log_3 x} = 5log_3 x$$ $$2log_3 3 = 5(log_3 x)^2$$ $$(log_3 x)^2 = \frac{2}{5}$$ $$log_3 x = \pm\sqrt{\frac{2}{5}}$$ $$x = 3^{\sqrt{\frac{2}{5}}} \; or \; 3^{-\sqrt{\frac{2}{5}}}$$ $$x = 2.0 \; or \; 0.50$$

3. (a) A curve has the equation $y = (p-1)x^2 + 2(p-3)$, where p is a constant. A line has the equation $y = 6x + 3$. Find the range of values of p if the curve lies completely above the line.

$$(p-1)x^2 + 2p - 6 = 6x + 3$$

$$(p-1)x^2 - 6x + 2p - 9 = 0$$

$$b^2 - 4ac < 0$$

$$(-6)^2 - 4(p-1)(2p-9) < 0$$

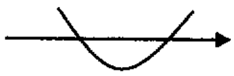
$$36 - (4p-4)(2p-9) < 0$$

$$36 - 8p^2 + 8p + 36p - 36 < 0$$

$$-8p^2 + 44p < 0$$

$$8p^2 - 44p > 0$$

$$4p(2p-11) > 0$$



$$p < 0 \; or \; p > 5.5$$

(rejected)

(b) By expressing $y = -2x^2 + 10x - 5$ in the form $y = a(x+b)^2 + c$, where a, b and c are constants, find the maximum value of y.

$$y = -2(x^2 - 5x) - 5$$

$$= -2\left[x^2 - 5x + \left(-\frac{5}{2}\right)^2 - \left(-\frac{5}{2}\right)^2\right] - 5$$

$$= -2[(x-2.5)^2 - 2.5^2] - 5$$

$$= -2(x-2.5)^2 + 7.5$$

Max value of $y = 7.5$

4.	(a)	In the binomial expansion of $\left(1-\frac{2}{7}x\right)^n$, the sum of the coefficient of the second and third term is zero. Calculate the value of n and hence, find the sixth term.

$$T_2 = \binom{n}{1}(1)^{n-1}\left(-\frac{2}{7}x\right)^1 = -\frac{2}{7}nx$$

$$T_3 = \binom{n}{2}(1)^{n-2}\left(-\frac{2}{7}x\right)^2 = \frac{n(n-1)}{2}\left(\frac{4}{49}x^2\right)$$

$$-\frac{2}{7}n + \frac{2n(n-1)}{49} = 0$$

$$-14n + 2n^2 - 2n = 0$$

$$2n^2 - 16n = 0$$

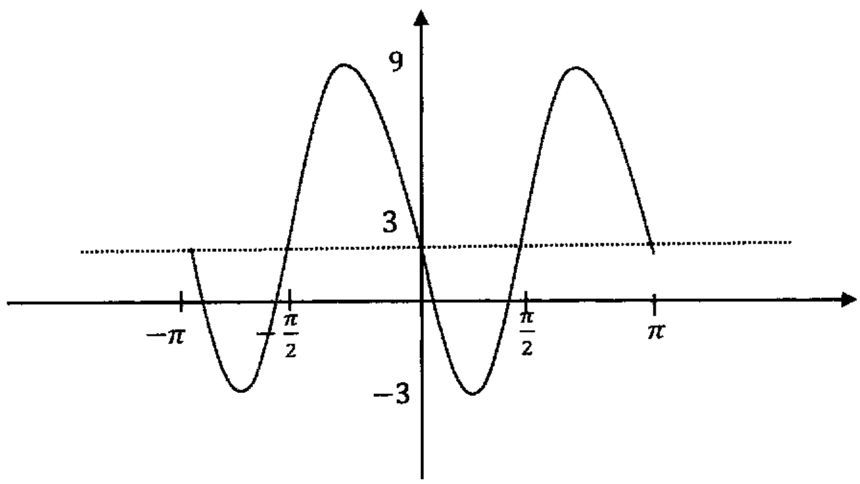
$$2n(n-8) = 0$$

$$n = 0 \quad or \quad n = 8$$

(rejected)

$$T_6 = \binom{8}{5}(1)^{8-5}\left(-\frac{2}{7}x\right)^5 = -\frac{256}{2401}x^5$$

	(b)	Write down the general term in the binomial expansion of $\left(\frac{1}{x^3} - 2x\right)^8$. Hence, find the value of the constant term in the expansion of $\left(3 + \frac{x^2}{2}\right)^2 \left(\frac{1}{x^3} - 2x\right)^8$.
		$T_{r+1} = \binom{8}{r}\left(\frac{1}{x^3}\right)^{8-r} (-2x)^r$ $= \binom{8}{r} x^{-24+3r}(-2)^r x^r$ $= \binom{8}{r}(-2)^r x^{4r-24}$ $\left(3 + \frac{x^2}{2}\right)^2 = 9 + 3x^2 + \frac{x^4}{4}$ $x^{4r-24} = x^0 \quad or\ x^{4r-24} = x^{-2} \quad or\ x^{4r-24} = x^{-4}$ $r = 6 \quad or\ no\ integer\ value \quad or \quad r = 5$ $T_{7=} \binom{8}{6}(-2)^6 = 1792$ $T_{6=} \binom{8}{5}(-2)^5 = -1792$ Constant term = $(1792 \times 9) + \left(\frac{1}{4} \times -1792\right) = 15680$

5.	(a)	The function f is defined as $f(x) = p - q\sin(rx)$, for $-\pi \le x \le \pi$, where p, q and r are positive integers. Given that the amplitude of the function is 6, the period is π and the maximum value of is 9.
		(i) State the values of p, q and r.
		$p = 3$ $q = 6$ $r = 2$
		(ii) Hence, sketch the graph of $f(x)$.
		 - Shape of curve and range - Points plotted correctly

	(b)	The acute angles A and B are such that $\cot(A-B)=\frac{1}{3}$ and $\cot A=\frac{1}{5}$. Without using a calculator, find the exact value of $\cos B$.

$$\cot A=\frac{1}{5}$$

$$\tan A=5$$

$$\cot(A-B)=\frac{1}{3}$$

$$\tan(A-B)=3$$

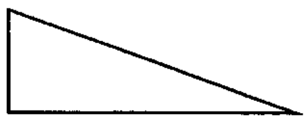
$$\frac{\tan A-\tan B}{1+\tan A\tan B}=3$$

$$\tan A-\tan B=3+3\tan A\tan B$$

$$5-\tan B=3+15\tan B$$

$$16\tan B=2$$

$$\tan B=\frac{1}{8}$$



$$\cos B=\frac{8}{\sqrt{65}}=\frac{8\sqrt{65}}{65}$$

6.		A circle passes through the points $(-5, 12)$ and $(9, 14)$. The centre of the circle lies on the line $2y + x = 15$.
	(a)	Find the equation of the circle.
		$m = \frac{14-12}{9-(-5)} = \frac{1}{7}$ $m_{\perp} = -7$ $midpoint = \left(\frac{-5+9}{2}, \frac{12+14}{2}\right) = (2, 13)$ $13 = -7(2) + c$ $c = 27$ Eqn of perpendicular bisector $y = -7x + 27$ $2(-7x + 27) + x = 15$ $-14x + 54 + x = 15$ $-13x = -39$ $x = 3$ $y = 6$ Centre of circle (3,6) Radius = $\sqrt{(9-3)^2 + (14-6)^2} = 10\ units$ Equation of circle $(x-3)^2 + (y-6)^2 = 100$ Or $x^2 - 6x + y^2 - 12y - 55 = 0$

	(b)	Explain why the line $y = mx + 6$ intersects the circle at 2 distinct points for all values of m.
		$(x-3)^2 + (mx+6-6)^2 = 100$ $x^2 - 6x + 9 + m^2x^2 - 100 = 0$ $(1+m^2)x^2 - 6x - 91 = 0$ $b^2 - 4ac = (-6)^2 - 4(1+m^2)(-91)$ $= 400 + 364m^2$ $400 + 364m^2 > 0$ for all values of m $\therefore$line cuts circle at 2 distinct points
7.	(a)	A curve has the equation $y = \frac{3x-5}{4x+1}$ for $x > 0$. Explain, with working, why the curve has no stationary points.
		$\frac{dy}{dx} = \frac{3(4x+1) - 4(3x-5)}{(4x+1)^2}$ $= \frac{12x + 3 - 12x + 20}{(4x+1)^2}$ $= \frac{23}{(4x+1)^2}$ Since $(4x+1)^2 \geq 0$ for all x, $\frac{dy}{dx} > 0$ Curve has no stationary point as $\frac{dy}{dx} \neq 0$.

(b) It is given that $f(x)$ is such that $f'(x) = \cos 4x - 3\sin 2x$. Given also that $f(\pi) = 0$, show that $f''(x) + 4f(x) = -3(\sin 4x + 2)$.

$$f(x) = \frac{1}{4}sin4x + \frac{3}{2}cos2x + c$$

$$f(\pi) = 0$$

$$\frac{3}{2} + c = 0$$

$$c = -\frac{3}{2}$$

$$f(x) = \frac{1}{4}\sin 4x + \frac{3}{2}\cos 2x - \frac{3}{2}$$

$$f''(x) = -4\sin 4x - 6\cos 2x$$

$$f''(x) + 4f(x) = -4\sin 4x - 6\cos 2x + 4\left(\frac{1}{4}\sin 4x + \frac{3}{2}\cos 2x - \frac{3}{2}\right)$$

$$= -4\sin 4x - 6\cos 2x + \sin 4x + 6\cos 2x - 6$$

$$= -3\sin 4x - 6$$

$$= -3(\sin 4x + 2)\ (shown)$$

(c) Given that $\frac{d}{dx}\left(\frac{2-x}{\sqrt{1-2x}}\right) = \frac{ax+b}{\sqrt{(1-2x)^3}}$, find the value of a and b.

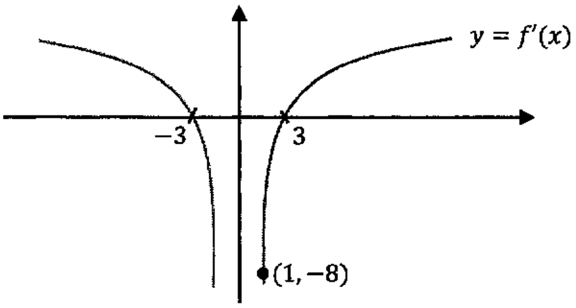
$$\frac{d}{dx}\left(\frac{2-x}{\sqrt{1-2x}}\right) = \frac{(1-2x)^{\frac{1}{2}}(-1) - (2-x)\left(\frac{1}{2}\right)(1-2x)^{-\frac{1}{2}}(-2)}{(1-2x)}$$

$$= \frac{(1-2x)^{-\frac{1}{2}}[(-1)(1-2x) - (2-x)\left(\frac{1}{2}\right)(-2)}{(1-2x)}$$

$$= \frac{(1-2x)^{-\frac{1}{2}}[-1 + 2x + 2 - x]}{(1-2x)}$$

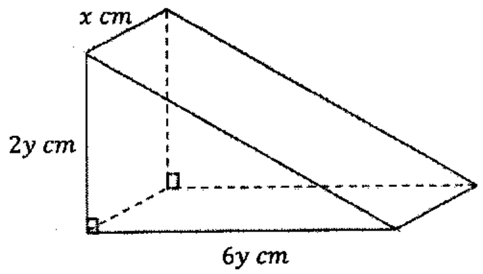
$$= \frac{1+x}{(1-2x)^{\frac{3}{2}}}$$

$$a = 1,\ b = 1$$

8.		A curve $y = f(x)$ passes through the point $(1, 10)$.
		
		The graph shown above is $y = f'(x)$.
	(a)	State the x-coordinates of the stationary points of the curve $y = f(x)$ and hence, determine their nature.
		$x = -3$ *and* $x = 3$
	(b)	Find the equation of the normal to the curve at the point $(1, 10)$.
		$x = 1, f'(x) = -8$ Gradient of tangent = -8 Gradient of normal = $\frac{1}{8}$ $10 = \frac{1}{8}(1) + c$ $c = \frac{79}{8}$ $y = \frac{1}{8}x + \frac{79}{8}$ or $8y = x + 79$

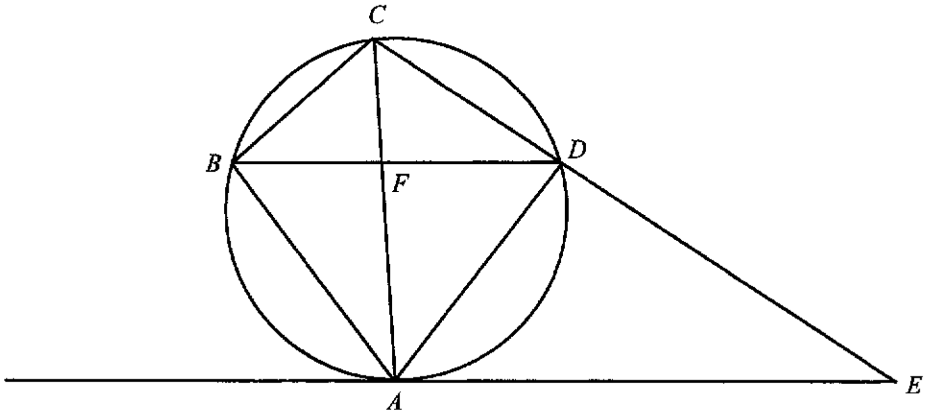
x	-3.1	-3	-2.9		2.9	3	3.1
$f'(x)$	+	0	−		−	0	+

Maximum point — Minimum point

9.		The diagram below shows a wooden door stopper in the shape of a right prism with a volume of 60 cm^3. The cross-section of the prism is a triangle, with side lengths of $2y$ and $6y$ cm respectively, with a width of x cm. 
	(a)	Express x in terms of y and show that the total surface area of the door stopper A, is given as $A = 12y^2 + \frac{20}{y}(\sqrt{10}+4)\ cm^2$.
		$\frac{1}{2} \times 2y \times 6y \times x = 60$ $x = \frac{10}{y^2}$ Total S.A $= 2\left(\frac{1}{2} \times 2y \times 6y\right) + 2xy + 6xy + \left(\sqrt{(2y)^2+(6y)^2}\right)x$ $= 12y^2 + 8xy + \left(\sqrt{40y^2}\right)x$ $= 12y^2 + 8y\left(\frac{10}{y^2}\right) + 2y\sqrt{10}\left(\frac{10}{y^2}\right)$ $= 12y^2 + \frac{80}{y} + \frac{20\sqrt{10}}{y}$ $= 12y^2 + \frac{20}{y}(\sqrt{10}+4)\ cm^2$

	(b)	Given that y can vary, find the value of y for which A has a minimum value.
		$$\frac{dA}{dy} = 24y - \left(\frac{20}{y^2}\right)(\sqrt{10}+4)$$ $$24y - \left(\frac{20}{y^2}\right)(\sqrt{10}+4) = 0$$ $$24y^3 = 20(\sqrt{10}+4)$$ $$y^3 = \frac{5}{6}(\sqrt{10}+4)$$ $$y = 1.8139$$ $$\frac{d^2A}{dy^2} = 24 + \frac{40}{y^3}(\sqrt{10}+4)$$ $$= 72 > 0$$ Value of A is at a minimum.
10.	(a)	Find all angles between 0 and 2π which satisfy $3\cos 2x + 4\sin x = 3$.
		$$3\cos 2x + 4\sin x = 3$$ $$3(1 - 2\sin^2 x) + 4sinx - 3 = 0$$ $$3 - 6\sin^2 x + 4sinx - 3 = 0$$ $$-6\sin^2 x + 4sinx = 0$$ $$-2\sin x(3\sin x - 2) = 0$$ $$-2\sin x = 0 \quad or \quad 3\sin x - 2 = 0$$ $$sinx = 0 \quad or \quad sinx = \frac{2}{3}$$ $$Basic\ angle = 0 \quad or\ 0.72972$$ $$x = \pi \quad or \quad x = 0.72972\ or\ 2.4118$$ $$x = 0.730, 2.41, \pi$$ (minus 1 mark for each wrong value)

	(b)	Without using a calculator,
	(i)	Show that $\cos\frac{7\pi}{12} = \frac{\sqrt{2}-\sqrt{6}}{4}$.
		$$\cos\frac{7\pi}{12} = \cos\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right)$$ $$= \cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$$ $$= \left(\cos\frac{\pi}{4}\right)\left(\cos\frac{\pi}{3}\right) - \left(\sin\frac{\pi}{4}\right)\left(\sin\frac{\pi}{3}\right)$$ $$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$$ $$= \frac{1-\sqrt{3}}{2\sqrt{2}}$$ $$= \frac{\sqrt{2}-\sqrt{6}}{4} \quad (shown)$$
	(ii)	Hence, find the exact value of $\sin^2\frac{7\pi}{12}$.
		$$\sin^2\frac{7\pi}{12} = 1 - \cos^2\frac{7\pi}{12}$$ $$= 1 - \left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)^2$$ $$= 1 - \frac{2-2\sqrt{2}\sqrt{6}+6}{16}$$ $$= \frac{16-(2-2\sqrt{12}+6)}{16}$$ $$= \frac{16-2+4\sqrt{3}-6}{16}$$ $$= \frac{8+4\sqrt{3}}{16}$$ $$= \frac{2+\sqrt{3}}{4}$$

12.		
		Points A, B, C and D is inscribed in a circle such that $AB = AD$. A tangent to the circle at point A meets the line CD produced at E. The lines AC and BD intersects at point F.
	(i)	Prove that the line BFD is parallel to line AE.
		$\angle ABD = \angle DAE(alt. seg. thm)$ $\angle ABD = \angle ADB\ (base\ angle\ of\ iso\ \Delta)$ $\angle ABD = \angle DAE$ BFD is parallel to AE as $\angle ABD\ and\ \angle DAE$ are equal, alternate angles in parallel lines.
	(ii)	Show that triangle ABC is similar to triangle EDA.
		$\angle BCA = \angle BDA\ (\measuredangle in\ same\ segment)$ $\angle BCA = \angle DAE\ (part\ (i))$ $\angle ABC + \angle CDA = 180\ (\measuredangle in\ opp\ segment)$ $\angle CDA + \angle EDA = 180\ (adj. \measuredangle on\ a\ str. line)$ $\angle ABC = \angle EDA$ $\Delta ABC\ is\ similar\ to\ \Delta EDA$ (angle-angle theorem).